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ENERGY LOSS AND STRAGGLING OF ELECTRONS

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Abstract. A computer program, written in FORTRAN, able to simulate the energy distribution of an initial monoenergetic electron beam up to 100 MeV incident energy was conceived. The mean energy loss by collisions and by bremsstrahlung emission is calculated for different media or mixtures by using very accurate theoretical formulas. The distribution function of the energy losses is determined within an approximation of the Landau's function and using the Bethe and Heitler formula for the radiative loss. The straggling distribution function is renormalized to reproduce the average value of the energy loss.

Key words: computer codes, energy distribution, energy losses, collisions Bremsstrahlung radiation, straggling of electrons.

1. INTRODUCTION

During the past several years, a wide number of applications for the bremsstrahlung radiation were developed, beginning with the production of neutron rich nuclei using the gamma-induced fission [1] process and finishing with the radiation therapy. A numerical code was conceived for modeling the bremsstrahlung emission from different targets [1] up to an incident electron energy of 100 MeV. In this program, only the average energy of the electron beam is determined in order to calculate the bremsstrahlung cross section, the distribution function being considered as a delta function centered in the average energy of the electron beam. The problem of the spectral distribution was disregarded. In the following, an estimation of the electron beam straggling is realised. In traversing a given thickness of the scattering medium, the electron undergo discrete energy loss. The variation in energy losses is referred as energy straggling.

2. ELECTRON SLOWING DOWN AND STRAGGLING

The slowing down of the electron is produced mainly by two effects: the inelastic collision and the bremsstrahlung production. The mean energy loss due to inelastic collisions is given by the following numerical formulas appropriate for low and high incident energies [2], respectively.

$$-\frac{\mathrm{d}\overline{E}_c}{\mathrm{d}x} = 0.306\rho\left(\frac{Z}{A}\right)\beta^{-2}\ln\left(1.16\frac{E_k}{I}\right) \qquad (1)$$

The Bohr expression for the mean energy loss reads

for β <0.5 and the available Bethe-Bloch expression in the relativistic region is

$$\frac{d\overline{E}_c}{dx} = 0.153\rho\frac{Z}{A}\beta^{-2}\left[\ln\frac{E_k(E_k+mc^2)^2\beta^2}{2I^2mc^2} + (1-\beta^2) - 2\sqrt{1-\beta^2} - 1 + \beta^2\frac{1}{8}(1-\sqrt{1-\beta^2})^2 - \delta\right] \qquad (2)$$

for $\beta \geq 0.5$. The mean energy loss is given in MeV/cm, the density ρ is considered in gcm^2, here E_k is the kinetic energy in MeV, mc^2=0.51 MeV, and I are the ionization potentials which are listed in the literature [3]. The parameter δ is known as a density effect. The semi-empirical form of δ originally proposed by
Sternheimer [4] is still used:

$$\delta = \begin{cases} 0 \quad \text{for} \quad X < X_0 \\ 4.606X + C + a(X_1 - X)^m \quad \text{for} \quad X_0 < X < X_1 \\ 4.606X + C \quad \text{for} \quad X_1 < X \end{cases} \qquad (3)$$

Where

$$X = \log_{10}\left(\frac{\beta}{(1-\beta^2)^{1/2}}\right) \tag{4}$$

is a velocity dependent parameter,

$$C = -2\ln\left(\frac{I}{\hbar\omega_p}\right) - 1 \tag{5}$$

is a material dependent parameter, with $\omega_p^2=4\pi Ne^2/m$ being the bulk plasma frequency. Numerically $\hbar\omega_p=30.47(Z\rho/A)^{1/2}$ in eV. The density ρ is expressed in g/cm^3, m=3,

$$a = -(C + 4.606X_0)/(X_1 - X_0)^m \tag{6}$$

$$X_0 = \begin{cases} 0.2 \quad \text{for} \quad I < 100 \text{ eV} \quad \text{and} \quad |C| < 3.681 \\ 0.326|C| - 1 \quad \text{for} \quad I < 100 \text{ eV} \quad \text{and} \quad |C| \geq 3.681 \\ 0.2 \quad \text{for} \quad I \geq 100 \text{ eV} \quad \text{and} \quad |C| < 5.215 \\ 0.326|C| - 1.5 \quad \text{for} \quad I \geq 100 \text{ eV} \quad \text{and} \quad |C| \geq 5.215 \end{cases} \tag{7}$$

$$X_1 = \begin{cases} 2 \quad \text{for} \quad I < 100 \text{ eV} \quad \text{and} \quad |C| < 3.681 \\ 3 \quad \text{for} \quad I \geq 100 \text{ eV} \quad \text{and} \quad |C| < 5.215 \end{cases} \tag{8}$$

