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REFLECTIONS CONCERNING THE GENUINE ORIGIN OF GRAVITATION

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Abstract. The purpose of this study is to evaluate the possibility of releasing again Seeliger's theory of finite range gravitation, in view of avoiding the paradox of the cosmical pressure, even in the case of a vanishing curvature of the space. The specific features of this theory are cautiously analysed, pursuing the following objectives: 1) compliance between the finite range gravitation and the doctrine of general relativity, 2) reliable interpretations of the finite range in geometric and physical terms, 3) recovering the main results of cosmological interest, 4) proposal for a mechanism of gravitational finite range interaction, in terms of a virtual interchange of quanta, 5) proposal for a scalar gravitation theory covering both the domain of the Solar system and the metagalactic domain.

Introduction

The mechanical stability of matter is a task of fundamental research, implying a reappraisal of the basic ideas of Physics, and interesting both the Physical Science and the Philosophy of Nature. Roughly, we may divide the theme into two distinct objectives: 1) stability of the atoms and 2) stability of the Universe as a whole. The first aspect pertains rather to Electrodynamics. We dealt with this problem long time ago, when we investigated the stability of the Hydrogen Atom [1]. The second aspect pertains, generally speaking, to the Gravitation Theory (including the classical and the relativistic parts, as well as the great number of cosmological models and alternative theories) [2]. The stability of the entire Universe makes the theme of the present research. The problem, as it originally stood, is contained in a "philosophical astonishment", formulated in 1895 by the German astronomer Hugo Seeliger [3]: Why are we not crashed, under the infinite pressure, yielded by the infinite number of stars of the Universe, as predicted by the Newtonian Mechanics? Seeliger did not hesitate to orientate the research towards the revision of Newtonian Mechanics. Meanwhile, the advent of the Special Relativity Theory (1905) [4] and of the General Relativity Theory (1916) [5], placed the stability problem on quite different grounds, throwing into the shade the old line of reasoning, opened by Seeliger. The forthcoming stage in the history of the cosmic stability coincides, to a great extent, with the history of the new branch of Physics – the Relativistic Cosmology [6]. The inter-war period brought a valuable theoretical acquisition, namely, the Robertson & Walker metric of the Universe (1935) [7]. It was in vogue until nowadays. The basic concept, exploited in the framework of this metric, is the space curvature. The mechanical stability stands to reason for a positive curvature, but, after a few decades of permanent efforts, paid for evaluating the metagalactic curvature, the astrophysicists came to a disappointing conclusion – with a great reliability, it is vanishing [8]. Nor the ambitious theories of gravitation, due to Fred Hoyle & Jayant Narlikar (1963) [9] and to C. Brans & R. H. Dicke (1961) [10], and applicable to an open Universe, had a more fortunate destiny. There is no solution to the paradox of the cosmical pressure put forward during the whole XX -th century, which should be formulated exclusively relying on the physical properties of the ordinary (*i.e.* atomic - molecular) matter. Given the situation, we decided to take again the early hypothesis of Seeliger, about the exponential attenuation of the gravitational potential, in view of harmonizing it with the doctrine of Relativity, of finding reliable motivation for it, and of putting it as a genuine explanation of the matter stability. The kind of difficulties we came across in this entering upon, will appear by pursuing the demonstrations. At all events, we hope to have succeeded the framing of Seeliger's Gravitation, beside the other relativistic theories in the collection of viable theories.

I. Schwarzschild - type metric and its cosmological consequences

The static gravitational field of a spherically symmetric mass distribution inside a sphere of radius R, may be conveniently described by resorting to Einstein's field equations, and by chosing an inertial frame with the origin in the center of the sphere and a spherical system of coordinates (r,θ,φ). Outside the source of the field, *i.e* for $r > R$, the equations to be solved are [11]:

$$R_{\mu\nu} = 0, \quad r > R, \quad (\mu,\nu = 0,1,2,3) \tag{1}$$

The general solution, under the specified conditions, is [12]:

$$(dS)^2 = \left(1 - 2\frac{\mu}{f}\right)(cdt)^2 - \frac{f'^2 (dr)^2}{\left(1 - 2\frac{\mu}{f}\right)} - f^2 d\Omega \tag{2a}$$

where:

$$\begin{aligned} & f = f(r), \quad R < r < \infty; \quad \mu = \frac{GM_0}{c^2}, \quad \lim_{r\to 0} \frac{f(r)}{r} = 1, \\ & d\Omega = (d\theta)^2 + \sin^2\theta\,(d\varphi)^2 \end{aligned} \tag{2b}$$

The choice of Einstein's gravitational theory instead of any ether alternative theory, is not made at random, it is deliberately made to rigorously comply with the equivalence between inertial and gravitational forces, outside the source. As we can see, the equivalence of forces is not able to completely determine the metric. Although this ambiguity has no affect upon the evaluation of the relativistic tests [12], its avoidance is of a real interest for our purpose - the building up of a self-consistent finite range theory of gravitation.

To determine the function $f(r)$ in (2;a,b), we need to take into account the other facet of the equivalence principle, namely, the equivalence between inertial mass and gravitational mass. But this may be achieved only by enlarging the doctrinal basis of General Relativity, in view of including the gravitational energy, among the other species of energy, into a global energetic balance. Thus, the completion we have to perform to General Relativity is somewhat similar to Rosen's bimetrism [13].

Let us consider a gravitational system made up of two point-like bodies, with rest masses (m_{01}, m_{02}), located at a relative distance r from one another. Deriving advantage from the Relativistic Analytical Mechanics, which predicts equal sharing of the potential energy between the two partners of the aggregate (irrespective of their rest masses), we write [14]:

$$\begin{aligned} & E = Mc^2, \quad M = M_1 + M_2, \\ & M_1 = m_{01} + \frac{1}{2}\frac{U(r)}{c^2}, \quad M_2 = m_{02} + \frac{1}{2}\frac{U(r)}{c^2}, \\ & U(r) = -G \cdot M_1 \cdot M_2 \cdot F(r) \end{aligned} \tag{3a}$$

The explicit expression of the (static) energy turns out to be:

$$E=\left(m_{01}+m_{02}\right)\cdot c^2-\frac{2Gm_{01}\cdot m_{02}F(r)}{1+J+\frac{1}{2}\frac{G}{c^2}\left(m_{01}+m_{02}\right)F(r)}, \tag{3b}$$

$$J\equiv\left[1+\frac{G}{c^2}\left(m_{01}+m_{02}\right)F(r)+\frac{1}{4}\frac{G^2}{c^4}\left(m_{01}-m_{02}\right)^2F^2(r)\right]^{\frac{1}{2}} \tag{3c}$$

Now, we go over from the two body case to the one body case, by writing:

$$m_{01}=m_0,\quad m_{02}=M_0,\quad m_0<<M_0,\quad J\rightarrow 1+\frac{1}{2}\mu\cdot F(r)$$

$$E\rightarrow\left(M_0+m_0\right)\cdot c^2-\frac{Gm_0M_0F(r)}{1+\frac{1}{2}\mu\cdot F(r)}, \tag{4a}$$

whence:

$$\gamma_{00}^{\frac{1}{2}}=\frac{E-M_0c^2}{m_0c^2}=\frac{1-\frac{1}{2}\mu\cdot F(r)}{1+\frac{1}{2}\mu\cdot F(r)}=\left(1-2\frac{\mu}{f}\right)^{\frac{1}{2}} \tag{4b}$$

Now, we may express the undetermined function $f(r)$ of the Schwarzschild - type metric through the function $F(r)$ of the potential energy:

$$f(r)=\frac{\left[1+\frac{1}{2}\mu\cdot F(r)\right]^2}{F(r)} \tag{5a}$$

The Schwarzschild - type metric, written in terms of the function $F(r)$ becomes:

$$(dS)^2=\left(\frac{1-\frac{1}{2}\mu\cdot F}{1+\frac{1}{2}\mu\cdot F}\right)^2(cdt)^2-\left(1+\frac{1}{2}\mu\cdot F\right)^4\left\{\left(1/F\right)'^2(dr)^2+\left(1/F\right)^2d\Omega\right\} \tag{5c}$$

Further on, since F^{-1} has the physical dimensions of a length, it is convenient to define an interaction length as:

$$r_{\text{inter}}=\frac{1}{F(r)} \tag{6a}$$

The metric acquires its final form:

$$(dS)^2 = \left(\frac{1 - \frac{1}{2}\frac{\mu}{r_{\text{inter}}}}{1 + \frac{1}{2}\frac{\mu}{r_{\text{inter}}}} \right)^2 (cdt)^2 - \left(1 + \frac{1}{2}\frac{\mu}{r_{\text{inter}}} \right)^4 \left\{ (dr_{\text{inter}})^2 + r_{\text{inter}}^2 d\Omega \right\} \tag{6b}$$

Putting the mass of the source to vanish $(\mu \to 0)$, we come across a universal chronotopic metric:

$$\left(dS_{U_2}\right)^2 = (cdt)^2 - \left\{ (dr_{\text{inter}})^2 + r_{\text{inter}}^2 d\Omega \right\} \tag{7}$$

which may be identified with the Minkowski metric, provided that $r_{\text{inter}} = r$, *i.e.* the gravitation is an infinite range gravitation. But, in view of avoiding the paradox of the cosmical pressure, we need $r_{\text{inter}} > r$. Therefore, the metric (7) is not Minkowskian.

To determine the function $r_{\text{inter}} = l(r)$ we resort to Seeliger's gravitation theory. Accordingly, we write:

$$U(r) = -G\frac{M_1 \cdot M_2}{r_{\text{inter}}} = -G\frac{M_1 \cdot M_2}{r} \cdot e^{-Kr}, \tag{8a}$$

whence:

$$r_{\text{inter}} = r \cdot \exp(Kr) \tag{8b}$$

The universal metric $\left(dS_{U_2}\right)^2$ is now completely determined:

$$\left(dS_{U_2}\right)^2 = (cdt)^2 - e^{2Kr}\left\{ (1+Kr)^2 (dr)^2 + r^2 d\Omega \right\} \tag{9a}$$

Out of this metric, we regain the Minkowski metric as a limiting case:

$$\begin{aligned} &\lim_{K\to 0}\left(dS_{U_2}\right)^2 = \left(dS_{U_1}\right)^2 \\ &\left(dS_{U_1}\right)^2 = (cdt)^2 - \left\{ (dr)^2 + r^2 d\Omega \right\} \end{aligned} \tag{9b}$$

The two metrics, $\left(dS_{U_1}\right)^2$ and $\left(dS_{U_2}\right)^2$ make up a "universal bimetrism". The metric $\left(dS_{U_1}\right)^2$ is used to define the coordinate system as well as the two scales – of length and of time

$$\begin{aligned} \left(dS_{U_1}\right)^2 &= (cdt)^2 - \left\{ (dx)^2 + (dy)^2 + (dz)^2 \right\} \\ &= (cdt)^2 - \left\{ \frac{\left(\vec{r} \cdot d\vec{r}\right)^2}{r^2} + \frac{\left(\vec{r} \times d\vec{r}\right)^2}{r^2} \right\} \\ &= (cdt)^2 - \left\{ (dr)^2 + r^2\left[(d\theta)^2 + \sin^2\theta (d\varphi)^2 \right] \right\} \end{aligned} \tag{10a}$$

where:

$$\vec{r} = x\cdot\vec{i} + y\cdot\vec{j} + z\cdot\vec{k},\quad r = \left(x^2+y^2+z^2\right)^{\frac{1}{2}},$$
$$\theta = \arccos\frac{z}{r},\quad \varphi = \arccos\frac{x}{\sqrt{x^2+y^2}} \tag{10b}$$

Unlike $(dS_{U_1})^2$, which stands for a Universe empty of atomo-molecular matter, as well as of any other form of subtle matter, able to yield inertial forces, $(dS_{U_2})^2$ is the metric of a Universe filled with a continuous fluid characterized by Seeliger's constant K:

$$\begin{aligned}(dS_{U_2})^2 &= (cdt)^2 - e^{2Kr}\left\{(1+Kr)^2\frac{(\vec{r}\cdot d\vec{r})^2}{r^2} + \frac{(\vec{r}\times d\vec{r})^2}{r^2}\right\}\\ &= (cdt)^2 - e^{2Kr}\left\{(1+Kr)^2(dr)^2 + r^2 d\Omega\right\}\end{aligned} \tag{11a}$$

