DOUBLE-BETA DECAY, NUCLEAR PHYSICS AND NEUTRINO MASS

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Abstract

Recent results in the domain of the nuclear double-beta ($\beta\beta$) decay are presented. Topics include the potential of this process to search for beyond Standard Model (SM) physics, the calculation of the relevant nuclear matrix elements (NME) and the derivation of the neutrino mass both from $\beta\beta$ decay and from data provided by the neutrino oscillation and cosmological experiments. It is strengthened the key position that the nuclear $\beta\beta$ decay has in determining some fundamental properties of the neutrino like its nature (Dirac or Majorana) and its absolute mass.

1 Introduction

Nuclear double-beta decay is a natural decay of an even-even nucleus which transforms into another even-even nucleus with the same mass but with its charge changed by two units. This process occurs whenever the single-beta decay of the nucleus can not occur due to energetical resons or if it is highly forbidden by angular momentum selection rules. It is a very rare process with half-lives in the range of $10^{19} - 10^{24}$ y. The $\beta\beta$ decay process by which a nucleus increases its charge by two units is the most accesible experimentally (and thus the most studied) due to larger $Q_{\beta\beta}$ values and we will refer in this paper only to this process. Usually on speaks of the following decay modes:

$$a) \qquad (A,Z) \rightarrow (A,Z+2) + 2e^- + 2\bar{\nu}$$

$$b) \qquad (A,Z) \rightarrow (A,Z+2) + 2e^-$$

$$c) \qquad (A,Z) \rightarrow (A,Z+2) + 2e^- + \chi$$

The first process called $2\nu\beta\beta$ decay mode is allowed within the SM and so far there are experimental results for ten nuclei decaying through this mode. The other two decay modes are only predicted by beyond SM theories which allow for the lepton number violation. The second process (b) is called neutrinoless double-beta ($0\nu\beta\beta$) decay mode where one finds only two electrons in the final states, while (c) is another neutrinoless decay mode but with emission (besides the two electrons) of other particles, called Majoron. It was demostrated [1] that the occurence of (b) implies that neutrino is a Majorana particle with a non-zero mass. This is why, the observability of such a decay mode will be crucial for understanding the neutrino properties. Consequently, there is a great interest to study the $\beta\beta$ decay process both theoretically and experimentally. These eforts are well presented in several reviews like [2]-[7].

After the first experimental evidence of the $2\nu\beta\beta$ decay [8] there was a continous effort to refine the theoretical calculations in order to get agreement with the experimental half-lives. Since the calculation of the phase space factors entering the $\beta\beta$ decay half-lives was considered settled, the nuclear structure methods tried to adjust their model parameters according to the data in order to predict accurately other $2\nu\beta\beta$ decay half-lives. The agreement was obtained in the framework of the pnQRPA-type methods [9]-[16]. In parallel, there was a continous progress of the $\beta\beta$ decay experiments in attemps to get evidence for the $0\nu\beta\beta$ decay mode. The knowledge of the NME and phase factors from one side, and of experimental lower limits for the $0\nu\beta\beta$ decay half-lives on the other side, allow us to extract upper limits for the neutrino mass.

On the other side, there have been built other theoretical scenarious for $0\nu\beta\beta$ decay half-lives than the usual decay mode by exchange of Majorana neutrinos between two nucleons inside the nucleus. These revealed the broader potential of the $\beta\beta$ decay process to search for beyond SM physics [17]-[21]. In the last time other neutrino related experiments produced essential results in understanding neutrino properties. Neutrino oscillation experiments confirmed today that at least one flavor of neutrino has non-zero mass. In addition cosmological data coming from measurements of cosmic microwaves background provided us with very stringent limits for neutrino masses. Making use of these data and of the knowledge of NME one can get information about the observability of the $0\nu\beta\beta$ decay mode. This is today a crucial point since only $\beta\beta$ decay experiments will be able in the next future to provide us with information about the absolute mass of neutrino and about its the nature: Dirac or Majorana? This places the $\beta\beta$ decay in a key position among other experiments aiming to determine the neutrino properties.