Since the experimental systematic of bremsstrahlung spectra is not accurate, and the coverage is too sparse, one must rely on the theoretical results. The bremsstrahlung differential cross-section in photon energy formulas are obtained by using the prescriptions given by Koch and Motz [5]. The same forms were used in the electron slowing down evaluations of Ref. [6]:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}k}=\begin{cases} f_E \frac{\mathrm{d}\sigma^{3\mathrm{BNa}}}{\mathrm{d}k} & \text{for } 0.01\,\mathrm{MeV}< E_e < 0.1\,\mathrm{MeV} \text{ and } k \geq 0.01 T_0 \\ f_E \frac{\mathrm{d}\sigma^{3\mathrm{BN}}}{\mathrm{d}k} & \text{for } 0.1\,\mathrm{MeV}\leq E_e < 2\,\mathrm{MeV} \text{ and } k \geq 0.01 T_0 \\ \frac{\mathrm{d}\sigma^{3\mathrm{BN}}}{\mathrm{d}k} & \text{for } 2\,\mathrm{MeV}\leq E_e < 15\,\mathrm{MeV} \text{ and } \gamma \geq 15 \\ \frac{\mathrm{d}\sigma^{3\mathrm{BSd}}}{\mathrm{d}k} & \text{for } 2\,\mathrm{MeV}\leq E_e < 15\,\mathrm{MeV} \text{ and } 2<\gamma < 15 \\ \frac{\mathrm{d}\sigma^{3\mathrm{BSc}}}{\mathrm{d}k} & \text{for } 2\,\mathrm{MeV}\leq E_e < 15\,\mathrm{MeV} \text{ and } \gamma \leq 2 \\ \frac{\mathrm{d}\sigma^{3\mathrm{BN}}}{\mathrm{d}k} & \text{for } 15\,\mathrm{MeV}\leq E_e < 50\,\mathrm{MeV} \text{ and } \gamma \geq 15 \\ \frac{\mathrm{d}\sigma^{3\mathrm{BSd}}}{\mathrm{d}k} & \text{for } 15\,\mathrm{MeV}\leq E_e < 50\,\mathrm{MeV} \text{ and } 2<\gamma < 15 \\ \frac{\mathrm{d}\sigma^{3\mathrm{BSc}}}{\mathrm{d}k} & \text{for } 15\,\mathrm{MeV}\leq E_e < 50\,\mathrm{MeV} \text{ and } \gamma \leq 2 \\ \frac{\mathrm{d}\sigma^{3\mathrm{BN}}}{\mathrm{d}k} & \text{for } 50\,\mathrm{MeV}\leq E_e < 500\,\mathrm{MeV} \text{ and } \gamma \geq 15 \\ \frac{\mathrm{d}\sigma^{3\mathrm{CSa}}}{\mathrm{d}k} & \text{for } 50\,\mathrm{MeV}\leq E_e < 500\,\mathrm{MeV} \text{ and } 2<\gamma < 15 \\ \frac{\mathrm{d}\sigma^{3\mathrm{CSb}}}{\mathrm{d}k} & \text{for } 50\,\mathrm{MeV}\leq E_e < 500\,\mathrm{MeV} \text{ and } \gamma \leq 2 \end{cases} \quad (9)$$