This hypothetical fluid (that we call conventionally "Ether") is responsible both for the exponential attenuation of the gravitational potential, and for the non-Minkowskian character of the metric $(dS_{U_2})^2$. As far as the forces, yielded by the respective fluid, are geometrized through the agency of the metric $(dS_{U_2})^2$, we expect a universal coupling (of geometric nature), between ether and ordinary matter, *via* the Covariance principle of the Mathematical Physics Equations with respect to this metric, provided that the ratio between gravitational and electrical potentials is not influenced by the ether. In other words, we expect the same value for the two attenuation constants – that of Seeliger and that of Proca [15]. For a system made up of an electron and a proton we obtain:

$$\frac{U_g}{U_e} = 4.4063\times10^{-40} \to K_S = K_P = K \tag{12}$$

Now, we have to answer the (difficult) question: what is the common value of the two constants (K_S, K_P)? The answer may be given resorting to the quantity r_{inter}, whose expression may be written as:

$$r_{\text{inter}} = r + \frac{cK}{H}(Hr)\cdot\left(\frac{r}{c}\right) + O\left(\frac{1}{2}K^2r^3\right) \tag{13a}$$

But this result may be interpreted in terms of the famous Hubble phenomenon [16]. Let us, for this purpose, consider two point-like bodies A and B located at the points $(0,0,0)$ and (r,θ_0,φ_0), respectively. The inter-body distance is:

$$r_{\text{inter}} = \int_{r=0}^{r=r}\sqrt{-(dS_{U_2})^2_{\delta t=0}} = \int_0^r e^{Kr}\,(1+Kr)dr = r\cdot\exp(Kr) \tag{13b}$$

Ignoring the very cause of the inequality $r_{\text{inter}} > r$ (which, in our opinion, is the cosmical ether), Edwin Hubble put forward the following explanation: during the quantum travel from A to B, the body B moved away relatively to A with a velocity $\mathrm{v}_H(r) = Hr$. The two explanations, that based on ether, and that based on matter expansion, do coincide (in their predictions concerning the length r_{inter}), provided that:

$$K = \frac{H}{c} \tag{14a}$$

To strengthen this conclusion, we calculated r_{inter} in Hubble's hypothesis, but resorting to Special Relativity for composing the motions, rather than to Classical Mechanics. The result is accurately the expected one: $r_{\text{inter}} = r \cdot \exp\left(\frac{H}{c} r\right)$ - in full compliance with the already established relation among the three constants of Nature (K, H, c). (see Addendum). The available astrophysical data agree (within known uncertitude limits) with the new relationship [16] $\left(H = cK = \sqrt{\frac{\pi}{3} G \rho_M}, \qquad \rho_M = 7.797 \times 10^{-29}\, g/cm^3 \right)$

$$K = 7.785 \times 10^{-29}\, cm^{-1}, \quad H = 2.334 \times 10^{-18}\, s^{-1} = 72.02 \frac{Km}{s \cdot Mp}, \quad c = 2.998 \times 10^{10}\, \frac{cm}{s}. \tag{14b}$$

The time spent by a quantum to cover the distance $\overline{AB} = r_{\text{inter}}$ may be calculated from the condition $\left(dS_{U_2}\right)^2 = 0$, whence $cdt = dr_{\text{inter}}$, and we obtain:

$$\tau_{\text{inter}} = \int_0^{\tau_{\text{inter}}} dt = \frac{1}{c} \int_0^r \frac{dr_{\text{inter}}}{dr} dr = \frac{1}{c} r_{\text{inter}} \tag{15}$$

The quantum travel occurs similarly to the case when the two point-like bodies are separated by a relative distance $\overline{AB} = r_{\text{inter}}$ and the virtual quanta, carrying out the negative energy from a body to another, travel at the constant speed c (the light velocity in empty space). This formal equivalence is achieved by the coordinate change $\vec{r} \exp(Kr) = \vec{\rho}$, which transforms the metric $\left(dS_{U_2}\right)^2$ into a Minkowsky-type one:

$$\left(dS_{U_2}\right)^2 \to \left(dS_{U_1}'\right)^2 = (cdt)^2 - (d\vec{\rho})^2, \quad \vec{\rho} = \vec{r} \exp(Kr) \tag{16}$$

But, in the Universe with a metric $\left(dS_{U_1}'\right)^2$, Synge's theory, about the gravitational potential [17], may be transposed without change, and we can write at once $\rho U(\rho) = const$, whence $U \propto \rho^{-1}$, *i.e.* the Seeliger's result $U(r) \propto \frac{1}{r} e^{-Kr}$.

The time spent by a virtual quantum to cover the distance AB is $> \frac{r}{c}$, because it is coupled to the universal ether and, accordingly, during its travel, it undergoes the "hindering" influence of this medium. The coupling of a virtual quantum to ether may be achieved by asking the propagation of a scalar wave in the Universe whose metric is $\left(dS_{U_2}\right)^2$, and by assuming the same features of the travel, irrespective of the sign (positive or negative) of the carried out energy. From the standpoint of the Minkowskian Universe, the ether behaves just like a dielectric medium, compelling the quantum to propagate through space at a speed $\mathrm{v}_L(r) < c$. Formally, we may speak of a refractive index, yielded by the cosmical ether:

$n(r) = c / \mathrm{v}_L(r) > 1$.

To obtain the universal metric $\left(dS_{U_2}\right)^2$, we resorted to the mass equivalence principle and to a certain procedure necessary to put in action the principle. But the respective procedure contains reasoning elements, taken over from Special Relativity, whose transposing into the General Relativity Theory may be doubtful. To avoid such kind of uncertainties, we apply the same procedure to a static gravitational system made up of N point-like bodies and come to the following system of recursive equations:

$$m_k^{(s+1)} = m_k^{(0)} - \frac{1}{2}\frac{1}{c^2}\sum_j \Phi_{jk}^{(s)}; \qquad j \neq k$$

$$\Phi_{jk}^{(s)} = G \cdot m_j^{(s)} \cdot m_k^{(s)} \cdot F\left(r_{jk}\right); \qquad F\left(r_{jk}\right) = G\frac{e^{-Kr_{jk}}}{r_{jk}},$$

$$E^{(s)} = c^2 \sum_k m_k^{(s)}, \quad s = 0, 1, 2, \ldots \infty \tag{17}$$

After two successive iterations, we obtain the result:

$$E \approx c^2 \sum_k m_k^{(0)} - \frac{1}{2}\sum_{j\neq k}\left\{\Phi_{jk}^{(0)} - \frac{1}{2}\frac{1}{c^2}\left[\frac{1}{m_j^{(0)}} + \frac{1}{m_k^{(0)}}\right]\Phi_{jk}^{(0)^2}\right\} + \frac{1}{4}\frac{1}{c^2}\sum_{\Delta}^{jkl} \Phi_{jk}^{(0)}\left\{\frac{1}{m_j^{(0)}}\Phi_{jl}^{(0)} + \frac{1}{m_k^{(0)}}\Phi_{kl}^{(0)}\right\} \tag{18}$$

The main characteristic of this result is the infringement of the principle of superposition for gravitational interactions. This feature becomes self-evident for a three-body system:

$$E \approx c^2\left(m_A^{(0)} + m_B^{(0)} + m_C^{(0)}\right) - G\left(m_A^{(0)}m_B^{(0)}F_{AB} + m_A^{(0)}m_C^{(0)}F_{AC} + m_B^{(0)}m_C^{(0)}F_{BC}\right) + \frac{1}{2}\frac{1}{c^2}G^2\left\{\left(m_A^{(0)} + m_B^{(0)}\right)m_A^{(0)}m_B^{(0)}F_{AB}^2 + \right.$$
$$\left. + \left(m_A^{(0)} + m_C^{(0)}\right)m_A^{(0)}m_C^{(0)}F_{AC}^2 + \left(m_B^{(0)} + m_C^{(0)}\right)m_B^{(0)}m_C^{(0)}F_{BC}^2\right\} + \frac{1}{c^2}m_A^{(0)}m_B^{(0)}m_C^{(0)}G^2\left\{F_{AB}F_{AC} + F_{AB}F_{BC} + F_{AC}F_{BC}\right\} \tag{19}$$

For $K \to 0$, the results (18) and (19) go into the results derived by V. Fock, based on the General Relativity Theory of Albert Einstein [18]. Thus, the procedure used by us, in view of deriving the universal metric $\left(dS_{U_2}\right)^2$, as a necessary stage towards a theory of finite range gravitation, proves to be in compliance with the General Relativity doctrine [19]. The noticeable conclusion is that Seeliger's gravitation theory may be adequately modified, so that it should harmonize with the Einsteinian construction, and should deliver a solution to the paradox of cosmical pressure, in the case of the vanishing curvature of the Universe.

II. The universal bimetrism and the interaction length r_{inter}

So far, starting our research with the task of completely determining the Schwartzschild-like metric, and endeavouring to solve the problem in the framework of the mass equivalence principle, by adopting a Cartesian system of coordinates, an inertia frame and a Seeliger-type potential, we came to some valuable conclusions of cosmological interest, namely:

1) There is a certain subtle matter, that we call conventionally ether, filling the whole cosmic space, and acting upon all the kinds of ordinary matter, through the intermediary of a geometric coupling, entailing the replacement of the infinite range interactions by long finite range interactions. Of course, this new ether, which is in compliance with the relativistic doctrine, has nothing to do with the luminiferous ether of the XIX -th century, put forward by Augustin Fresnel.
2) The distance, between two point-like bodies A and B, located at the positions (0, 0, 0) and $\left(r, \theta_0, \varphi_0\right)$, is not r but $r_{\text{inter}} = r \cdot \exp(Kr) > r$. The lengthening of distance from r to

r_{inter} is assigned to the hypothetic ether, and is derived from a chronotopic metric, whose departure from the Minkowski metric is put equally in the ether's charge.

3) A universal bimetrism is set up. It implies simultaneous covariant formulation of the Mathematical Physics Equations both with respect to the Minkowski metric $\left(dS_{U_1}\right)^2$, and with respect to the metric $\left(dS_{U_2}\right)^2$, distorted by the cosmic ether. It is a noteworthy occasion to remember now an old opinion of A. S. Eddington, according to which, if a certain kind of ether should exist, it will necessary be reduced to geometry.

The universal bimetrism is not necessarily a relativistic effect. Accordingly, its area of action extends beyond the constraint of Special Relativity, in the purely classical domain. To achieve this extension, we have to break the metrical linkage between position space and time, and to replace the metrical universes U_1 and U_2 by affine universes, defined as Cartesian products between time and the same position three-dimensional spaces:

$$\left(dS_{U_1}\right)^2 \to T \times P_3\left(U_1\right) \tag{1a}$$

$$\left(dS_{U_2}\right)^2 \to T \times P_3\left(U_2\right) \tag{1b}$$

In this way, the Newtonian Mechanics must be modified, in a similar manner to that adopted by H. Seeliger, in view of applying it to over-galactic regions of the position space. The Newtonian Mechanics, as it stands, remains to be used for infrarelativistic velocities $\left(|\mathrm{v}| << c\right)$ and, at the same time, for infragalactic space regions $\left(r << \dfrac{1}{K}\right)$.

The quantity $\tau = \dfrac{1}{c} r_{\text{inter}}$, called "interaction time", plays an essential role in the emission-absorption theory of Gravitation. It is proportional to the potential yielded by a point-like body. The following two equivalent dynamic equations are fulfilled by τ:

$$\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\frac{1}{\tau}\right) - K^2\left(\frac{1}{\tau}\right)\right\} = -4\cdot\pi\cdot c\cdot\delta(\vec{r}) \tag{2a}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{e^{Kr}}{1+Kr}\cdot r^2\cdot\frac{\partial}{\partial r}\frac{1}{\tau}\right) = -4\cdot\pi\cdot c\cdot\delta(\vec{r}) \tag{2b}$$

while equation (2,a) suggests rather an interpretation in terms of the meson-theory, entailing us to write:

$$K = \frac{m_0 c}{\hbar}, \qquad m_0 \text{ - rest mass of the exchanged particle.} \tag{3a}$$

equation (2,b) specifies the geometric nature of the two partners of the interaction. But, as we have already pointed out, the physical reason, responsible for the finiteness of the interaction range, is the cosmic ether (whose presence is manifested as a distorted geometry). So, we are entitled to denominate the exchanged particle as "etheron". The mass of the etheron is obtained by combining (II, 3a) with (I, 14a). It turns out to be:

$$m_0 = \frac{\hbar H}{c^2}, \quad \to \quad m_0 c^2 = \hbar H, \quad \varepsilon = \pm\hbar H \tag{3b}$$

We point out that the term "etheron", and the previous formula of mass, were, for the first time, proposed by the Romanian physicist Ioan Iovitzu - Popescu, long time ago, when the compliance between the ether concept and the General Relativity Theory was by no means evident. By this remarkable intuition, Iovitzu - Popescu turns out to be a forerunner of the kind of gravitational theory we enter upon in this scientific work [20].