The paper will be organized as follows: in section 2 I will discuss the broader potential of the $\beta\beta$ decay to search for beyond SM physics and Section 3 is devoted to the nuclear structure methods used to calculate the NME relevant for $\beta\beta$ decay. In section 4 I will refer to the neutrino mass problem in connection to the $\beta\beta$ decay and other ν-related experiments. Finally, section 5 will be dedicated to some conclusions and prospectives of the domain.

2 Connections to the beyond SM theories

The occurence of the $\beta\beta$ decay by exchange of Majorana neutrinos is the most usual scenarious. In this scenarious in the $0\nu\beta\beta$ decay half-lives there are also other contributions coming from RH components in the weak interaction. Combined studies of the $0\nu\beta\beta$ decay to excited states and of $\beta^+\beta^+$ decay modes may give us information about the possible existence of such currents. However, within other GUTs where the lepton number violation is allowed, one can imagine alternative possibilities for the occurence of this process. In this way the study of the nuclear $\beta\beta$ decay may provide us with information about the correctness of the hypothesis and put upper limits on the coupling constants and the masses of various exotic particles.

Within L-R theories $0\nu\beta\beta$ decay may be mediated by RH W bosons and the absence of such a decay mode give us lower limits for the mass of these bosons [18], [20].

In SUSY theories $0\nu\beta\beta$ decay appears via exchage of supersymmetric particles as: gluinos, photinos, neutralinos, etc. Since the conservation of the R-parity is made by an ad-hoc basis, the non-observability of this decay mode can be used for restricting R-parity violating SUSY models [17].

One can also imagine the occurence of the $0\nu\beta\beta$ decay as a spontaneous breaking of a global B-L symmetry due to a light mass or massless boson called Majoron which can couple to the neutrino [19].

Leptoquarks, bosons carrying both lepton and baryon quantum numbers, which appear in some GUTs can also mediate $0\nu\beta\beta$ decay and one can put restrictions on their mass.

Compositness, a possible substructure of quarks at a scale of $\sim$ TeV is another interesting hypothesis. The $0\nu\beta\beta$ decay may be a possible low energy manifestation of it, and the non-observability of this decay mode give us indications on the scale [21].

All these demonstrate the broader potential of the nuclear $\beta\beta$ decay to search for beyond SM physics than that retated to the nature and mass of the neutrino. These are additional arguments for studying this process both theoretically and experimentally.

3 Nuclear structure methods

In the theoretical estimations of the $\beta\beta$ decay half-lives the evaluation of the involved nuclear ME plays an important role in extracting the neutrino mass. However, their calculation represents one source of uncertainty. This is because the nuclei undergoing such a decay are generally medium- and heavy-mass open-shell nuclei with a rather complicate nuclear structure. In addition, the sensitive part is connected to an accurate description of the intermediate odd-odd nuclei participating in the $\beta\beta$ decay, which is still more complicate.

Generally, there are two types of nuclear structure methods used in $\beta\beta$ decay: a) QRPA- and b) Shell Model-based methods. Shell Model-based methods are very attractive since they are more precise. Unfortunately, calculations of the NME with these methods remain unfeasible for the majority of the nuclei decaying $\beta\beta$. Thus, there are a few calculations performed with such methods [34].

The QRPA-based methods [9]-[27] have been the most employed for computing NME for a wide class of nuclei.

The pnQRPA [9], a version of the usual QRPA but developed for charge-exchange proceses, was the first method widely used for calculations of nuclear charge-exchanging processes.