Where

$$\frac{\mathrm{d}\sigma^{3\mathrm{BNa}}}{\mathrm{d}k}=\frac{Z^2 r_0^2}{137}\frac{16}{3}\frac{1}{p^2}\ln\left(\frac{p+p'}{p-p'}\right)\frac{1}{k} \quad (10)$$

$$\frac{\mathrm{d}\sigma^{3\mathrm{BN}}}{\mathrm{d}k}=\frac{Z^2 r_0^2}{137}\frac{p'}{p}\left[\frac{4}{3}-2EE'\left(\frac{p'^2+p^2}{p^2 p'^2}\right)+\frac{\mathrm{E}E'}{p^3}+\frac{\mathrm{E'}E}{p'^3}+\frac{\mathrm{E}\mathrm{E'}}{pp'}+\mathrm{L}\mathrm{U}\right]\frac{1}{k} \quad (11)$$

$$\mathrm{E}=\ln\frac{E+p}{E-p} \quad (12)$$

$$\mathrm{E'}=\ln\frac{E'+p'}{E'-p'} \quad (13)$$

$$L = 2\ln\left(\frac{EE'+pp'-1}{k}\right) \tag{14}$$

$$U = \frac{8EE'}{3pp'} + k^2\,\frac{E^2E'^2+p^2p'^2}{p^3p'^3} + \frac{k}{2pp'}\left[\left(\frac{EE'+p^2}{p^3}\right)\varepsilon - \left(\frac{EE'+p'^2}{p'^3}\right)\varepsilon' + \frac{2kEE'}{p'^2\,p^2}\right] \tag{15}$$

$$\frac{d\sigma^{3BSd}}{dk} = \frac{4Z^2r_0^2}{137}\left[1+\left(\frac{E'}{E}\right)^2 - \frac{2}{3}\frac{E'}{E}\right]\left[\ln\frac{2EE'}{k} - \frac{1}{2} - c(\gamma)\right]\frac{1}{k} \tag{16}$$

$$c(\gamma) = 0.102\exp(-0.151\gamma) + 0.47(-19.8\gamma) \tag{17}$$

$$\frac{d\sigma^{3CSa}}{dk} = \frac{4Z^2r_0^2}{137}\left\{\left[1+\left(\frac{E'}{E}\right)^2 - \frac{2E'}{3E}\right]\left[\ln\left(\frac{183}{Z^{1/3}}\right) - f(Z)\right] + \frac{E'}{9E}\right\}\frac{1}{k} \tag{18}$$

$$\phi_1(\gamma) = \phi_2(\gamma) + 0.5\exp(-2.31\gamma) + 0.12\exp(-19.8\gamma)$$

$$\phi_2(\gamma) = 20.14\exp(-0.151\gamma) \tag{19}$$

$$\frac{d\sigma^{3BSc}}{dk} = \frac{4Z^2r_0^2}{137}\left\{\left[1+\left(\frac{E'}{E}\right)^2\right]\left[\frac{\phi_1(\gamma)}{4} - \frac{\ln Z}{3}\right] - \frac{2E'}{3E}\left[\frac{\phi_2(\gamma)}{4} - \frac{\ln Z}{3}\right]\right\}\frac{1}{k} \tag{20}$$

$$f(Z) = \left[\frac{1}{1+(Z/137)^2} + 0.2020\right]\left(\frac{Z}{137}\right)^2 \tag{21}$$

$$\frac{d\sigma^{3CSc}}{dk} = \frac{4Z^2r_0^2}{137}\left\{\left[1+\left(\frac{E'}{E}\right)^2\right]\left[\frac{\phi_1(\gamma)}{4} - \frac{\ln Z}{3} - f(Z)\right] - \frac{2E'}{3E}\left[\frac{\phi_2(\gamma)}{4} - \frac{\ln Z}{3} - f(Z)\right]\right\} \tag{22}$$

In the previous relations, r_0 denotes the classical electron radius, E is the total electron energy in mc^2=0.51 MeV units, $E'=E-k$ is the final energy of the electron after the emission of a quanta of energy k, p and p' are the initial and final momentum of the electron in mc units:

$$\begin{aligned} p &= \sqrt{(E-1)(E+1)} \\ p' &= \sqrt{(E'-1)(E'+1)} \\ \beta &= \frac{p}{E} \end{aligned} \tag{23}$$

Also, A represents a corrective factor for the Born approximation obtained by
interpolating the data of Koch and Motz [7], f_E is the Elwert factor restricted to nonrelativistic electron energies:

$$f_E = \frac{\beta\{1+\exp[-2\pi Z/(137\beta)]\}}{\beta'\{1+\exp[-2\pi Z/(137\beta')]\}} \qquad (24)$$

and γ=100k(EE'Z$^{1/3}$)$^{-1}$ is a screening factor.