Taking the energy $\varepsilon = m_0 c^2 = \hbar H$ as the exchange energy of a single etheron, we conclude, just as in the meson theory, that the virtual exchange is not a causal process. This aspect, although strange, is however acceptable, in terms of the uncertainty principle of W. Heisenberg $\Delta E \cdot \Delta t \sim \hbar$; $\Delta p \cdot \Delta x \sim \hbar$; $\Delta E = \varepsilon$, $\Delta t = \frac{1}{H}$, $\Delta p = \frac{\varepsilon}{c}$, $\Delta x = \frac{c}{H}$, $\frac{\Delta x}{\Delta t} \sim c$. A more convenient picture, of the virtual change of energy, may be achieved resorting to the universe U_2, and writing the equation for the propagation of a spherical outgoing scalar wave:

$$e^{-Kr}(1+Kr)\frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{e^{Kr}}{1+Kr}\cdot r^2 \cdot \frac{\partial \Psi}{\partial r}\right) - \frac{1}{\mathrm{v}_L^2(r)}\frac{\partial^2 \Psi}{\partial t^2} = 0, \tag{4a}$$

$$\mathrm{v}_L(r) = c \cdot \frac{e^{-Kr}}{1+Kr} \tag{4b}$$

The exact solution to the equation (4,a) (with the specification 4,b) is:

$$\Psi(r) = \frac{A}{r} e^{-Kr} \cos\Phi \tag{5a}$$

$$\Phi = 2\pi\left(\nu_0 t - \frac{r}{\lambda_0} e^{Kr}\right) \tag{5b}$$

where A, ν_0, λ_0 are specific constants of the wave, and the constants (ν_0, λ_0) are connected to one another through the relationship:

$$\nu_0 \cdot \lambda_0 = c, \qquad (\omega \equiv 2\pi\nu_0), \tag{5c}$$

c being the light velocity in empty space. By close analogy with real quanta, the virtual quanta must exhibit the dual wave-particle aspect. Resorting to the wave-like picture, the negative energy is carried out, from point A to point B, in the Euclidean space, over a distance r, by the spherical wave, which propagates through space according to equation (4,a), with a velocity (4,b). The energy ε and the momentum p_r, carried out by the wave, are:

$$-\varepsilon = \hbar \cdot \frac{\partial \Phi}{\partial t} = \hbar\omega, \qquad -p_r = -\hbar \cdot \frac{\partial \Phi}{\partial r} = \frac{\hbar\omega}{\mathrm{v}_L(r)}, \tag{6a}$$

$$\left(\mathrm{v}_L(r) \equiv \dot{r} = \frac{c \cdot e^{-Kr}}{1+Kr}\right), \tag{6b}$$

whence:

$$\dot{r} \cdot p_r = \mathrm{H}, \qquad \mathrm{H} = \varepsilon, \qquad \varepsilon = -\hbar\omega \tag{7}$$

Assuming now:

$$\omega = H, \qquad \Phi = H\left(t - \frac{1}{c} r \cdot \exp Kr\right) \tag{8}$$

the energy, carried out by the wave, in the time $\tau = \frac{1}{c} r_{\text{int}}$, is just:

$$\varepsilon = -\hbar H \,, \tag{9}$$

as expected. (The sign minus is necessary to obtain attractive gravitation).

Alternatively, resorting to the particle-like picture, the constant negative energy is carried out through the intermediary of a vanishing rest mass particle, moving onto the null geodetic lines of the metric $\left(dS_{U_2}\right)^2$, whence:

$$\left(dS_{U_2}\right)^2 = 0, \quad \to \Lambda = m_o c^2 \left(1 - n^2 \frac{\dot{r}^2}{c^2}\right)^{\frac{1}{2}}, \qquad n = (1 + Kr) \cdot e^{Kr} \tag{10a}$$

$$\varepsilon = \lim_{\substack{m_o \to 0 \\ \dot{r} \to \frac{c}{n}}} \frac{-m_0 c^2}{\left(1 - n^2 \frac{\dot{r}^2}{c^2}\right)^{\frac{1}{2}}} = -\hbar H \tag{10b}$$

Comparing to one another the equations (3,b) and (10,b) we ascertain a striking difference: while $\hbar H$ in (3,b) is the rest energy of a particle, the same quantity in (10,b) is the motion energy of a quantum. This strange situation is a consequence of the double dynamic picture of the quantity $\frac{1}{\tau}$ in (2,a) and (2,b) – specific only to (finite) long range interactions. To avoid the obvious contradiction, we assume the quantum motion to be accompanied by an inner process, in which kinetic energy is gradually transformed into rest mass energy, to the extent of the removing away of the quantum from its emitting source. At the instant of the emission, $\mathrm{v}_L(r) = \mathrm{v}_L(0) = c$, while after an infinite time thereafter, $\mathrm{v}_L(\infty) = 0$. Accordingly, we may write:

$$\varepsilon = -\hbar H = \frac{-m_0(r)c^2}{\left(1 - \frac{V^2(r)}{c^2}\right)^{\frac{1}{2}}}, \qquad V(r) = c \cdot \frac{e^{-Kr}}{1 + Kr} \tag{11a}$$

whence:

$$m_0(r) = \frac{\hbar H}{c^2}\left(1 - \frac{V^2(r)}{c^2}\right)^{\frac{1}{2}}, \qquad \to \tag{11b}$$

$$m_0(0) = 0, \qquad m_0(\infty) = \frac{\hbar H}{c^2} \tag{12}$$

So, the etheron is a quantum at the emission instant and becomes a particle of negative rest mass at a distance $\sim \frac{1}{K}$ far from the source.
This peculiar process may be equally assigned to the cosmical ether.

Denoting by (e_A, e_B) the emissions and by (a_A, a_B) the absorptions of the two point-like bodies, which reciprocally exchange negative energy quanta $(\varepsilon = -\hbar H)$, we can write down the equations of the energy conservation as:

$$e_A(t) = a_B(t+\tau), \tag{13a}$$

$$a_A(t) = e_B(t-\tau), \tag{13b}$$

where:

$$\tau = \int_0^r \frac{dr}{\mathrm{v}_L(r)}, \qquad \mathrm{v}_L(r) = c \cdot \frac{e^{-Kr}}{1+Kr}, \tag{14a}$$

$$\tau = \frac{1}{c} r_{\text{inter}}, \qquad r_{\text{inter}} = r \cdot \exp Kr \tag{14b}$$

$$\varepsilon = -\hbar H \tag{15}$$

Now, we are prepared to build up a sub-Mechanics, intended to explain the genuine mechanism of the universal attraction. At first, we devise a static model and, thereafter, we go over from statics to dynamics, by asking the covariance of the field equations and of the motion equations, with respect to those coordinate transformations leaving unchanged the metric $(dU_2)^2$. Among these transformations, we point out a set of transformations which are a generalization of the known Lorentz formulas

$$e^{Kr'} \cdot \vec{r}\,' = e^{Kr}\left\{\vec{r} + \frac{\gamma^2}{\gamma+1}\frac{(\vec{V}\cdot\vec{r})\cdot\vec{V}}{c^2}\right\} + \gamma\cdot\vec{V}\cdot t$$

$$t' = \gamma\cdot\left\{t + \frac{1}{c^2}e^{Kr}\cdot\vec{r}\cdot\vec{V}\right\}, \qquad \gamma = \left(1 - \frac{\vec{V}^2}{c^2}\right)^{-1/2} \tag{16a}$$

By applying these formulas, we come to the invant quantity $J = t^2 - r^2 \cdot \exp(2Kr)$. For a photon (or other quanta), J does vanish for $t = \tau$, where

$$\tau = \pm\frac{1}{c}\cdot r_{\text{inter}} \tag{16b}$$

III. The Classical Seeliger's model of Gravitation

To illustrate how concepts as covariance, equivalence, bimetrism and equilibrium do apply to the matter of the Universe at a metagalactic scale, we consider an ideal fluid with inner gravitation and Euler-type hydrodynamic forces, defined in the affine Universe $T\times P_3$ and submitted to a variational principle of action, *via* a Lagrange function L. At first the metric of the position space is left unspecified. This actually means two different actions:

1) using arbitrary position coordinates and resorting to Analytic Geometry in P_3. Thus, we can write:

$$(ds)^2 = a_{jk} dx^j dx^k, \quad (j,k = 1,2,3)$$

$$b^{jl} \cdot a_{lk} = \delta_k^j, \quad Det \left\| a_{jk} \right\| = a, \tag{1}$$

$$G_{jk}^l = \frac{1}{2} b^{ls} (a_{js,k} + a_{ks,j} - a_{jk,s})$$

2) the fact whether the metric $(ds)^2$ belongs to the Universe U_1 or to the Universe U_2 will be decided in the last stage of the theoretical analysis, when an inertia frame is considered and Cartesian coordinates are adopted. The action integral is defined as

$$\mathrm{A} = \int L \sqrt{a} \left(d^3 x\right) dt,$$

$$L = \frac{1}{2} \cdot \rho \cdot a_{jk} \cdot \mathrm{v}^j \cdot \mathrm{v}^k - (\rho \cdot H - p) - \frac{m_0^{(g)}}{m_0^{(i)}} \cdot \rho \cdot \Phi \tag{2}$$

$$-\frac{1}{8\pi G} \left(b^{kj} \Phi_{,k} \Phi_{,j} + K_1^2 \cdot \Phi^2 \right), \qquad H = \int_o^{p(\rho)} \frac{dp}{\rho(p)}$$

where the denotations are the usual ones, *i.e.*:

ρ - invariant mass density,
p - invariant pressure,
Φ - gravitational potential,
H – Helmholtz (hydrodynamic) potential,
G - Newton's constant,
K_1- undetermined constant (with physical dimensions $length^{-1}$). The field equations are obtained by asking the vanishing of the action variation against the gravitational potential:

$$\delta_\Phi \mathrm{A} = 0,$$

$$\frac{1}{\sqrt{a}} \cdot \frac{\partial}{\partial x^k} \left(\sqrt{a} \cdot b^{jk} \cdot \frac{\partial \Phi}{\partial x^j} \right) - K_1^2 \cdot \Phi = 4\pi \cdot G \cdot \frac{m_o^{(g)}}{m_o^{(i)}} \cdot \rho \tag{3}$$

The ratio $m_o^{(g)} / m_o^{(i)}$, between gravitational and inertial mass of the source, is introduced in view of having a proportionality between Φ and $m_o^{(g)}$, knowing that ρ is the density of the inertial mass.

To obtain the motion equations, there are three but equivalent methods. The first method is based on a Lagrange function of motion 2 derived out of the Lagrange function L of the model:

$$\Lambda = m_{op}^{(i)} \cdot \frac{\partial L}{\partial \rho} = \frac{1}{2} \cdot m_{op}^{(i)} \cdot a_{jk} \cdot \mathrm{v}^j \cdot \mathrm{v}^k - m_{op}^{(i)} \left(H + \frac{m_o^{(g)}}{m_o^{(i)}} \cdot \Phi \right)$$

$$\delta\int\Lambda\, dt = 0, \rightarrow \frac{d}{dt}\left(\frac{\partial\Lambda}{\partial \mathrm{v}^k}\right) - \frac{\partial\Lambda}{\partial x^k} = 0 \tag{4}$$

$$\frac{d^2x^s}{dt^2} + G^s_{jk}\cdot\frac{dx^j}{dt}\cdot\frac{dx^k}{dt} = -b^{ls}\cdot\left(\frac{m_o^{(g)}}{m_0^{(i)}}\frac{\partial\Phi}{\partial x^l} + \frac{1}{\rho}\frac{\partial p}{\partial x^l}\right)$$

The second method relies on performing variations, against the metrical functions b^{jk} in the position space P_3, of the hydrodynamic elements of the model.

$$\delta_g\sqrt{a} = -\frac{1}{2}\sqrt{a}\cdot a_{jk}\cdot\delta b^{jk}, \qquad \delta_g \mathrm{v}^k = 0,$$

$$\delta_g\rho = +\frac{1}{2}\rho\, a_{jk}\cdot\delta b^{jk}, \qquad \delta_g\left(\rho\sqrt{a}\right) = 0, \tag{5}$$

$$\delta_g\left(a_{jk}\cdot \mathrm{v}^j\cdot \mathrm{v}^k\right) = -\mathrm{v}_j\cdot \mathrm{v}_k\cdot\delta b^{jk}$$

Thereafter, a canonical tensor of energy is defined as:

$$\tau_{jk} = \frac{-2}{\sqrt{a}}\cdot\frac{\delta\left(\sqrt{a}L\right)}{\delta b^{jk}} = \rho\cdot \mathrm{v}_j\cdot \mathrm{v}_k + p\cdot a_{jk} + \frac{1}{4\pi\cdot G}\left(\Phi_{,j}\Phi_{,k} - \frac{1}{2}a_{jk}b^{ls}\Phi_{,l}\Phi_{,s} + \frac{1}{2}K_1^2\Phi^2 a_{jk}\right); \tag{6}$$