The phonon operators are defined:

$$\Gamma^{\dagger}_{1\mu}(k) = \sum_{l=(j_p,j_n)} \left[X^1_k(l)A^{+}_{1\mu}(l) + Y^1_k(l)\tilde{A}_{1\mu}(l)\right] \tag{3.1}$$

such that

$$\Gamma_{1\mu}(k)|0\rangle_{i,f} = 0$$

X and Y are the forward- and backward-going pnQRPA amplitudes and k labels the positive solutions of the pnQRPA equations. The A, $A^{\dagger}$ are the bi-fermion quasiparticle operators coupled to angular momentum J=1 and projection μ:

$$A^{\dagger}_{1\mu}(l) = \sum_{m_p,m_n} C^{j_p j_n 1}_{m_p m_n \mu} a^{\dagger}_{j_p m_p} a^{\dagger}_{j_n m_n} \ : \ \tilde{A}_{1\mu} = (-)^{1-\mu} A_{1,-\mu} \tag{3.2}$$

In the QBA the operators $A^{\dagger}$, A as well as the phonon operators $\Gamma^{\dagger}$, Γ fulfil the boson-type commutator relations.

In the formalism we also need the bi-fermion density-type operators:

$$B^{\dagger}_{1\mu}(l) = \sum_{m_p,m_n} C^{j_p j_n J}_{m_p -m_n \mu} a^{\dagger}_{j_p m_p} a_{j_n m_n} (-)^{j_n - m_n} \ ; \ \tilde{B}_{1\mu}(l) = (-)^{1-\mu} B_{1\mu}(l) \tag{3.3}$$

Using the equation of motion method one can derive the pnQRPA equations which may be written in the matrix representation as follows:

$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{B} & \mathcal{A} \end{pmatrix}_{J^\pi} \begin{pmatrix} X^m \\ Y^m \end{pmatrix} = \Omega^m_{J^\pi} \begin{pmatrix} \mathcal{U} & 0 \\ 0 & -\mathcal{U} \end{pmatrix}_{J^\pi} \begin{pmatrix} X^m \\ Y^m \end{pmatrix} \tag{3.4}$$

Solving this equation one gets the X and Y amplitudes as well as the pnQRPA energies $\Omega^m_{J^\pi}$.

Further, in the particle representation the transition operators of interest $\beta^{\pm}$ are defined as follows:

$$\beta^-_\mu(l) = \sum_{m_p m_n} \langle j_p m_p|\sigma_\mu|j_n m_n\rangle c^\dagger_{j_p m_p} c_{j_n m_n} \ ; \ \beta^+_\mu(l) = (-)^\mu \left(\beta^-_{-\mu}(l)\right)^\dagger , \tag{3.5}$$

were σ_μ denotes the μ-th spherical component of the Pauli spin operator. Their expressions in the quasiparticle representation read [6]:

$$\beta^-_\mu(l) = \theta_l A^\dagger_{1\mu}(l) + \bar{\theta}_l \tilde{A}_{1\mu}(l) + \eta_l B^\dagger_{1\mu}(l) + \bar{\eta}_l \tilde{B}_{1\mu}(l)$$

$$\beta^+_\mu(l) = -\left(\bar{\theta}_l A^\dagger_{1\mu}(l) + \theta_l \tilde{A}_{1\mu}(l) + \bar{\eta}_l B^\dagger_{1\mu}(l) + \eta_l \tilde{B}_{1\mu}(l)\right), \tag{3.6}$$

where the following notations are used:

$$\theta_l = \frac{\hat{j}_p}{\sqrt{3}}\langle j_p||\sigma||j_n\rangle U_p V_n;\ \bar{\theta}_l = \frac{\hat{j}_p}{\sqrt{3}}\langle j_p||\sigma||j_n\rangle U_n V_p;\ \hat{j} = \sqrt{2j+1}$$

$$\eta_l = \frac{\hat{j}_p}{\sqrt{3}}\langle j_p||\sigma||j_n\rangle U_p U_n;\ \bar{\eta}_l = \frac{\hat{j}_p}{\sqrt{3}}\langle j_p||\sigma||j_n\rangle V_p V_n \tag{3.7}$$

