Matter-antimatter asymmetry of the early Universe and some elementary considerations about the space-time properties

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Starting from the observed cosmological matter-antimatter asymmetry, some idea about the antiparticle concept in the view of the space-time properties and their consequence in the quantum gravity are discussed qualitatively. A possible scenario for the matter/antimatter generation and the asymmetry in the primordial Universe is suggested.

Keywords: matter-antimatter, universe, quantum gravity, antiparticles

1. Introduction

While electroweak and strong interactions propagate through space-time, gravity turns out to be a property of the space-time itself.

By the year 1974, quantum field theories have been developed to describe the weak, electromagnetic and strong forces, but there is still no accepted quantum theory for the remaining force, gravity. The underlying difference between gravitation and the other forces is that in the case of gravitation, the structure of space-time itself is to be quantified.

The combination of gravity, quantum theory and relativity, considered by their respective fundamental constants: G- Newtonian gravitational constant, \( \hbar \) -reduced Planck constant and the speed of light, gives rise to the Planck length \( L_p = 1.62 \times 10^{-35} \) m, the Planck energy \( E_p = 1.22 \times 10^{19} \) GeV and the Planck time:

\[ t_p = 5.31 \times 10^{-44} \text{ s} \]

The geometry of space comes from the quantum gravitational field. This simple statement has profound implications in that our belief that gravity could be turned into a quantum theory immediately implies that the structure of space-time has quantum fluctuations. Another way of rephrasing this concept is that space-time is expected to have a granular (or foamy) structure, where however the size of space-time cells fluctuates stochastically, thereby causing an intrinsic uncertainty in the measurements of space-time lengths, and indirectly of energy and momentum of a particle moving through space-time.

This talk is an essay about some aspects in the quantum gravity phenomenology. If we consider, in accord with Guang-jiong Ni and co-workers [1] that space-time reversal \( \vec{r} \rightarrow -\vec{r}, t \rightarrow -t \) is equivalent to the transformation between particle and antiparticle, thus, the solution proposed by Ahluwalia and Kirchbach [2] to explain the matter-antimatter asymmetry in the Universe must be modified. Qualitative considerations about implications for early Universe are given.

2. The matter-antimatter asymmetry problem

2.1 Preliminaries

At the present time, the Universe appears (from the observation) matter dominated, with little or no antimatter. For a detailed discussion, see for example reference [3]. The observations in cosmic rays gave a ratio antiprotons to protons \( \frac{\bar{p}}{p} \approx 10^{-4} \), and are entirely consistent with the flux coming from pair production. Furthermore, no characteristic \( \gamma \)-rays are seen in the sky which arise from proton – antiproton annihilations. If the Universe had islands of antimatter, one would expect such signals to be present.

In addition, there are also theoretical difficulties in assuming that the Universe was matter-antimatter symmetric in its late evolution. In this case, one can estimate the amount of matter that would remain after the proton and antiproton in the Universe go out of equilibrium, at around \( T \approx O(1 \text{ GeV}) \). Below this

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temperature, inverse annihilations \((2\gamma \rightarrow p + \bar{p})\) are blocked and the direct process \(p + \bar{p} \rightarrow 2\gamma\) considerably reduces the number of protons (and antiprotons) compared to that of photons to values of \(\eta \approx 10^{-18}\).

For these reasons, \(\eta = O(10^{-10})\), as observed, is evidence that there was some primordial baryon asymmetry. That is, really,

\[
\eta = \frac{(n_\beta - n_\bar{\beta})}{n_\gamma} \quad (1)
\]

Recent measurements obtained by the AMS experiment reported that the ratio antihelium –helium [4] is:

\[
\frac{\Phi_{\bar{\text{He}}}}{\Phi_{\text{He}}} < 1.7 \times 10^{-6} \quad (2)
\]

The AMS Collaboration detected no antihelium, or any \(Z > 2\) anti-nuclei.

### 2.2. The antiparticles and the space-time symmetry

When a particle is excited from the vacuum, it becomes observable. In an experiment, the momentum or energy is the existence form of a particle. In some sense, we would also say that the space-time is the existence form of the vacuum. The connection between the particle and the vacuum can be established by the Heisenberg uncertainty principle.

Every particle has its antiparticle.

Currently, there is nearly no clear explanation for the wave function for antiparticles. One often said that in the vacuum all the negative states (the sea) are filled. From excitation, a hole is created in the sea. Therefore, when an electron or other particle is created, the hole created in the sea corresponds to a positron, or generally an antiparticle.