Writing this tensor in its contravariant aspect, and then performing the covariant divergence we come to the result

$$\nabla_k\tau^{jk} + \frac{\partial}{\partial t}\left(\rho\cdot \mathrm{v}^j\right) = \rho\cdot\left\{\left(\frac{d^2x^j}{dt^2} + G^j_{kl}\frac{dx^k}{dt}\cdot\frac{dx^l}{dt}\right) + b^{jl}\cdot\left(\frac{m_0^{(g)}}{m_0^{(i)}}\cdot\frac{\partial\Phi}{\partial x^l} + \frac{1}{\rho}\cdot\frac{\partial p}{\partial x^l}\right)\right\} = 0\,; \tag{7}$$

The entire meaning of the tensor τ^{jk} is revealed by devising formally a four-dimensional tensor $T^{\alpha\beta}$ whose components are:

$$T^{00} = C^2\rho, \qquad T^{0j} = T^{j0} = C\cdot\rho\cdot \mathrm{v}^j, \qquad T^{jk} = \tau^{jk}, \tag{8}$$

and by introducing the time as an additional zero-th coordinate $x^0 = Ct$. Now, both motion equations and the mass conservation equation may be cast in a compact form as a vanishing divergence condition for the tensor $T^{\alpha\beta}$

$$T^{jk}_{\ \ |k} + T^{jo}_{,0} = 0 \qquad \text{(motion equations)} \tag{9a}$$

$$T^{0k}_{\ \ |k} + T^{00}_{,0} = C\cdot\left\{\frac{\partial\rho}{\partial t} + \frac{1}{\sqrt{a}}\cdot\frac{\partial}{\partial x^j}\left(\sqrt{a}\cdot\rho\cdot \mathrm{v}^j\right)\right\} = 0 \qquad \text{(continuity equation)} \tag{9b}$$

The conservative character of the tensor $T^{\alpha\beta}$ strongly suggests to go over from the affine Universe $T\times P_3$ to a metrical one U_1 or U_2. Here C is a constant, with physical dimensions of a velocity, playing the essential role in establishing the geometric linkage between the position space and time.

The third method requires to perform variations of the density and of the fluid velocity, induced by variations in the position coordinates and in the time of a fluid particle:

$$\delta_x \rho = -\nabla_k \cdot \left(\rho \cdot \delta q^k\right)$$

$$\delta_x \mathrm{v}^k = \frac{\partial}{\partial t} \cdot \delta q^k + \left(\mathrm{v}^j \cdot \nabla_j \delta q^k - \delta q^j \cdot \nabla_j \mathrm{v}^k\right) \tag{10}$$

$$\delta q^k \equiv \delta x^k - \mathrm{v}^k \delta t$$

Here δx^k is a vector and δt a scalar in P_3 space,

$$\delta x^k = \varepsilon f^k\left(x^l, t\right), \quad \delta t = \varepsilon f^0\left(x^l, t\right) \tag{11a}$$

submitted to suitable constraints at the bundaries of position space and of time:

$$f^k(\pm\infty, t) = f^k\left(x^l, \pm\infty\right) = 0 \ ,$$

$$f^0(\pm\infty, t) = f^0\left(x^l, \pm\infty\right) = 0 \ ; \tag{11b}$$

ε is a dimensionless arbitrarily small parameter, and $\left(f^k, f_0\right)$ are unspecified functions of their arguments. The variation induced in the Lagrange function of the model turns out to be:

$$\delta_x L = \delta_x \left[\frac{1}{2}\rho \cdot a_{jk} \cdot \mathrm{v}^j \cdot \mathrm{v}^k + p - \rho \cdot \left(H + \frac{m_o^{(g)}}{m_0^{(i)}} \cdot \Phi\right)\right] =$$

$$= -\rho \cdot \left(\delta q^k\right) \cdot \left\{ a_{jk} \cdot \left(\frac{d^2 x^j}{dt^2} + G^j_{ls} \cdot \frac{dx^l}{dt} \cdot \frac{dx^s}{dt}\right) + \left(\frac{m_0^{(g)}}{m_0^{(i)}} \cdot \frac{\partial \Phi}{\partial x^k} + \frac{1}{\rho} \cdot \frac{\partial p}{\partial x^k}\right)\right\} + \tag{12a}$$

$$+ \nabla_k \cdot \left\{\rho \cdot \mathrm{v}_j \cdot \left(\mathrm{v}^k \cdot \delta q^j - \frac{1}{2}\mathrm{v}^j \cdot \delta q^k\right) + \rho \cdot \delta q^k \cdot \left(\frac{m_0^{(g)}}{m_0^{(i)}} \cdot \Phi + H\right)\right\} + \frac{\partial}{\partial t}\left(\rho \cdot \mathrm{v}_k \cdot \delta q^k\right)$$

Taking into account the boundary conditions and asking the vanishing of the variation $\delta_x \mathrm{A}$, *i.d.* est:

$$\int \left(\delta_x L\right) \cdot \sqrt{a} \cdot \left(d^3 x\right) dt = 0 , \tag{12b}$$

we obtain, in this manner too, the motion equations (4).

Further on, the variational calculation being already carried out, to maintain the generality in defining the position coordinates is of no utility. On the contrary the physical situations rather impose to specify the metrical structure of P_3 ,when an inertia frame and Cartesian coordinates are chosen. There are only two possible options of physical interest:1) P_3 pertains to U_1 and 2) P_3 pertains to U_2. Conditioned by the choice between U_1 and U_2, and by the purpose to have an exponential attenuation of the potential, is the determination of the constant K_1, namely: $K_1 = K$ if $P_3 \subset U_1$ and $K_1 = 0$ if $P_3 \subset U_2$.

A. The Seeliger model as it stands

In this case , $K_1 = K$. Adopting an inertia frame and a system of Cartesian coordinates, we can write:

$$a_{jk} = \delta_{jk}, \qquad b^{jk} = \delta^{jk}, \qquad G^{l}_{jk} = 0 \tag{13a}$$

In addition, we have the condition $m_o^{(g)} = m_0^{(i)}$, as well as the specification that ρ is just the mass density defined in the framework of Newtonian Mechanics. Accordingly, the field equations and the motion equations acquire the classical form:

$$\Delta\Phi - K^2\Phi = 4\pi \cdot G \cdot \rho$$
$$\ddot{\vec{r}} = -\nabla\left(\Phi + H\right) \tag{13b}$$

Now, let us consider a spherical source, of radius *R*, whose center coincides with the origin of the inertial frame, and whose mass density is:

$$\rho = \rho(r), \qquad 0 < r < R, \qquad \rho = 0, \qquad R < r < \infty . \tag{14a}$$

Moreover, we assume the source to reach its inner mechanical equilibrium, under the simultaneous action of the gravitational and hydrodynamical forces. The equations (13b) become:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi}{dr}\right) - K^2 \cdot \Phi = 4\pi \cdot G \cdot \rho(r) \tag{14b}$$

$$\ddot{\vec{r}} = 0, \quad \rightarrow \Phi(r) + H(r) = \Phi(R), \qquad 0 < r < R$$

(We assumed $H(R) = 0$). Taking into account the definition of H in (2), and combining (2) with (14b), we can write:

$$\int_r^R \frac{1}{\rho}\frac{dp}{dr}dr = \Phi(r) - \Phi(R), \qquad 0 < r < R \tag{14c}$$

(where we assumed $p = p(r)$, $p(R) = 0$). For ρ constant $\rho = \rho_0$, we obtain the pressure distribution inside the sphere as:

$$p(r) = \rho_0\left[\Phi(R) - \Phi(r)\right] \tag{15a}$$

For $K = 0$, $\Phi(r) = -G\dfrac{M_0}{R}\left[\dfrac{3}{2} - \dfrac{1}{2}\left(\dfrac{r}{R}\right)^2\right]$, $\Phi(R) = -G\dfrac{M_0}{R}$, and obtain the result:

$$p(r) = \frac{1}{2}\rho_0 c^2\left(\frac{GM_0}{c^2R}\right)\cdot\left[1 - \frac{r^2}{R^2}\right] \tag{15b}$$

The maximal equilibrium pressure inside the sphere is located just in the center:

$$p_{Max} = \frac{1}{2}\rho_0 c^2\left(\frac{GM_0}{c^2R}\right) \tag{16a}$$

Now, it is worthwhile to dwelling upon a little, on this simple and remarkable formula, which is the origin of the so-called "pressure paradox". For Seeliger, who lived at the end of the XIX -th century, the expression (16a) suggested just nothing, so that he replaced it by an equivalent one:

$$p_{Max} = \frac{2}{3} \cdot \pi \cdot G \cdot \rho_0^2 \cdot R^2 \tag{16b}$$

Then, he considered the whole Universe as generated by addition of successive spherical shells of constant density, ρ_0, to an initial sphere, so that the mass and the size of the Universe grow limitless, but the mass density is kept constant. So, Seelinger came across its famous paradox of the infinite pressure, and what remains to us is to be puzzled, because we are still alive, not crashed by the giant pressure, yielded by milliards and milliards of stars. To save our souls, Seeliger introduced his constant *K*. In our times, R. H. Dicke put forward a remarkable conjecture, which may be, at the same time, a solution to Seeliger's paradox [21]. He assumed that the whole Universe behaves like a giant servo-system which permanently adjusts its mass and size to have:

$$\frac{GM_0}{c^2 R} = k_D, \qquad k_D = 0\,(1) \tag{17a}$$

A model of such adjustment is delivered by the Scalar-Tensorial Theory of Gravitation, due to C. Brans and R.H. Diche (1961) [10]. The starting point of this theory is the hypothesis that G^{-1} is actually a scalar field, whose smoothed out value, at a metagalactic scale, is $\sim M_0 / c^2 R$. Combining their theory with the hypothesis that the Universe is a giant Black-Hole, the authors obtain:

$$\frac{GM_0}{c^2 R} = 2\left(\frac{\omega + 2}{\omega + 3/2}\right)^{1/2}, \tag{17b}$$

where ω is a dimensionless parameter . According to our estimation, we have $\omega \geq 48$, so that the factor containing ω in (17,b) may be disregarded. For $\omega = \infty$, one obtains $k_D = 2$ and $p_{\max} = \rho_0 c^2$. This may be considered as the prediction delivered by the Black-Hole model of the Universe. It is interesting, to point out that the same prediction is obtained from the Seeliger model of an infinite and flat Universe, provided that the gravitational energy is taken into account (see Addendum)

$$p_{\max} = \rho_0 c^2 \tag{18}$$

At the same time, the result(18) is in compliance whit Einstein's General Relativity Theory, for an open Universe. Indeed, out of the field equations:

$$R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} \cdot R = -\frac{8\pi \cdot G}{c^4} \cdot \left\{ \rho \cdot c^2 \cdot U_\mu \cdot U_\upsilon + p \cdot \left(\frac{\gamma}{\gamma - 1} \cdot U_\mu \cdot U_\upsilon - g_{\mu\upsilon} \right) + \frac{c^4 \Lambda}{8\pi \cdot G} \cdot g_{\mu\upsilon} \right\}, \tag{19a}$$

we obtain at first

$$R = \frac{8\pi \cdot G}{c^4} \cdot \left(\rho \cdot c^2 - \frac{3\gamma - 4}{\gamma - 1} \cdot p + \frac{c^4 \Lambda}{2\pi \cdot G} \right) \tag{19b}$$

and, thereafter,

$$\lim_{R\to 0} R = 0, \quad \to p_{Max} = \frac{\gamma - 1}{3\gamma - 4}\left(\rho_0 c^2 + \frac{c^4 \Lambda}{2\pi G}\right) \tag{19c}$$

The single value of γ, agreeing with a constant mass density in an infinite flat space, is $\gamma = \infty$. In this case, the recovering of the formula (18) is obtained for

$$\Lambda c^2 = 4\pi G \rho_0 \tag{20a}$$

Moreover, a connection may be established between Einstein's cosmological constant Λ and Seeliger's cosmological constant K. This may be done by equating the equilibrium pressure

$$p_{Equi} = 2\pi G \left(\frac{\rho_0}{K}\right)^2 \tag{20b}$$

to the maximal pressure (18), *i.d.* est [22]

$$p_{Equi} = p_{Max} \to 2(cK)^2 = 4\pi G \rho_0$$

and by taking into account the formula (20,a). So, we come to the relationship

$$\Lambda = 2K^2 \tag{20c}$$

The result (20c) strengthens the idea of associating Seeliger's constant K to a certain subtle matter named ether, as far as Einstein's constant Λ is already associated with such matter [22]. Besides this, the equality (20c) enlightens the sense of the statement about the equivalent role of the two constants Λ and K, in spite of the very different mathematical formalisms entailing them. A formula similar to (20c) may be obtained through the intermediary of the Hoyle & Narlikar "Creation Theory" [9]. This theory postulates the maintaining of a constant mass density, in an infinite expanding Universe, by compensating the density decrease, due to expansion, by a constant rate creation of the ordinary matter, at the expense of a cosmical scalar field. The creation rate proves not being dependent on the variance properties of the creation field, as far as it is the same for both a scalar field and a vectorial field. Taking advantage of the Hoyle & Narlikar formula:

$$3H^2 = 4\pi G \rho$$

and combining it with the relation $H = c \cdot K$, derived in the framework of our analysis about Seeliger's theory and still taking into account the condition (20a), we come to the result $\Lambda = 3K^2$ (not too remote from the previous result (20c)).