The NME relevant for the $2\nu\beta\beta$ decay mode within pnQRPA has the expression:

$$M^{2\nu}_{GT} = \sum_{l,k} \frac{\langle 0^+_f||\sigma\tau^-||1^+_k\rangle\langle 1^+_k|1^+_l\rangle\langle 1^+_k||\sigma\tau^-||0^+_i\rangle}{E_l + Q_{\beta\beta}/2 + m_e - E_0} \tag{3.8}$$

where the $|0^+_{i,f}\rangle$ are the ground states (g.s.) of the initial and final nuclei participating in the $\beta\beta$ decay, $|1^+_{k,l}\rangle$ and E_l are the intermediate states and their energies generated by two different pnQRPA procedures applied to the parent and the daughter nuclei and E_0 is the initial g.s. energy. .

The success of pnQRPA method in explaining the suppression mechanism of the $(2\nu\beta\beta)$ decay ME was achieved later on, by the inclusion of the particle-particle correlations [11]-[15]. However, this inclusion leads to a strong sensitivity of these NME on the particle-particle component of the pn residual interaction. Namely, the $2\nu\beta\beta$ decay ME as functions of the particle-particle interaction strength (usually denoted by g_{pp}) decrease rapidly and change sign, within a very narrow interval of values of g_{pp} and this causes difficulties for fixing this parameter adequately.

Trying to overcome this drawback several improvements of this method have been proposed during the last decade: the appropriate treatment of the particle-number non-conservation [23], the inclusion of the pn pairing [28]-[29], the double commutator method, [24], etc.. However, more successful have been the extensions of the pnQRPA beyond the quasi-boson approximation (QBA) developed in Refs. [22], [26], [30]-[31]. Their main achievement is that the ME become more stable against g_{pp} and the instability of the pnQRPA is shifted towards the region of unphysical values of this parameter.

The first method including higher-order corrections to pnQRPA was developed in Refs. [22], [25]. In this approach the pnQRPA phonon operator and the transition $\beta^{\pm}$ operators were expressed as boson expansions of appropriate pair operators and there were kept the next order terms from these series beyond the QBA:

$$A^\dagger_{1\mu}(pn) = \sum_k \left(A^{(1,0)}_{k_1}\Gamma^+_{1\mu}(k) + A^{(0,1)}_{k_1}\tilde{\Gamma}^+_{1\mu}(k)\right) \tag{3.9}$$

$$B^{\dagger}_{1\mu}(pn) = \sum_{k_1k_2} \left(B^{(2,0)}_{k_1k_2}(pn)[\Gamma^{\dagger}_1(k_1)\Gamma^{\dagger}_2(k_2)]_{1\mu} + B^{(0,2)}_{k_1k_2}(pn)[\Gamma_1(k_1)\Gamma_2(k_2)]_{1\mu} \right) \tag{3.10}$$

where

$$\Gamma^{\dagger}_{2\mu}(k') = \sum_{l'=(j_p,j'_p;j_n,j'_n)} \left[X^2_{k'}(l')A^{+}_{2\mu}(l') + Y^2_{k'}(l')\tilde{A}_{2\mu}(l') \right] \tag{3.11}$$

Retaining in the above series up to two-phonon states we got the version of this method called SQRPA.

The boson expansion coefficients $A^{(1,0)}$, $A^{(1,0)}$, $B^{(2,0)}$, $B^{(0,2)}$ are determined so that the equations (3.9)-(3.10) are also valid for the corresponding ME in the boson basis.

Then, SQRPA, has been employed, with some extensions, both for 2ν- and $0\nu\beta\beta$ decays, for a wide class of isotopes nuclei, as well as for transitions to excited states in Refs. [25]-[27].