The mean energy loss due to the radiative process is

$$\frac{d\bar{E}_b}{dx} = \frac{6.02490\times10^{23}\, mc^2}{A}\int_0^{E_k} k\frac{d\sigma}{dk}dk \qquad (25)$$

so that the total energy loss is given by the relation

$$\frac{d\bar{E}}{dx} = \frac{d\bar{E}_c}{dx} + \frac{d\bar{E}_b}{dx} \qquad (26)$$

The average energy at the depth x is:

$$E_a = E_i - \int_0^x \frac{d\bar{E}}{dx}dx \qquad (27)$$

where E_i is the incident energy. Our numerical results agree within few percents with the previous published values [11].

The spectral distribution of the electron energy loss of fast electrons is ruled by two main Coulomb interactions: (i) collisions with atomic electrons, the so called ionization processes and (ii) collisions with atomic nuclei, which produces the bremsstrahlung radiation. The variation in energy loss due to both type of collisions is commonly referred as energy straggling. In the following, $f(E_i,E_f,x)$ refers to the energy loss straggling distribution in MeV^{-1} units. An electron initially with kinetic energy E_i, after passing through a target of thickness x, will have a probability $f(E_i,E_f,x)dE_f$ of being in the kinetic energy interval between E_f and E_f+dE_f. The energy loss straggling distribution derived by Landau is evaluated by taking into account only the energy loss due to ionization. Therefore, this distribution must be furthermore corrected to take into account the radiative processes. Within the approximation of the universal function appearing in the Landau theory reported in Ref. [7], the Landau distribution can be represented as follows:

$$f_{\rm ion}(E_i, E_f, x) = \frac{1}{\xi}\varphi(\lambda) \qquad (28)$$

where

$$\xi = \frac{2\pi\mu N r_0^2}{\beta^2}\frac{Z}{A} \qquad (29)$$

$$\lambda = \frac{1}{\xi}\left\{E_i - E_f - \xi\left[\ln\left(\frac{2\xi\mu\beta^2}{(1-\beta)^2 I^2}\right) - \beta^2 + 0.37 - \delta\right]\right\} \qquad (30)$$

$$\varphi(\lambda) = \begin{cases} \dfrac{a_0 + a_1(\lambda-\lambda_p)^2}{1 + b_1(\lambda-\lambda_p)^2 + b_2(\lambda-\lambda_p)^4}, & \lambda_1 \le \lambda < \lambda_p \\ \dfrac{c_0}{1 + d_1(\lambda-\lambda_p) + d_2(\lambda-\lambda_p)^2} & \lambda_p \le \lambda < \lambda \end{cases} \qquad (31)$$

Here N=6.0249x10^{23} is Avogadro's number, r_0=2.82x10-[11] the classical electron radius in cm, $\mu=m_0c^2$=0.51 MeV the electron rest energy, x is the depth in g/cm^2 (so that ξ is in MeV units), I is the ionization potential and δ the density effect. The parameters in the function $\varphi(\lambda)$ have the following values: λ_1=-2.9, $\lambda_2\to\infty$, λ_p=-0.2225, a_0=0.17904, a_1=-0.0253, b_1=0.0482, b_2=0.1132, c_0=0.17904, d_1=0.0134 and d_2=0.1355. This approximation reproduces the Landau formula within an absolute error of 2x10^{-3}.

The radiative energy loss straggling can be parametrised with the Bethe and Heitler formula. The form given in Ref. [8] was retained:

$$f_{\text{rad}}(E, E_f, x) = \frac{1}{(E_i+\mu)\Gamma(l)}\left(\ln\frac{E_i+\mu}{E_f+\mu}\right)^{l-1} \qquad (32)$$

where $l=x/(X_0\ln 2)$ is the thickness in units of radiation length X_0. The radiation length can be obtained with the parametrization found in Ref. [9] which gives values in g/cm^2:

$$X_0 = \frac{716.405A}{Z^2(L_r - f) + zL'_r} \qquad (33)$$

where $L_r=\ln(184.15Z^{-1/3})$, $L_r'=\ln(1194Z^{-2/3})$, $f=1.202z-1.0369z^2+1.008z^2/(1+z)$ and $z=(Z/137)^2$.