Due to the relationship between Λ and K, leaving aside of the Λ constant in the field equations of the General Relativity Theory, requires a similar treatment for the K constant in the field equation of Seeliger's theory. However, for the sake of mechanical equilibrium, the constant K cannot be altogether ignored – it is only transferred to the background metric, ensuring in this way the fulfilment of Eddington's asymptotic version of the mass equivalence principle as well [24].

Further on, we come back to the Seeliger's theory and write down the solution of equation (14b):

$$\Phi = -G \cdot \left\{ \frac{e^{-Kr}}{r} \cdot \int_0^r \frac{shKr}{Kr} \cdot 4\pi r^2 \rho(r) dr + \frac{shKr}{Kr} \cdot \int_r^R \frac{e^{-Kr}}{r} \cdot 4\pi r^2 \rho(r) dr \right\}; \qquad 0 < r < R$$

$$\Phi = -G \cdot \frac{m}{r} \cdot e^{-Kr}, \qquad m = \int_0^R \frac{shKr}{Kr} \cdot 4\pi r^2 \rho(r) dr, \qquad R < r < \infty \tag{21}$$

The infringement of the mass equivalence principle is ascertained, because $\Phi \propto m$, for $R < r < \infty$, instead of $\Phi \propto M_0$, where:

$$M_0 = \int_0^R 4\pi r^2 \rho(r) dr \tag{22}$$

For a concrete application to an interaction, we consider a source particle of mass M_0 and a test particle of mass m_0, $m_0 << M_0$. Denoting the position vectors by $\vec{r}_0$ and $\vec{r}_p$, respectively, and taking $r = |\vec{r}_p - \vec{r}_0|$ we obtain

$$\Phi = -G \frac{m}{|\vec{r}_p - \vec{r}_0|} e^{-K|\vec{r}_p - \vec{r}_0|} \tag{23}$$

For a homogeneous sphere of radius R, one obtains

$$\Phi_R(r) = -\frac{4\pi G \rho_0}{K^2} \left\{ 1 - \frac{shKr}{Kr} (1 + KR) e^{-KR} \right\}$$

$$p_R(r) = 4\pi G \left(\frac{\rho_0}{K} \right)^2 (1 + KR) e^{-KR} \left(\frac{shKR}{KR} - \frac{shKr}{Kr} \right) \tag{24}$$

The limiting behaviour of these functions is

$$\lim_{K \to 0} \Phi_R(r) = -G \frac{M_0}{R} \left(\frac{3}{2} - \frac{1}{2} \frac{r^2}{R^2} \right)$$

$$\lim_{R \to \infty} \Phi_R(r) = -\frac{4\pi G \rho_0}{K^2}$$

$$\lim_{K \to 0} p_R(r) = \frac{2}{3} \pi G \rho_0^2 R^2 \left(1 - \frac{r^2}{R^2} \right) \tag{25}$$

$$\lim_{R \to \infty} p_R(r) = 2\pi G \left(\frac{\rho_0}{K} \right)^2 = p_{Equi}$$

The equilibrium pressure in the infinite Universe of constant mass density is constant. Equating now p_{Max} in (18) is p_{Equi} in (25) we obtain

$$cK = H = \sqrt{2\pi G\rho_0} \tag{26a}$$

For $\rho_0 = 1.299\times10^{-29}\, g/cm^3$, we have

$$K \approx 0.778\times10^{-28}\, cm^{-1}, \qquad cK \approx 2.334\times10^{-18}\, s^{-1} \tag{26b}$$

(To compare with $H \sim 72\ Km/s\cdot Mp = 2.334\times10^{-18}\, s^{-1}$)

B. The modified Seeliger model for complying with the equivalence principle

In this version, the position space P_3 is a subspace of the Universe U_2. Adopting an inertia frame (with the origin in a point-like source) and a Cartesian system of coordinates, the metric of P_3 takes the form

$$(ds)^2 = a_{jk}dx^j dx^k, \qquad (j,k = 1,2,3)$$

$$a_{jk} = e^{+2Kr}\cdot\left\{\delta_{jk} + \left[(1+Kr)^{+2} - 1\right]\cdot\frac{x_j x_k}{r^2}\right\},$$

$$x_1 = x^1 = x, \qquad x_2 = x^2 = y, \qquad x_3 = x^3 = z \tag{27a}$$

$$r = \left(x^2 + y^2 + z^2\right)^{\frac{1}{2}}$$

The contravariant aspect of the metric is:

$$(ds)^2 = b^{jk}dx_j dx_k, \qquad (j,k = 1,2,3),$$

$$b^{jk} = e^{-2Kr}\cdot\left\{\delta^{jk} + \left[(1+Kr)^{-2} - 1\right]\cdot\frac{x^j x^k}{r^2}\right\}, \qquad b^{jl}a_{lk} = \delta^j_k \tag{27b}$$

This time, the shortening of the interaction range is coming rather from geometry than from (ordinary) matter. Accordingly, we take in (3) $K_1 = 0$. Other details to be taken into account are the following ones:

$$m_0^{(g)} = m_0^{(i)}, \qquad \sqrt{a} = (1+Kr)\cdot e^{3Kr}\cdot\sqrt{a_M}, \qquad \rho\sqrt{a} = \rho_M\sqrt{a_M},$$

$$\sqrt{a} = Det\left\|a_{jk}\right\|, \qquad \sqrt{a_M} = Det\left\|\delta_{jk}\right\|. \tag{27c}$$

The field equation, preserves its Laplace - Beltrami - Poisson form even after the specifications regarding the frame and the coordinates. For a spherical source of radius *R*, the respective equation is:

$$\frac{1}{r^2}\cdot\frac{\partial}{\partial r}\cdot\left(r^2\cdot\frac{e^{Kr}}{1+Kr}\cdot\frac{\partial\Phi}{\partial r}\right)=4\pi G\rho_M \tag{28}$$

The label M stands for quantities defined in the Minkowski Universe (U_1). The solution to equation (28) is (*):

$$\Phi_R(r)=-G\cdot\left\{\frac{e^{-Kr}}{r}\cdot\int_0^r 4\pi\cdot r^2\cdot\rho_M(r)dr+\int_r^R\frac{e^{-Kr}}{r}\cdot 4\pi\cdot r^2\cdot\rho_M(r)dr\right\};\qquad 0<r<R$$

$$\Phi_R(r)=-G\cdot\frac{M_0}{r}\cdot e^{-Kr},\qquad M_0=\int_0^R 4\pi\cdot r^2\cdot\rho_M(r)dr,\qquad R<r<\infty \tag{29}$$

*) The case of arbitrary mass distribution is not yet studied.

The fulfilment of the mass equivalence principle is now self-evident because $\Phi_R(r)\propto M_0$ for $r>R$.

Equation (28) may equally be written as a Seeliger-type equation with additional (non-local) sources:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right)-K^2\cdot\Phi=4\pi\cdot G\cdot(\rho_M+\rho_{add}),$$

$$\rho_{add.}=\rho_M\cdot\left[(1+Kr)\cdot e^{-Kr}-1\right]+K^2\int_r^\infty r\cdot e^{-Kr}\cdot\rho_M dr \tag{30}$$

By direct calculation, we may verify now the equality:

$$\int_0^R\frac{shKr}{Kr}\cdot 4\pi r^2\cdot(\rho_M+\rho_{add.})dr=\int_0^R 4\pi r^2\rho_M dr\,, \tag{31}$$

necessary for compliance with the mass equivalence principle. The additional sources may be assigned formally to the interaction between the ordinary matter and the cosmical ether.

Further on, our task is to calculate the cosmic pressure yielded by a constant distribution of mass in the Universe U_1:

$$\rho_M=\rho_0,\quad\to\quad\rho(r)=\rho_M\frac{\sqrt{a_M}}{\sqrt{a}}=\rho_0\cdot\frac{e^{-3Kr}}{1+Kr} \tag{32a}$$

With that end in view, we first calculate the pressure in a sphere of radius R and constant mass density $\rho_M=\rho_0$, resorting to formulas:

$$\Phi_R(r)=-4\pi G\frac{\rho_0}{K^2}\cdot\left\{\left(1+Kr+\frac{1}{3}K^2r^2\right)\cdot e^{-Kr}-(1+KR)\cdot e^{-KR}\right\},$$

$$\frac{\partial p}{\partial r}+\rho\frac{\partial\Phi_R(r)}{\partial r}=0, \tag{32b}$$

$$\frac{\partial}{\partial r}\Phi_R(r)=\frac{4}{3}\cdot\pi G\rho_0 r(1+Kr)\cdot e^{-Kr},$$

$$p_R(r)=\int_r^R \rho_M\cdot\frac{\sqrt{a_M}}{\sqrt{a}}\cdot\frac{\partial\Phi_R(r)}{\partial r}dr$$

So, we obtain:

$$p_R(r)=\frac{4}{3}\pi G\rho_0^2\cdot\int_r^R e^{-4Kr}\cdot rdr=\frac{\pi}{12}\cdot G\cdot\left(\frac{\rho_0}{K}\right)^2\cdot\left\{(1+4Kr)\cdot e^{-4Kr}-(1+4KR)\cdot e^{-4KR}\right\} \tag{33a}$$

The quantity $p_R(r)$ fulfils the limiting conditions:

$$\lim_{K\to 0} p_R(r)=\frac{2}{3}\cdot\pi\cdot G\cdot\rho_0^2\cdot R^2\cdot\left(1-\frac{r^2}{R^2}\right) \tag{33b}$$

$$\lim_{R\to\infty} p_R(r)=\frac{\pi}{12}\cdot G\cdot\left(\frac{\rho_0}{K}\right)^2\cdot(1+4Kr)\cdot e^{-4Kr}=p_\infty(r)$$

The maximal value of the cosmical pressure $p_\infty(r)$ is obtained for r = 0, it is:

$$p_{Max}=\frac{\pi}{12}\cdot G\cdot\left(\frac{\rho_0}{K}\right)^2 \tag{34a}$$

Equating now $p_{Max}=\frac{1}{4}\cdot\rho_0\cdot c^2$ to p_{Max} in (34,a) we obtain:

$$cK=H=\sqrt{\frac{\pi}{3}G\rho_0} \tag{34b}$$

The value predicted for H (the Hubble constant) by the modified Seeliger model is $\sqrt{6}$ times smaller as compared to the value predicted by the original Seeliger model for the same value of the (smoothed out) cosmical mass density. Unfortunately, the error in the astrophysical determination of the mass density is, at present, too large to conclude whether a factor $\sqrt{6}$ is relevant or not. So, we cannot reject the possibility of an infringement of the mass equivalence principle at a metagalactic scale, if ρ_0 is indeed the density of the total matter (excepting the ether). To point out that the mass equivalence principle is saved if we admit the existence of the so called "hidden mass". Then, ρ_0 is to be replaced by $\rho_0+\rho_{hid.}$, $\rho_{hid.}>>\rho_0$, in (34,b), and the value $H\sim 72\ Km/s\cdot Mp$ may be recovered. (This happens exactly for $\rho_{hid.}=5\rho_0$).

By the infringement of the mass equivalence principle, we mean only the conflictual situation regarding the asymptotic definition of the equivalence, due to A. S. Eddington [24]. This by no means preclude any formulation of the respective principle. On the contrarily, for both versions of the Seeliger's theory, we may verify a formulation based on the field inside the source:

$$M_{0g}=\frac{1}{G}\int_0^R\left\{\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi}{dr}\right)-K^2\Phi\right\}\cdot r^2dr$$

(35a)

$$M_{0i}=\int_0^R 4\pi\rho_M(r)\cdot r^2dr,\quad\rightarrow\quad M_{0i}=M_{0g};$$

$$M_{0g}=\frac{1}{G}\int_0^R\left\{\frac{1}{r^2}\frac{d}{dr}\left(r^2\cdot\frac{e^{Kr}}{1+Kr}\frac{d\Phi}{dr}\right)\right\}\cdot r^2dr$$

(35b)

$$M_{0i}=\int_0^R 4\pi\rho_M(r)\cdot r^2dr,\quad\rightarrow\quad M_{0i}=M_{0g};$$

Eddington's definition of the gravitational mass referred only to the infinite range gravitation. The adaptation of his asymptotic definition to Seeliger's theory is made, for the first time, in this work.