An alternative approach for extending pnQRPA is based on the idea of partial restoration of the Pauli exclusion principle by taking into account the next terms in the commutator of the like-nucleon operators involved in the derivation of the QRPA equations:

$$\left[A_{\mu\nu}(k,l,J,M), A^{\dagger}_{\mu'\nu'}(k',l',J,M) \right] =$$
$$\mathcal{N}(k\mu,l\nu)\mathcal{N}(k'\mu',l'\nu') \left(\delta_{\mu\mu'}\delta_{\nu\nu'}\delta_{kk'}\delta_{ll'} - \delta_{\mu\nu'}\delta_{\nu\mu'}\delta_{lk'}\delta_{kl'}(-)^{j_k+j_l-J} \right) \tag{3.12}$$

Within the RQRPA method the above commutator is calculated more precisely by adding in the expression (3.12), besides the scalar term, the next terms which are just the proton and neutron number operators. The value of this commutator is replaced by its expectation values in the RPA g.s. Further, one observes that one can mimic the boson behavior of the A, $A^{\dagger}$ operators if one renormalizes them as follows [30], [31]:

$$\bar{A}_{\mu\mu'}(k,l,J,M) = D^{-1/2}_{\mu k\nu k' J^{\pi}}\ A_{\mu\mu'}(k,l,J,M) \tag{3.13}$$

where the $D_{\mu k\nu k'}$ matrices are defined as follows:

$$D_{\mu k\nu k' J^{\pi}} = \mathcal{N}(k\mu,l\nu)N(k'\mu',l'\nu') \left(\delta_{\mu\mu'}\delta_{\nu\nu'}\delta_{kk'}\delta_{ll'} - \delta_{\mu\nu'}\delta_{\nu\mu'}\delta_{lk'}\delta_{kl'}(-)^{j_k+j_l-J} \right)$$
$$\left[1 - j_l^{-1}\langle 0^{+}_{RPA}|[a^{\dagger}_{\nu l}a_{\nu l'}]_{00}|0^{+}_{RPA}\rangle - j_k^{-1}\langle 0^{+}_{RPA}|[a^{\dagger}_{\mu k}a_{\mu k'}]_{00}|0+_{RPA}\rangle \right] \tag{3.14}$$

By also renormalizing the QRPA amplitudes, $\mathcal{A}, \mathcal{B}$ matrices and the QRPA phonon operator :

$$\bar{X}^m = D^{1/2}X^m\ ;\ \bar{Y}^m = D^{1/2}Y^m\ ;\ \bar{\mathcal{A}}^m = D^{-1/2}\mathcal{A}D^{-1/2}\ ;\ \bar{\mathcal{B}}^m = D^{-1/2}\mathcal{B}D^{-1/2} \tag{3.15}$$

$$\Gamma^{m+}_{JM^{\pi}} = \sum_{k,l,\mu\le\mu'} \left[\bar{X}^m_{\mu\mu'}(k,l,J^{\pi})\bar{A}^{\dagger}_{\mu\mu'}(k,l,J,M) + \bar{Y}^m_{\mu\mu'}(k,l,J^{\pi})\tilde{\bar{A}}_{\mu\mu'}(k,l,J,M) \right] \tag{3.16}$$

one observes that the RQRPA equations have the same form as in the ordinary QRPA, but now the quantities of eq. (3.4) are replaced by the renormalized ones.

To calculate $\bar{\mathcal{A}}$ and $\bar{\mathcal{B}}$ we need to determine the renormalization matrices D. This is done by solving numerically a system of non-linear equations by an iterative procedure. As input values one can use their expressions in which the averages of the number operators are replaced by the backward-going amplitudes obtained as initial solutions of the QRPA equation.

Before starting the QRPA procedure, we need the occupation amplitudes (u, v) and the quasiparticle energies, in order to get the quasiparticle representation of the RPA operators.

This is done by solving the HFB equations, in which one may include, in the general case, both like- and unlike-nucleon pairing interactions. When one includes only like-nucleon pairing in these equations, the QRPA procedure described above was named pnRQRPA [30], [31]. Later on this method was extended by the inclusion of both types of pairing interaction [32] and this version was called full-RQRPA. In this work by the full-RQRPA method we also understand this extension.