The total probability that an electron of the initial energy E_i reaches the energy E_f while traversing a material of thickness x is obtained by a folding procedure:

$$f(E_i, E_f, x) = \int_{E_f}^{E_i} f_{\text{ion}}(E_i, y, x)\, f_{\text{rad}}(y, E_f, x)\,\text{d}y \qquad (34)$$

This integral contains a singularity in the limit $y=E_f$ if l<1. To handle this problem in numerical computations, by following the prescriptions of Ref. [8], the folding integral is approximated with the expression:

$$f(E_i,E_f,x)=\int_{y_0}^{E_i} f_{\mathrm{ion}}(E_i,y,x)\,f_{\mathrm{rad}}(y,E_f,x)\mathrm{d}y+\mathrm{f}_{\mathrm{ion}}(E_i,E_f,x)\frac{\left[\ln\left(\frac{y_0+\mu}{E_f+\mu}\right)\right]}{\Gamma(l+1)} \qquad (35)$$

where $y_0=\varepsilon E_f$ and ε is a number slightly larger than one. In the present calculation ε=1.002. In principle, the radiative distribution is not normalized:

$$\int_0^{E_i} f(E_i,E_f,x)\mathrm{d}E_f = C \qquad (36)$$

$$\overline{E}=\frac{1}{C}\int_0^{E_i} E'\,f(E_i,E_f,x)\mathrm{d}E_f \qquad (37)$$

so that the mean value of the energy is obtained by performing the renormalization with the constant C

This average energy is derived within the Landau distribution function which assumes that the ionization losses are small compared to the initial energy. Therefore, this kind of simulation does not take into account the attenuation of the electron beam due to energy loss comparable to the electron energy before collisions. In these circumstances, the average energy of the beam will be larger than that obtained within accurate formalism of the electron slowing down. To reproduce the average energy obtained with the theory used in computing the electron slowing down and to take into account the attenuation of the beam, the function distribution is multiplied with the ratio $r=E_a/\bar{E}$, so that our distribution function becomes $rf(E_i,E_f,x)$.

To compute the slowing down and the bremsstrahlung cross section for mixtures, the effective values of the target parameters can be deduced using the Bragg rule [10] which stipulates that the stopping of the individual elements are approximately additive.

3. **RESULTS**

The dependence of the collision energy loss upon the layer thickness was investigated. The Fig. 1 shows the calculated electron kinetic energy for 30 MeV electrons travelling through various path lengths in carbon, beginning with zero up to the maximal range in steps of 0.5 cm. In the upper part, the Fig. 1(a), the spectral distribution is due to ionization processes only. The shape of the distribution is asymmetric around the maximum probable value of the energy electron, with a longer tail towards low energies. The increase of the path length leads to an increase of the width of the distributions. The Fig. 1(b) the folded distributions between those due to radiative and ionization processes are displayed. The radiative process has an visible effect on the shape of the spectral distribution increasing the number of lower energy electrons at the expense of the higher energy electrons. The asymmetry of the distribution function increases while the most probable energy decreases. These behaviors are more visible for larger travelled depths. Finally, in Fig. 1(c), the attenuation of the electron beam is taken into account by performing the renormalisation procedure with the mean energy obtained from the stopping power theories. An decrease of the overall distributions can be remarked for path lengths in the vicinity of the end of the electron range.

This computer code can be used for an accurate description of electron transport phenomena in matter, as needed for the bremsstrahlung modeling described in ref. [1,11].

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FIGURE CAPTION

FIG. 1 Spectral distributions $f(E_i,E_f,x)$ due to collision loss for 30 MeV electrons traversing various lengths in carbon are represented as function of the final kinetic energy E_f. The distributions are calculated for different absorber thicknesses in steps of 0.5 cm up to the range of the electrons. (a) The Landau ionization distribution. (b) The ionization distribution folded with the radiative distribution. (c) The total distribution normalised to take into account the attenuation.