IV. Search for a Gravitational sub-Mechanics

The theory, so far outlined, is rather a phenomenology in which universal laws of Theoretical Mechanics, as covariance, inertia, conservation, to which we add the two facets of the equivalence principle – that of force and that of mass – are observed, but the genuine mechanism of interaction is systematically overlooked. For this reason, the obtained formula of the potential energy, for a two body gravitational system, correctly accounts for the inter-body distance dependence, but gives no information about the dependence on masses. To accomplish the complete formula on theoretical grounds, we extend now the Synge-type exchange theory [17], but resorting to an emission – absorption mechanism of interaction.

The gravitational potential energy of a two-body system may be written as:

$$U(r)=\tau\left(\overline{W}_{A\to B}+\overline{W}_{B\to A}\right) \tag{1}$$

Here, $\tau=\frac{1}{c}r_{\text{inter}}$ is the time spent by the negative quanta for covering the inter-body distance $\overline{AB}$; $\overline{W}_{A\to B}$ is the average power emitted by the body $\underline{a}$ and travelling towards the body $\underline{b}$; $\overline{W}_{B\to A}$ is the average power emitted by the body $\underline{b}$ and travelling towards the body $\underline{a}$. We point out that we deal with *virtual processes*, implying negative energy quanta, as far as the number of such quanta, emitted by a certain body, depends on the absorption capability of the other body. So, the conventional causality is infringed, because the emission is conditioned by *preliminary information* about the interaction partner, prior to the spending of the *causal duration* τ. We call this strangeness we come across in sub-Mechanics, *virtual causality*. Rendering in mathematical terms the basic idea of the virtual causality, we write:

$$\overline{W}_{A\to B}=W_A\cdot P_B\quad,\qquad \overline{W}_{B\to A}=W_B\cdot P_A \tag{2}$$

where (W_A,W_B) are the emitted powers by the body *a* and body *b* respectively, while (P_B,P_A) stand for the absorption probabilities of the body *b* and body *a*, respectively. Formula (1) already contains

Synge's statement that potenti al energy is actually the energy of negative quanta in transit between the two partners of the interaction.

To derive the mass factor in the potential energy formula, we need to put forward some statements, outlining a rather microscopic picture of the gravitational interaction. Accordingly, we adopt a "principle of universality" relying on three basic statements:

Statement 1: Any mass unit of matter emits negative virtual quanta towards the whole matter of the Universe, at a constant rate:

$N_e = const.$ (number of emitted quanta per unit mass and unit time).

Statement 2: Any mass unit of matter absorbs negative virtual quanta coming from the whole matter of the Universe, with a constant cross-section:

$\sigma_a = const. \qquad \left(cm^2 \cdot g^{-1}\right).$

Statement 3: For any mass unit of matter, the virtual emission and the virtual absorption do balance at any place and any instant of the natural history of the Universe, resulting a universal and everlasting "pulsation of matter".

This constant and universal rhythm may be identified with the genuine cause of gravitation.

Based on the previous hypotheses, we may render in explicit form the factors entering the formulas (2), namely:

$$W_A = -(\hbar\omega_0)\cdot\left(m_{0_A} N_e\right), \qquad W_B = -(\hbar\omega_0)\cdot\left(m_{0_B} N_e\right),$$

$$P_B = \frac{m_{0_B}\sigma_a}{4\pi \cdot r_{\text{inter}}^2}, \qquad P_A = \frac{m_{0_A}\sigma_a}{4\pi \cdot r_{\text{inter}}^2}, \tag{3}$$

Out of formulas (2) and (3), we derive now the equality:

$$\overline{W}_{A\to B} = \overline{W}_{B\to A} \tag{4}$$

whence:

$$\overline{W}_{A\to B} = \overline{W}_{B\to A} = \frac{1}{2}\left(\overline{W}_{A\to B} + \overline{W}_{B\to A}\right) \equiv \frac{1}{2}\cdot\overline{W} = \frac{1}{2}\cdot\frac{U(r)}{r_{\text{inter}}}\cdot c \tag{5}$$

Another useful formula may be derived as follows:

$$W = -\frac{dU(r)}{dt} = -\frac{dU(r)}{dr}\frac{dr}{dt} = -c\cdot\frac{dU(r)}{dr} = -c\cdot\frac{dU}{dr_{\text{inter}}}\cdot\frac{dr_{\text{inter}}}{dr} = c\cdot\frac{U(r)}{r_{\text{inter}}}\cdot\frac{dr_{\text{inter}}}{dr} = \overline{W}\cdot\frac{dr_{\text{inter}}}{dr} \tag{6}$$

The potential energy formula acquires now the completely explicit form (*i.e.* concerning both the geometric and the material part), taking in view the already derived expression of r_{inter}, namely:

$$r_{\text{inter}} = r\cdot\exp(Kr), \tag{7}$$

as well as the formula (1) (or formula (5)):

$$U(r) = -\left(\frac{\hbar\omega_0}{2\pi\cdot c}\cdot N_e\sigma_a\right)\cdot\frac{m_{0_A}\cdot m_{0_B}}{r_{\text{inter}}} \tag{8}$$

The result (8) is just Seeliger' s formula, provided that we identify the constant factor with the Newtonian constant *G*.

$$U(r) = -G \cdot \frac{m_{0_A} \cdot m_{0_B}}{r} \cdot e^{-Kr}, \tag{9a}$$

$$G \equiv \hbar\omega_0 \cdot \frac{N_e \sigma_a}{2\pi c}, \qquad K \equiv \frac{H}{c} \tag{9b}$$

According to the statement (3), we write the equality between absorption and emission of negative virtual quanta by a body of mass m_{0_A}.

$$\left(\frac{dE}{dt}\right)_{Abs.} = \left(\frac{dE}{dt}\right)_{Emi.} \tag{10}$$

The emission is, by definition, expressed as:

$$\left(\frac{dE}{dt}\right)_{Emi.} = -(\hbar\omega_0) \cdot m_{0_A} N_e, \tag{11}$$

while, for estimating the absorption, we resort to the result (6):

$$\left(\frac{dE}{dt}\right)_{Abs.} = \int_{(B)} dW_{B\to A} = \int_{(B)} \frac{dr_{\text{inter}}}{dr} \cdot d\overline{W}_{B\to A},$$

$$d\overline{W}_{B\to A} = \left\{-(\hbar\omega_0) \cdot N_e \cdot \left(\rho_0 4\pi r^2 dr\right)\right\} \cdot \frac{m_{0_A} \cdot \sigma_a}{4\pi \cdot r^2_{\text{inter}}} \tag{12}$$

As far as no gravitational shielding exists in the Universe, the integration over the emitting sources B extends over the infinite position space. Assuming the mass of the Universe as uniformly and homogeneously distributed, with constant density ρ_0, we may write:

$$\begin{aligned}\left(\frac{dE}{dt}\right)_{Abs.} &= \int_0^\infty \left\{-(\hbar\omega_0) \cdot N_e \cdot \left(\rho_0 4\pi r^2 dr\right)\right\} \cdot \frac{m_{0_A}\sigma_a}{4\pi r^2_{\text{inter}}} \cdot \frac{dr_{\text{inter}}}{dr} = \\ &= -(\hbar\omega_0) \cdot \left(m_{0_A}\sigma_a\right) \cdot (N_e \rho_0) \cdot \int_0^\infty e^{-Kr} \cdot (1 + Kr) dr = \\ &= -\frac{2}{K} \cdot (\hbar\omega_0) \cdot \left(m_{0_A}\sigma_a\right) \cdot (N_e \rho_0)\end{aligned} \tag{13}$$

Out of (10), (11) and (13), we come to a constraint among the various universal constants:

$$\sigma_a = \frac{1}{2} \cdot \frac{K}{\rho_0} = \frac{1}{2} \cdot \frac{H}{\rho_0 c} \tag{14}$$

Adopting the observational values for H and ρ_0, namely: $H \sim 1.6195 \cdot 10^{-18} s^{-1}$ $\left(\sim 50 Km/s \cdot M_p\right)$ and $\rho_0 \sim 2.0855 \cdot 10^{-30} g \cdot cm^{-3}$, we may estimate for σ_a a value of the magnitude order of:

$$\sigma_a \sim \begin{cases} 12.95 \quad cm^2 \cdot g^{-1} \\ 2.166 \cdot 10^{-23} \quad cm^2 / proton \end{cases}$$

To estimate the other basic constant of the model, namely N_e, we resort to the equality:

$$\hbar\omega_0 = \hbar H \qquad (15)$$

(whose reliability is weaker as compared to that of the relation $cK = H$. In this way, we get the formula:

$$G = N_e \cdot \frac{\hbar}{4\pi} \cdot \frac{H^2}{\rho_0 c^2}, \qquad (16)$$

which may be reverted for evaluating the constant N_e:

$$N_e \sim \begin{cases} 5.6820 \cdot 10^{47} \quad quanta / g \cdot s \\ 9.5032 \cdot 10^{23} \quad quanta / proton \cdot s \end{cases}$$

At the same time, the energy of a virtual quantum (the smallest possible in Nature) is:

$$\hbar\omega_0 = \hbar H \sim 1.7078 \cdot 10^{-45} \quad ergs$$

Accordingly, the virtual emissivity of matter turns out to be:

$$w = -(\hbar\omega_0) \cdot N_e \sim -970.37 \quad erg \cdot g^{-1} \cdot s^{-1}$$

Concluding remarks

1) The indetermination, in the radial metric function of the Schwarzschild-type invariant, allows, in conjunction with a frame of inertia and with the mass equivalence principle, to include the finite range Seeliger's potential into the General Relativity doctrine.

2) The completion of the Schwarzschild metric, in the specified sense, delivers us a certain universal metric $(dS_{U_2})^2$ with vanishing curvature, but not reducible (under the conditions of choosing a frame of inertia and a Cartesian system of coordinates) to the Minkowskian metric $(dS_{U_1})^2$.

3) The existence of the two metrics $(dS_{U_1})^2$ and $(dS_{U_2})^2$ makes up a universal bimetrism, interpretable in terms of a cosmic ether, coupled with all the kinds of physical interactions, *via* the covariance of the Mathematical Physics equations with respect to (dS_{U_2}). The consequences of this special covariance are both the finite range of the gravitational interaction and the fulfilment of the mass equivalence principle according to Eddington's asymptotic formulation [24].

4) A considerable effort is paid to argue that the so called "interaction length", in the case of the finite range interaction, is greater than the geometric inter-particle distance r - $r_{\text{inter}} = r \cdot \exp(Kr)$. The result of this effort is the reaching of the relationship $cK = H$, between Seeliger's constant K and the Hubble constant H. Further on, the quantity r_{inter} is a basic concept,

entering the virtual interchange model of gravitational interaction (due to J. L. Synge [1935] [17]) and extended by us for finite range gravitation).

5) The entering upon of the mechanical equilibrium of the entire Universe, under the combined action of the gravitational forces (of the Newton – Seeliger type) and of the hydrodynamical forces (of the Euler type), depends on the option regarding the origin of the range finiteness (either we accept the new bimetrism, *i.e.* we assign to the graviton a rest mass of geometric origin, or we reject it, and assume a close analogy between finite range gravitation and the mesonic theory). The equilibrium pressure in the two cases has different values (although the magnitude order is almost the same).

6) An alternative way, for calculating the maximal pressure in the infinite Universe with smoothed out mass density, is to ask the vanishing of the trace of the (canonical) energy tensor. In physical terms, this means either the vanishing of the scalar curvature, or the vanishing of the cosmical gravitational forces. By equating the two expressions of the cosmical pressure, we reach an alternative expression for the constant K (in terms of the smoothed out density). Finally, out of the two expressions for K, we derive a relationship between the Hubble constant and the cosmical mass density, comparable with the similar formulas of some relativistic cosmological models.

7) A connection between Seeliger's constant K and Einstein's constant Λ may be established as well, bringing arguments for the similar role played by the two types of ether – that of Seeliger and that of Einstein – in the problem of matter stability.

8) The mechanism of emission and absorption of virtual quanta, carrying out negative energy between the two partners of a gravitational interaction, leads to the formula of potential energy proposed by Seeliger, provided that the gravitational constant G is expressed in terms of the virtual emissivity constant and of virtual absorption cross section.