However, one remarks that the RQRPA-type methods face an undesirable drawback namely, a significant degree of non-conservation of the Ikeda Sum Rule (ISR). Although refinements in the way of calculating the averages of the quasiparticle number operator are proposed [31] the result was a rather small reduction of the violation. Another challenging issue of this method is the dependence of the calculated ME on the size of the s.p. basis, especially for the neutrinoless mode.

Analyzing the calculations of the $\beta\beta$ decay ME existent in literature one still observes discrepancies between the values of the same ME, which may differ up to a factor three. On the other hand, it is difficult to compare results obtained with different versions of the QRPA-based methods and using different parameters and codes. In order to reduce such discrepancies and have an idea about the magnitude of the deviations between different calculations we made a study of both two-neutrino and neutrinoless $\beta\beta$ decay ME for the experimentally interesting nuclei ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, $^{128,130}Te$, and ^{136}Xe, with the pnQRPA, pnRQRPA, full-RQRPA and SQRPA methods [27]. For each method are used two different s.p. basis in order to see the dependence of the results on the size of the Hilbert space. For a better comparison between the results the calculations are performed for each method with the same set of parameters regarding all steps of the QRPA codes (the construction of the s.p. basis, of the BCS and QRPA wave functions, as well as the renormalization of the G-matrix elements).

In table 1 are presented the results obtained for the $0\nu\beta\beta$ NME for several isotopes, calculated with four QRPA-based methods and for the two basis sets, at the values of g_{pp} fixed, for each method, as it was explained previously. By inspection, one observes that in all cases the differences between the results obtained with the same method but with different basis are not so big around the physical values of g_{pp}. With a few exceptions one would say that, the uncertainty in the calculations of the neutrinoless ME performed with all these methods, coming from the use of different s.p. basis, is within a factor of two. The largest discrepancies between the results occur for pnQRPA and RQRPA and possible explanations for this may be related to the shortcomings of these methods which were already discussed previously. One also observes that the full-RQRPA and SQRPA values do not differ from each other by more that 50%. So, in the worst case one may have confidence in the calculated NME for the $0\nu\beta\beta$ decay mode within a factor two but, having in view that the SQRPA displays both a better stability against the change of the basis and a good fulfillment of the ISR, we may reduce the uncertainties in the predictions of these NME within QRPA-based methods (due to the uncertainties discussed above), to about 50%.

4 Neutrino mass

Until recently the most strigent limits for the neutrino mass were extracted from $\beta\beta$ decay calculations under the following procedure: the $0\nu\beta\beta$ decay half-lives are written in the usual factorize form (see for instance [6]):

$$\left[T^{0\nu}_{1/2}\right]^{-1} = C_{mm}\left(\frac{\langle m_\nu\rangle}{m_e}\right)^2 + C_{\lambda\lambda}\lambda^2 + C_{\eta\eta}\eta^2 + C_{m\lambda}\frac{\langle m_\nu\rangle}{m_e}\lambda + C_{m\eta}\frac{\langle m_\nu\rangle}{m_e}\eta + C_{\lambda\eta}\lambda\eta \tag{4.1}$$