In quantum mechanics, the momentum and energy operators for the particle are:

\[
\bar{p} = -i\hbar \nabla \quad \text{and} \quad E = i\hbar \frac{\partial}{\partial t}.
\]

If the Pauli metric is considered, note that after space-time inversion, the values of the components of the four momenta of particles remain unaltered, but for antiparticles:

\[
\bar{p}_c = i\hbar \nabla \quad \text{and} \quad E_c = -i\hbar \frac{\partial}{\partial t} \quad \text{respectively.}
\]

For example, the plane waves:

\[
\Psi(x,t) = \exp \left[ \frac{i(p\cdot x - Et)}{\hbar} \right] \quad (3a)
\]

and

\[
\Psi_c(x,t) = \exp \left[ -i\left(\bar{p}\cdot x - Et\right) \right] \quad (3b)
\]

describe a particle and an antiparticle with the same momentum and positive energy respectively. The difference between the particle and antiparticle appears in the phase of the wave function.
The existence of the difference of phase between the particle and the antiparticle is a key problem in the clarification of the antiparticle concept. This conclusion is also suggested by Ahluwalia and Kirchbach[2], in a more rigorous work.

Any particle must be described by a relativistic field. The corresponding equations can be expressed as two coupled Schrödinger relativistic equations in the \( \phi(\tilde{x}, t) \) and \( \chi(\tilde{x}, t) \) components – see reference [1] for details. Thus, a particle is not a pure state. The two contradicted fields, \( \phi(\tilde{x}, t) \) and \( \chi(\tilde{x}, t) \) can be associated with particle and antiparticle respectively.

For a particle, an electron for example, \( |\phi| > |\chi| \), \( \phi(\chi) \) characterising the electron determines the phase of space-time, i.e. \( \chi = \phi \approx \exp\left(-i \frac{Et}{\hbar}\right) \) for \( (E > 0) \). For a positron, the situation is just opposite, \( |\chi| > |\phi| \), with \( \phi = \chi \approx \exp\left(i \frac{Et}{\hbar}\right) \) if \( (E > 0) \).

The discovery of parity violation and CP violation whereas CPT theorem remains valid, imposes the nonconservation of time T inversion.

Performing the inversion of C, P and T independently on the wave function for a particle, an electron for example, one sees that the combined effect of CPT inversion is essentially ascribed to the transformation \( \tilde{x} \rightarrow -\tilde{x} \) and \( t \rightarrow -t \).

When performing a space-time inversion to an electron wave function, \( \phi(\tilde{x}, t) \rightarrow \phi(-\tilde{x}, -t) = \chi_c(\tilde{x}, t) \), there remains a large component, whereas \( \chi(\tilde{x}, t) \rightarrow \chi(-\tilde{x}, -t) = \phi_c(\tilde{x}, t) \) remains small. Hence, an electron changes into a positron. Therefore, in some sense we can say that an electron contains some ingredient of positron implicitly and coherently too. The space-time inversion is equivalent to the transformation between particle and antiparticle.

The photons are insensitive to the arrow of time.

Recent results on neutrinoless double beta decay, as reported by Klapdor-Kleingrothaus et. co-workers [5], which provided a first direct evidence of the phenomenon, if confirmed, would take us for the first time into the realm of Majorana space-time structure. In this context, Ahluwalia and Kirchbach [6] developed a detailed formalism for the description of the non-trivial space-time structure supposing the metamorphosis neutrino \( \leftrightarrow \) antineutrino could be possible.

Practically, any discrimination between particle and antiparticle is relative. Thus, an antiparticle is obtained after the transformation:

\[
|\tilde{a}\rangle = \text{CPT} |a\rangle
\]  

(4)

as was proposed by Lee and Wu [7].

The phase of the wave could be read as \( \frac{\tilde{p} \cdot \tilde{x} - Et}{\hbar} = (\tilde{k} \cdot \tilde{x} - \omega \tilde{t}) \), while the phase of internal clock (a stationary wave associated with the particle) will be

\[
-\frac{Et}{\hbar} = -\omega \tilde{t}'.
\]  

(5a)

The phase is an invariant in the sense of being constant in respect to the change of \( \tilde{v} \):

\[
(\tilde{p} \cdot \tilde{x} - Et) = -E \omega \tilde{t}'.
\]  

(5b)

In the Minkowski space, the four-dimensional vectors \( \left( \tilde{p}, \frac{iE}{c}, \tilde{x}, ic\tilde{t} \right) \) are invariant to the transformations of co-ordinates between two different reference systems.
With the enhancement of momentum $\vec{p}$ for the particle wave state, $\phi(\vec{x}, t)$, it seems slower and slower as magnitude and the $\chi(\vec{x}, t)$ wave component increases from zero gradually till a limit to $\vec{p} \rightarrow \infty$, corresponding to $v \rightarrow c$, when

$$\lim_{v \rightarrow c} |\chi| = |\phi|. \quad (6)$$

$\phi$ requires the sense of evolution of space-time in its phase, whereas $\chi$ requires the opposite one.