9) The equal sharing of the potential energy between the two point-like partners of a gravitational interaction, irrespective of the ratio between the two rest masses, ensures, in conjunctions with some statements expressing the fulfilment of the inertia principle for the aggregate as a whole, the obtaining of the first order relativistic Lagrange function for a two-body system. For an increased reliability in the relativistic Seeliger's theory of gravitation, we devised a scalar field theory, correctly accounting for both gravitational tests and cosmical equilibrium.

Addendum 1

(Compliance with the metric dilation hypothesis)

Let (a, b) be two point-like bodies placed at the instant t = 0 in the positions (A, B), so that the distance $\overline{AB} = r$. At the instant t = 0, a real quantum starts from A towards B. Ignoring the existence of the metric $\left(dS_{U_2}\right)^2$, and aiming to explain why the quantum, travelling at the constant speed c, spends a time $\tau > r/c$ for covering the inter-body distance, the astronomers of the XX -th century came to the daring hypothesis about the "Expanding Universe" [2]. This actually me ans that the body $\underline{b}$ is slowly removing from the body $\underline{a}$, during the quantum travel, "due to the metric dilation". Accordingly, we can write :

$$r_{\text{inter}} = r + \overline{V}\tau_0 \ , \ \tau_0 = r/c, \ \ \overline{V} = \frac{c}{r}\int_0^{r/c} V(\tau)dt \qquad (1)$$

The transformations (1) should be corrected for going over from Classical to Relativistic Mechanics:

$$r_{\text{inter}} = \gamma \cdot \left(r + \overline{V}\tau_0\right), \ \tau = \gamma\left(\tau_0 + \frac{1}{c^2}\overline{V}r\right), \ \gamma \equiv \left(1 - \frac{\overline{V}^2}{c^2}\right)^{-1/2} \qquad (2)$$

Accounting for the light propagation at the constant velocity *c*, in both inertia frames, connected to one another through the Lorentz transformations (2), we can write again:

$$\tau = \frac{1}{c} r_{\text{inter}} \;,\; \tau_0 = \frac{1}{c} r \;, \qquad (3)$$

so that, out of the formulas (2) and (3) , we realize we have to retain a single independent relation:

$$r_{\text{inter}} = r \left(\frac{1 - \frac{1}{c}\overline{V}}{1 + \frac{1}{c}\overline{V}} \right)^{\frac{1}{2}} \qquad (4)$$

On the other hand , $\overline{V}$ is a function of *r* , as resulting from (1) :

$$\overline{V} = F(r) \qquad (5)$$

Since, during the travel, $\overline{V}$ is a constant quantity, entering the Lorentz transformations in the position of a velocity, and since the distances are additive quantities, the relativistic law of velocity composition delivers the functional equation:

$$F(r_1 + r_2) = \frac{F(r_1) + F(r_2)}{1 + \frac{1}{c^2} F(r_1) F(r_2)} \qquad (6)$$

whose solution is the expression [25] :

$$F(r) = c \cdot th\left(\frac{H}{c} r \right) \qquad (7)$$

The constant $H \equiv F'(0)$ may be identified as the observational Hubble constant. Now, from (4), (5) and (7) we obtain the total distance, traveled by the real quantum, as:

$$r_{\text{inter}} = r \cdot \exp\left(\frac{H}{c} r \right) \qquad (8)$$

This is just the interaction length, required by the extended Synge model, provided that the constant (*K*, *H*, *c*) are interconnected through the relationship:

$$cK = H \qquad (9)$$

The time τ, spent by the quantum to cover this distance is obtained from (3) and (8), namely:

$$\tau = \frac{1}{c} r \cdot \exp\left(\frac{H}{c} r \right) , \qquad (10)$$

Addendum 2

(Extended Seeliger's Theory)

The starting point is the action variational principle in flat space-time:

$$\mathrm{A} = \frac{1}{c}\int L\sqrt{-g}\,\left(d^4x\right)$$

$$L = \left(c^2 + H\right)\cdot \rho - p + \rho\cdot\Phi - \frac{1}{8\pi\cdot G}\cdot\left(g^{\alpha\beta}\Phi_{,\alpha}\Phi_{,\beta} - K^2\Phi^2\right), \tag{1}$$

$$H = \int_0^{p(\rho)}\frac{dp}{\rho(p)}$$

The field equation is obtained asking the vanishing of the action variation against the potential Φ:

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\beta}\left(\sqrt{-g}\cdot g^{\alpha\beta}\frac{\partial\Phi}{\partial x^\alpha}\right) + K^2\Phi = -4\pi\cdot G\cdot\rho \tag{2}$$

For a static field with spherical symmetry the field equation becomes:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi}{dr}\right) - K^2\Phi = 4\pi\cdot G\cdot\rho \tag{3}$$

The solution to the previous equation, inside a spherical source of constant mass density ρ_0 and radius R , is:

$$\Phi_R(r) = -\frac{4\pi\cdot G\cdot\rho_0}{K^2}\left[1 - \frac{shKr}{Kr}\cdot(1+KR)\cdot e^{-KR}\right] \tag{4}$$

The motion equations are given by the Euler & Lagrange variational principle:

$$\delta\int\frac{\partial L}{\partial\rho}\left(g_{\alpha\beta}\,dx^\alpha\,dx^\beta\right)^{\frac{1}{2}} = 0\ ,\quad \frac{\partial L}{\partial\rho} = c^2 + (\Phi + H) \tag{5}$$

The mechanical equilibrium condition, inside the spherical source is (accounting for $H_R(R)=0$):

$$\frac{\partial L}{\partial\rho} = const. \to H_R(r) = \Phi_R(R) - \Phi_R(r) \tag{6}$$

or, because $\rho = const.$,

$$p_R(r) = +\rho_0\left[\Phi_R(R) - \Phi_R(r)\right] \tag{7}$$

Out of (7) and (4) one obtains the pressure distribution inside the sphere as:

$$p_R(r) = 4\pi\cdot G\cdot\left(\frac{\rho_0}{K}\right)^2\cdot(1+KR)\cdot e^{-KR}\cdot\left(\frac{sh\,KR}{KR} - \frac{sh\,Kr}{Kr}\right) \tag{8}$$

The cosmical equilibrium pressure turns out to be constant throughout the space:

$$p_{Equi.} = \lim_{R\to\infty} p_R(r) = 2\pi \cdot G \cdot \left(\frac{\rho_0}{K}\right)^2 \tag{9}$$

Further on, we calculate the energy tensor:

$$\begin{aligned} T_{\mu\nu} &= \frac{-2}{\sqrt{-g}} \cdot \frac{\delta\left(\sqrt{-g} \cdot L\right)}{\delta \cdot g^{\mu\nu}} = \left[c^2 + (\Phi + H)\right] \cdot \rho \cdot U_\mu \cdot U_\nu - p \cdot g_{\mu\nu} + \\ &+ \frac{1}{4\pi \cdot G} \cdot \left(\Phi_{,\mu} \cdot \Phi_{,\nu} - \frac{1}{2} \cdot g_{\mu\nu} \cdot \Phi_{,\lambda} \cdot \Phi^{|\lambda} + \frac{1}{2} \cdot K^2 \cdot \Phi^2 \cdot g_{\mu\nu}\right) \end{aligned} \tag{10}$$

The *trace* of the energy tensor is $T = g^{\mu\nu} \cdot T_{\mu\nu}$, *i.e.:*

$$T = c^2 \cdot \rho + (\Phi + H) \cdot \rho - 4p + \frac{1}{2\pi \cdot G} \cdot \left(K^2 \cdot \Phi^2 - \frac{1}{2} \cdot g^{\alpha\beta} \cdot \Phi_{,\alpha} \cdot \Phi_{,\beta}\right) \tag{11}$$

The maximal pressure in the Universe is obtained by asking *T=0* , whence:

$$p_{Max} = \frac{1}{4} \cdot \rho \cdot c^2 + \frac{1}{4} \cdot (\Phi + H) \cdot \rho + \frac{1}{8\pi \cdot G} \cdot \left(K^2 \cdot \Phi^2 - \frac{1}{2} \cdot g^{\alpha\beta} \cdot \Phi_{,\alpha} \cdot \Phi_{,\beta}\right) \tag{12}$$

Further on, we have to insert the condition $\rho = \rho_0$ and to calculate the limit of the expression for $R \to \infty$. Thus, we obtain:

$$\lim_{R\to\infty} \Phi_R(r) = -\frac{4\pi \cdot G \cdot \rho_0}{K^2}$$

$$\lim_{R\to\infty} (\Phi_R + H_R) = \lim_{R\to\infty} \Phi_R(R) = -\frac{2\pi \cdot G \cdot \rho_0}{K^2} \quad , \tag{13}$$

$$\lim_{R\to\infty} g^{\alpha\beta} \Phi_{,\alpha} \Phi_{,\beta} = 0$$

and, accordingly:

$$p_{Max} = \frac{1}{4} \cdot \rho_0 \cdot c^2 - \frac{\pi}{2} \cdot G \cdot \left(\frac{\rho_0}{K}\right)^2 + 2\pi \cdot G \cdot \left(\frac{\rho_0}{K}\right)^2 \tag{14}$$

Out of (9) and (14) we write further:

$$p_{Max} - p_{Equi} = \frac{1}{4}\left[\rho_0 \cdot c^2 - 2\pi \cdot G \cdot \left(\frac{\rho_0}{K}\right)^2\right] \tag{15}$$

Finally, asking the condition $p_{Max} = p_{Equi}$, we come to the results:

$$p_{Max} = \rho_0 \cdot c^2 = 2\pi \cdot G \cdot \left(\frac{\rho_0}{K}\right)^2 , \tag{16}$$

$$cK = (2\pi \cdot G \cdot \rho_0)^{\frac{1}{2}}$$

Addendum 3
(Modified Seeliger's Theory)

This time, the constant K is transferred to the background metric $\left(dS_{U_2}\right)^2$. Accordingly, it no longer explicitly appears in the Lagrange function and in other expressions derived through the variational procedure. We write:

$$L = c^2 \cdot \rho + (\Phi + H) \cdot \rho - p - \frac{1}{8\pi \cdot G} \cdot g^{\alpha\beta} \cdot \Phi_{,\alpha} \cdot \Phi_{,\beta},$$

$$T^{\mu\nu} = \left(c^2 + \Phi + H\right) \cdot \rho \cdot U^{\mu} \cdot U^{\nu} - p \cdot g^{\mu\nu} + \frac{1}{4\pi \cdot G} \cdot \left(\Phi^{|\mu} \cdot \Phi^{|\nu} - \frac{1}{2} \cdot g^{\mu\nu} \cdot \Phi_{,\lambda} \cdot \Phi^{|\lambda}\right),$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \cdot \frac{e^{Kr}}{1+Kr} \cdot \frac{\partial \Phi}{\partial r}\right) = 4\pi \cdot G \cdot \rho_M$$

Thereafter, we account for the relativistic mass equivalence principle, by performing in the previous field equation the substitution $\rho_{M \to} \frac{\sqrt{a}}{\sqrt{a_M}} \frac{T}{c^2}$, where $T = g_{\alpha\beta} \cdot T^{\alpha\beta}$, *i.e.*:

$$T = c^2 \cdot \rho + (\Phi + H) \cdot \rho - 4p - \frac{1}{4\pi \cdot G} \cdot \Phi_{,\lambda} \cdot \Phi^{|\lambda}$$

But,

$$\Phi_R(r) + H_R(r) = \Phi_R(R) = -\frac{4\pi}{3} G \cdot \rho_0 \cdot R^2 \cdot e^{-KR},$$

$$\lim_{R\to\infty}\left(\Phi_R(r) + H_R(r)\right) = 0, \qquad \rho = \rho_M \frac{\sqrt{a_M}}{\sqrt{a}} \quad , \quad \rho_M = \rho_0 (const.)$$

Asking now the condition $\lim_{R\to\infty} T = 0$, we obtain:

$$\lim_{R\to\infty} \Phi_R(r) = \Phi_\infty(r) = const, \Phi'_\infty(r) = 0 \ ,$$

and the maximal cosmical pressure turns out to be:

$$p_{Max} = \frac{1}{4} \rho \cdot c^2 = \frac{1}{4} \rho_0 \cdot c^2 \cdot \frac{e^{-3Kr}}{1+Kr}$$

Addendum 4
(Relativistic effects)

Accounting for the interaction contribution to the rest masses of a two-body gravitational system (A, B), and denoting by (n_A, n_B) the numbers of the virtual interchanged quanta, we can write:

$$M_A^{(0)} = m_{0A} - \overline{n_A} \cdot \frac{\hbar\omega_0}{c^2}, \qquad M_B^{(0)} = m_{0B} - \overline{n_B} \cdot \frac{\hbar\omega_0}{c^2}.$$