The $\chi$ component tries to hold the particle back; hence, the inertial mass of the particle increases without limits, and the moving internal clock tends to stop with the enhancement of component $\chi$ imposed by the conservation of the density of probability.

The space-time reversal $\vec{x} \rightarrow -\vec{x}$, $t \rightarrow -t$ is equivalent to the transformation between particle and antiparticle. This is a basic symmetry; thus, a particle is always not pure. A particle state $\phi(\vec{x}, t)$ is always accompanied by its antiparticle state $\chi(\vec{x}, t)$. They are connected by the simple relation:

$$\phi(-\vec{x}, -t) \rightarrow \chi(\vec{x}, t) \quad (7)$$

This is a hidden symmetry.

The coupled equations for the state should be invariant in respect to the transformation. Under the combined space-time inversion, all particles with mutual interactions turn to their antiparticles respectively.

Thus the symmetry is:

$$\Psi_c(\vec{x}, t) \rightarrow \Psi_c(-\vec{x}, -t) \equiv \Psi_c(\vec{x}, t) \equiv e^{i\chi} \Psi_i(\vec{x}, t) \quad (8)$$

and in this interpretation, the antimatter exists everywhere, but under suppressed state.

The demand of C, P and T covariances, separately, fixes the phase to be $\pm 1$. If these conditions are relaxed, a natural CP violating kinematic structure emerges.

In this context, until now, the neutral kaon system is unique and represents a special case. In the standard language, $K^0$ / $\bar{K}^0$ are essentially eigenstates of strangeness, but when they propagate in ordinary matter, the eigenstates of the hamiltonian are mixed as short- and long-lived kaons: $K_s$ / $K_L$, apparently the single system where the particle-antiparticle components are in equal proportions and direct observable. Also this is the unique system where the CP violation as well as the regeneration is experimentally observed. The production, behaviour and fate of neutral kaons in dense matter in collision between particles or large nuclei at very high energy represents a source of understandings of the space-time structure in the energy range considered.

### 2.3 Phenomenology of quantum gravity and cosmological implications

If the quantum mechanics is considered, thus, the space-time appears smooth on large scale. On small scales, however, it is bubbly and foamy due to quantum fluctuations.

In accord with Ng [8], the quantum fluctuations act on the metric and thus $g_{\mu\nu} \rightarrow \left( g_{\mu\nu} + \delta g_{\mu\nu} \right)$ where

$$\delta g_{\mu\nu} \geq \left( \frac{L_p}{R} \right)^{\alpha}$$

with $\alpha = 1$ or $\frac{2}{3}$ if the classical evaluation [10] or the holographic principle is used. The coupling of the metric (with quantum fluctuation) to the energy-momentum tensor of the particle, the fluctuation manifests on the tensor:

$$\left( g_{\mu\nu} + \delta g_{\mu\nu} \right)_{\mu
u} = g_{\mu\nu} \left( \epsilon_{\mu\nu} + \delta \epsilon_{\mu\nu} \right). \quad (9)$$
For $\alpha = \frac{2}{3}$ results $\delta R \geq (RL_p)^{\frac{1}{3}}$ and $\delta \nu \geq (m_p)^{\frac{1}{3}}$ respectively.

An important phenomenon appears as a quantum effect: for a scalar particle of mass $m$ moving slowly and satisfying the Schrodinger-Klein-Gordon, the uncertainty in metric induces a multiplicative phase factor in the wave-function:

$$\Psi \rightarrow e^{i\delta \phi} \Psi$$  \hspace{1cm} (10a)

where

$$\delta \phi = \frac{1}{\hbar} \int dt \int \delta g \delta g^{(0)} dt.$$  \hspace{1cm} (10b)

For consistency the integral should be fairly insensitive to the lower integration limit as long as $t >> t_p$.

In essence, this can be the phenomenon by which the antiparticles can be produced.

Antimatter exists under suppressed state. If, consequence of quantum fluctuation in the metric, the phase factor transforms the particle moving from forward direction in space-time in backward axis, thus, the particle becomes an antiparticle and the antimatter can be observable. The observed matter-antimatter asymmetry may reside in asymmetric space-time quantum fluctuations.

The primordial matter-antimatter asymmetry of the Universe could be understood in two possible scenarios:

a) Photonic birth: After the Big Bang, only photons were produced. Photons are insensitive to the arrow of time. After their interactions, for $t > 0$ the quantum fluctuations permits to generate matter ant the antimatter.

b) Matter birth: At $t = 0$, it is not possible to define a backward space-temporal fluctuation. Antimatter exists only in a suppressed state and the primordial Universe begins with a maximal matter-antimatter asymmetry. only matter exist, but at the later time antimatter can be also generated.

The holographic principle requires that the degrees of freedom of a spatial region reside not in the interior as in an ordinary quantum field theory but on the surface of the region. Furthermore it requires the number of degrees of freedom per unit area to be no greater than 1 per Planck area. As a consequence, the entropy of a region must not exceed its area in Planck units.

In a Universe described by the Robertson-Walker cosmology with space being flat, because the quantum fluctuations, it is necessary to suppose the anisotropic behaviour [9], thus, the metric will be:

$$ds^2 = dt^2 - \sum_i t_i^{2\gamma} dx_i^2$$  \hspace{1cm} (11)

where the number of spatial dimensions will be kept general so $i$ runs from 1 to $d$. For any direction the condition impose $p_i > \frac{1}{\sqrt{d}}$. The ratio between entropy/area, $(S/A)$, for this case is easily evaluated to be:

$$S/A = \prod_i R_{H,i} \left[ \prod_i t_i^{p_i} t^{1-p_i} \right]^{d-1/d} = t^{-\sum p_i} < 1$$  \hspace{1cm} (12)

where $R_{H,i} = t_i^{p_i}$ is the co-ordinate size of the horizon in direction $i$. The denominator in equation is the proper area of the horizon. The exponents satisfy the following equations: $\sum_i p_i = 1$ and $\sum_i p_i^2 = 1$. During a radiation dominated era $S/A = 10^{-28} \left[ \frac{t_d}{t} \right]^2$; $t_d$ s the time of decoupling.

For a spatial region characterised by a total volume $V$ and a total energy $E$, in accord with Sasakura [10], can be established the bound condition between the entropy associated and the number of bits that characterise the region. If the total spatial volume is partitioned in bits (each bit is labelled by an integer), thus:
$$V = \sum_k N \delta V_k,$$

where \(\delta V\) is the fundamental degree of freedom in quantum gravity, associated with a finite spatial volume and which can be determined by:

$$\delta V \equiv 24\pi G\hbar t.$$  \hspace{1cm} (14)

From the uncertainty inequality for energy, \(\delta E \geq \hbar / t\) and assuming also that the total energy is partitioned by the bits, then on obtains:

$$E = \sum_k \delta E_k \geq \sum_k \frac{\hbar}{t} \propto \sum_k \frac{G\hbar}{N} \delta V_k \geq G\hbar^2 N^2 / V.$$  \hspace{1cm} (15)

The quantity \(\sum_k \frac{G\hbar^2}{N} \delta V_k\) takes its minimum value when the volume of each bit takes the same value obtained as \(V / N\): 

Thus, for a given value for energy and volume, the maximum number of bits is

$$N \leq \sqrt{\frac{EV}{G\hbar^2}}.$$  \hspace{1cm} (16)

Corresponding to this region, the entropy (dimensionless) is:

$$S(E,V) \equiv 1.4 \sqrt{\frac{EV}{G\hbar^2}} \leq 1.4N.$$  \hspace{1cm} (17)

The length scale of a space-time bit can be estimated as: \(l_{bit} \propto (24\pi G\hbar t)^{1/3}\), and in accord with [3] can be expressed in terms of the mean energy per photon \(2.7kT\):

$$l_{bit} \propto (24\pi G\hbar t)^{1/3} \approx \left(\frac{248\pi G\hbar}{8^{1/2}E^2}\right)^{1/3}$$  \hspace{1cm} (18)

where \(E\) is expressed in MeV. In the early Universe \(g\) (a factor which takes into account the number of independent spin states) would have been around 50. Considering the age of the Universe about \(10^{10}\) years, the present length scale of a space-time bit is

$$l_{bit} \approx 10^{-14} \text{ m} \approx \left(10 \text{ MeV}\right)^{-1}$$  \hspace{1cm} (19)

the Universe radius being of the order \(10^{26} \text{ m}\).

We also have the following relation for the Universe radius (lower index is for the present time) at any time moment:

$$\frac{R}{R_0} \approx g^{1/4} \cdot 1.81 \cdot 10^{-10} l^{1/2}.$$  \hspace{1cm} (20)
Correctly, the uncertainty relation to determine the space-time fluctuations by which the antiparticles can be generated, must be applied to a four-dimensional volume. A possible solution was proposed by Sasakura [11].

**Summary**

In the paper some ideas about the antiparticle concept are discussed qualitatively in the view of the space-time properties, as well as their consequences in the quantum gravity. The combined space-time inversion is equivalent to particle-antiparticle transformation. This transformation is equivalent to a difference of phase of value ±1 in the wave function. The quantum fluctuations in the quantum gravity phenomenology suggest a way to produce the antiparticles and possible scenarios for the generation of asymmetry in the primordial Universe.

**References**