But,

$$\overline{n_A} = \overline{n_B} = \frac{1}{2} \cdot (n_A + n_B), \qquad U(r) = -(n_A + n_B) \cdot \frac{\hbar\omega_0}{c^2},$$

so that

$$M_A^{(0)} = m_{0A} + \frac{1}{2} \cdot \frac{U(r)}{c^2}, \qquad M_B^{(0)} = m_{0B} + \frac{1}{2} \cdot \frac{U(r)}{c^2}.$$

When the two point-like bodies (A, B) are in motion, with respect to a certain frame of inertia, the kinetic contribution should be added, in view of obtaining the total inertial masses (M_A, M_B) . Accordingly, these quantities are defined as:

$$M_A = m_{0A} + \frac{1}{c^2} \cdot \left(\frac{1}{2} \cdot m_{0A} \cdot \vec{\mathrm{v}}_A^{\,2} + \frac{1}{2} \cdot U \right) + O\left(\frac{1}{c^4} \right) ,$$

$$M_B = m_{0B} + \frac{1}{c^2} \cdot \left(\frac{1}{2} \cdot m_{0B} \cdot \vec{\mathrm{v}}_B^{\,2} + \frac{1}{2} \cdot U \right) + O\left(\frac{1}{c^4} \right) ,$$

and enter the basic formulas of Theoretical Mechanics, in the "Invariantive version" [14]

$$\vec{R} = \frac{M_A \vec{r}_A + M_B \vec{r}_B}{M_A + M_B}, \quad M = M_A + M_B, \quad \vec{P} = M \frac{d\vec{R}}{dt}, \quad M = \frac{1}{c^2} E, \quad \vec{P} = const., \quad M = const$$

$$\vec{P} = M_A \cdot \vec{\mathrm{v}}_A + M_B \cdot \vec{\mathrm{v}}_B + \frac{1}{2} \cdot \left(\vec{r}_A - \vec{r}_B \right) \frac{d}{dt} (M_A - M_B) ,$$

$$\vec{P} = \frac{\partial \Lambda}{\partial \vec{\mathrm{v}}_A} + \frac{\partial \Lambda}{\partial \vec{\mathrm{v}}_B},$$

The total linear momentum $\vec{P}$ may be estimated as:

$$\vec{P} = \left(m_{0A} \cdot \vec{\mathrm{v}}_A + m_{0B} \cdot \vec{\mathrm{v}}_B \right) + \frac{1}{c^2} \left\{ \frac{1}{2} m_{0A} \cdot \vec{\mathrm{v}}_A^{\,2} \cdot \vec{\mathrm{v}}_A + \frac{1}{2} m_{0B} \cdot \vec{\mathrm{v}}_B^{\,2} \cdot \vec{\mathrm{v}}_B + \frac{1}{2} U(r) \cdot \left(\vec{\mathrm{v}}_A + \vec{\mathrm{v}}_B \right) + \frac{1}{4} s \cdot \vec{r} \right\} + O\left(\frac{1}{c^4} \right)$$

where

$\vec{r} = \vec{r}_A - \vec{r}_B$ and

$$s = \frac{d}{dt} \left(m_{0A} \cdot \vec{\mathrm{v}}_A^{\,2} - m_{0B} \cdot \vec{\mathrm{v}}_B^{\,2} \right) = 2 \left(m_{oA} \cdot \vec{\mathrm{v}}_A \cdot \vec{a}_A - m_{0B} \cdot \vec{\mathrm{v}}_B \cdot \vec{a}_B \right) = -U'(r) \cdot \frac{\left(\vec{\mathrm{v}}_A + \vec{\mathrm{v}}_B \right) \cdot \vec{r}}{r} + O\left(\frac{1}{c^2} \right)$$

To reach the previous result, we used the first order motion equations:

$$m_{0A} \cdot \vec{a}_A = -U'(r) \cdot \frac{\vec{r}}{r} + O\left(\frac{1}{c^2}\right) \quad , \quad m_{0B} \cdot \vec{a}_B = +U'(r) \cdot \frac{\vec{r}}{r} + O\left(\frac{1}{c^2}\right)$$

Now, the quantity $\vec{P}$ acquires the expression:

$$\vec{P} = m_{0A}\vec{\mathrm{v}}_A + m_{0B}\vec{\mathrm{v}}_B + \frac{1}{c^2}\left\{\frac{1}{2}m_{0A}\vec{\mathrm{v}}_A^{2}\vec{\mathrm{v}}_A + \frac{1}{2}m_{0B}\vec{\mathrm{v}}_B^{2}\vec{\mathrm{v}}_B + \frac{1}{2}U(r)\left(\vec{\mathrm{v}}_A + \vec{\mathrm{v}}_B\right) - \frac{1}{2}(rU')\frac{\left(\vec{\mathrm{v}}_A + \vec{\mathrm{v}}_B\right)\cdot\vec{r}}{r^2}\vec{r}\right\} + O\left(\frac{1}{c^4}\right)$$

By a vectorial integration, we obtain now the Lagrange function of motion:

$$\Lambda = -(m_{0A} + m_{0B})c^2 + \left(\frac{1}{2}m_{0A}\vec{\mathrm{v}}_A^{2} + \frac{1}{2}m_{oB}\vec{\mathrm{v}}_B^{2}\right) - U(r) + \frac{1}{c^2}\left\{\left[\frac{1}{8}m_{oA}\left(\vec{\mathrm{v}}_A^{2}\right)^2 + \frac{1}{8}m_{0B}\left(\vec{\mathrm{v}}_B^{2}\right)^2\right] + \frac{1}{2}U(r)\vec{\mathrm{v}}_A\vec{\mathrm{v}}_B - \right.$$

$$\left. -\frac{1}{2}(rU')\frac{\left(\vec{\mathrm{v}}_A\cdot\vec{r}\right)\left(\vec{\mathrm{v}}_B\cdot\vec{r}\right)}{r^2} + \frac{1}{4}k_1\cdot U(r)\cdot\vec{\mathrm{v}}^{2} - \frac{1}{4}k_2(r\cdot U')\frac{\left(\vec{\mathrm{v}}\cdot\vec{r}\right)^2}{r^2} - k_3\left(\frac{1}{m_{0_A}} + \frac{1}{m_{0B}}\right)\cdot U^2(r)\right\} + O\left(\frac{1}{c^4}\right)$$

where (k_1, k_2, k_3) are arbitrary dimensionless constants, and a zero energy term was added, to ensure a relativistic calibration of the energy. In addition:

$$\vec{\mathrm{v}} = \dot{\vec{r}} = \vec{\mathrm{v}}_A - \vec{\mathrm{v}}_B \quad , \quad U(r) = -G\frac{m_{0A}\cdot m_{oB}}{r}\cdot\exp(-Kr).$$

Addendum 5

(Relativistic field theory for the two-body problem)

The starting point is a variational principle based on an Action integral:

$$\mathrm{A} = \frac{1}{c}\cdot\int L\sqrt{-g}\,\left(d^4x\right)$$

$$L = \rho\cdot c^2 + (\rho\cdot H - p) + \rho\cdot c^2\cdot(\Lambda - 1) + \left[\left(22 + 18\frac{m_0}{M_0}\right)\cdot p - \rho\cdot H\right]\cdot\varphi -$$

$$-\frac{c^4}{12\pi\cdot G}\cdot\left(7 + 6\frac{m_0}{M_o}\right)\cdot K^2\cdot\varphi^3 - \frac{c^4}{8\pi\cdot G}\left(g^{\alpha\beta}\cdot\varphi_{,\alpha}\cdot\varphi_{,\beta} - K^2\cdot\varphi^2\right)$$

where:

$$\Lambda = \left[f_1(\varphi) - f_2(\varphi)\left(\varphi_{,\alpha}\cdot U^{\alpha}\right)^2\right]^{\frac{1}{2}} \quad , \quad f_1(\varphi) = 1 - 2\varphi - \left(2 + 3\frac{m_0}{M_0}\right)\cdot\varphi^2 \quad , \quad H = \int_0^{p(\rho)}\frac{dp}{\rho(p)}$$

$$\nabla_{\alpha}\left(\rho\cdot U^{\alpha}\right) = 0 \quad , \quad m_0 << M_0$$

The field equation is:

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^{\alpha}}\left(\sqrt{-g}\cdot g^{\alpha\beta}\cdot\frac{\partial\varphi}{\partial x^{\beta}}\right)+K^{2}\cdot\varphi=\frac{4\pi\cdot G}{c^{4}}\cdot\rho_{E} \quad ,$$

$$\rho_{E}\approx\left[\rho\cdot c^{2}\cdot\left(1-\frac{1}{2}\varphi\right)+\left(\rho\cdot H-p\right)\right]+\left(7+6\frac{m_{0}}{M_{0}}\right)\cdot\left(\frac{1}{2}\rho\cdot c^{2}\cdot\varphi+\frac{K^{2}\cdot c^{4}}{4\pi\cdot G}\cdot\varphi^{2}-3p\right)$$

The motion equations are given by the Euler & Lagrange variational principle:

$$\delta\int\Lambda\,dS=0,\qquad -\frac{1}{m_{0}}\Lambda=c^{2}\cdot\left(\gamma_{\mu\nu}U^{\mu}U^{\nu}\right)^{\frac{1}{2}}+\left\{H+\left[\left(21+18\frac{m_{0}}{M_{0}}\right)\frac{\partial p}{\partial\rho}-H\right]\cdot\varphi\right\}\cdot\left(g_{\mu\nu}U^{\mu}U^{\nu}\right)^{\frac{1}{2}}$$

$$\gamma_{\mu\nu}=f_{1}(\varphi)\cdot g_{\mu\nu}-f_{2}(\varphi)\cdot\varphi_{,\mu}\cdot\varphi_{,\nu}$$

(In the static case , $f_{2}(\varphi)\rightarrow 4\frac{\varphi}{\varphi'^{2}_{r}}$)

Calculating the energy tensor, one obtains:

$$T_{\mu\nu}=\left(c^{2}\frac{f_{1}}{\Lambda}+H\right)\cdot\rho\cdot U_{\mu}U_{\nu}-p\cdot g_{\mu\nu}+\left\{\left[\left(22+18\frac{m_{0}}{M_{0}}\right)\cdot p-\left(21+18\frac{m_{0}}{M_{0}}\right)\cdot\rho\cdot\frac{\partial p}{\partial\rho}\right]\cdot g_{\mu o}+\right.$$
$$\left.+\left[\left(21+18\frac{m_{0}}{M_{0}}\right)\cdot\rho\cdot\frac{\partial p}{\partial\rho}-\rho\cdot H\right]\cdot U_{\mu}\cdot U_{\nu}\right\}\cdot\varphi+$$
$$+\frac{c^{4}}{4\pi\cdot G}\cdot\left\{\left(\varphi_{,\mu}\cdot\varphi_{,\nu}-\frac{1}{2}\cdot g_{\mu\nu}\cdot\varphi_{,\lambda}\cdot\varphi^{|\lambda}\right)+\frac{1}{2}\cdot K^{2}\cdot\varphi^{2}\cdot g_{\mu\nu}-\frac{1}{3}\left(7+6\frac{m_{0}}{M_{0}}\right)\cdot K^{2}\cdot\varphi^{3}\cdot g_{\mu\nu}\right\}$$

In this version, Seeliger's Theory of Gravitation may predict correctly both the conditions for the mechanical equilibrium of the Universe as a whole, and the relativistic one - body effects at the scale of the Solar system. The two-body problem is, at the same time, presumably correct .

The acceleration, in the one - body case, is given by the expression of a central attraction:

$$\vec{a}=\left\{1-\frac{\vec{\mathrm{v}}^{2}}{c^{2}}-2\frac{\left(\vec{\mathrm{v}}\cdot\vec{r}\right)^{2}}{c^{2}r^{2}}\right\}\cdot F'_{r}\cdot\frac{\vec{r}}{r}-4\cdot\frac{\left(\vec{\mathrm{v}}\times\vec{r}\right)^{2}}{c^{2}r^{2}}\cdot\left(\frac{1}{r}F\right)\cdot\frac{\vec{r}}{r} \quad ,$$

$$F(r)=G\cdot\frac{M_{0}}{r}\cdot e^{-Kr}$$

For $K\rightarrow 0$, the previous expression becomes:

$$\vec{a}=-\left(1-3\frac{\mathrm{v}^{2}}{c^{2}}\cos 2\theta\right)\cdot G\cdot\frac{M_{0}}{r^{3}}\cdot\vec{r} \quad , \quad \theta=\bullet\ \left(\vec{\mathrm{v}},\vec{r}\right)$$

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