Nucleus	pnQRPA	pnRQRPA	full-RQRPA	SQRPA
^{76}Ge	1.71(l) ; 4.45(s)	1.87(l) ; 3.74(s)	2.40(l) ; 3.68(s)	3.21(l) ; 3.82(s)
	0.84(l) ; 0.33(s)	0.79(l) ; 0.40(s)	0.62(l) ; 0.41(s)	0.46(l) ; 0.39(s)
^{82}Se	4.71(l) ; 5.60(s)	2.70(l) ; 4.30(s)	2.63(l) ; 4.15(s)	3.54(l) ; 4.13(s)
	6.75(l) ; 5.67(s)	11.71(l) ; 7.38(s)	12.04(l) ; 7.64(s)	8.96(l) ; 7.68(s)
^{96}Zr	2.75(l) ; 4.16(s)	2.72(l) ; 3.01(s)	2.42(l) ; 2.99(s)	2.12(l) ; 2.70(s)
	24.61(l) ; 16.27(s)	24.88(l) ; 22.48(s)	27.97(l) ; 22.64(s)	31.92(l) ; 25.06(s)
^{100}Mo	3.81(l) ; 5.37(s)	3.40(l) ; 4.36(s)	3.35(l) ; 4.11(s)	4.23(l) ; 4.51(s)
	1.75(l) ; 1.24(s)	1.96(l) ; 1.53(s)	1.63(l) ; 1.53(s)	1.58(l) ; 1.48(s)
^{116}Cd	2.85(l) ; 3.99(s)	3.39(l) ; 3.61(s)	2.35(l) ; 2.62(s)	2.29(l) ; 2.67(s)
	3.13(l) ; 2.24(s)	2.63(l) ; 2.47(s)	3.80(l) ; 3.41(s)	3.90(l) ; 3.34(s)
^{128}Te	3.43(l) ; 4.84(s)	2.83(l) ; 4.29(s)	2.85(l) ; 3.75(s)	2.85(l) ; 3.38(s)
	1.31(l) ; 0.94(s)	1.60(l) ; 1.05(s)	1.59(l) ; 1.21(s)	1.58(l) ; 1.34(s)
^{130}Te	3.77(l) ; 4.73(s)	3.00(l) ; 4.55(s)	2.61(l) ; 3.49(s)	2.42(l) ; 2.53(s)
	2.81(l) ; 2.24(s)	3.54(l) ; 2.33(s)	4.07(l) ; 3.04(s)	4.39(l) ; 4.19(s)
^{136}Xe	1.35(l) ; 1.69(s)	1.02(l) ; 1.57(s)	0.89(l) ; 0.99(s)	0.98(l) ; 1.03(s)
	2.17(l) ; 2.73(s)	3.6(l) ; 2.35(s)	4.14(l) ; 3.72(s)	3.76(l) ; 3.57(s)

Table 1: The neutrinoless matrix elements and upper limits for the neutrino mass parameter calculated in this paper with pnQRPA, pnRQRPA, full-RQRPA and SQRPA methods using the experimental limits given in Table 2. The numbers in the first row of each nucleus are the matrix elements, while the numbers in the second row are the the neutrino mass parameters. The calculations are performed with the large basis (l) or with the small (s) one.

where $\langle m_\nu \rangle$ is the effective neutrino mass and λ and η are parameters related to the possible existence of RH currents in the weak interaction.

$$\langle m_\nu \rangle = \Sigma_i |U_{ei}|^2 e^{\alpha_i} m_i \tag{4.2}$$

is an average over the neutrino mass eigenstates m_i and U_{ei} is the neutrino mixing matrix. The coefficients C_{ij} are products of phase space and combinations of NME. As an example C_{mm} of the term giving the leading contribution to $0\nu\beta\beta$ by the mechanisms of exchange Majorana neutrinos between two nucleons inside the nucleus has the expression:

$$C_{mm} = F_1^{0\nu} \left(M_{GT}^{0\nu} - \left(\frac{g_v}{g_A} \right)^2 M_F^{0\nu} \right)^2 \tag{4.3}$$

$F_1^{0\nu}$ is the phase-space integral and $M_{GT}^{0\nu}$ and $M_F^{0\nu}$ are Gamow-Teller (GT) and Fermi (F) NME. If one neglects the effects of right-handed weak currents (see e.g. [4]) the only term that remains in the (4.1) is the first one and the corresponding (simplified) expression for the $0\nu\beta\beta$ decay half-life can be used to put limits on the neutrino mass. Once we calculated the NME for $0\nu\beta\beta$ we used the best presently available half-lives for this decay mode and deduced upper limits for the neutrino mass parameter. The results are presented in the last column of the Table 1.

Alternatively, one can derive $< m_\nu >$ from ν-related experiments and compare the obtained values with those from $\beta\beta$ decay calculations. In this way one can do predictions on the observability of the $0\nu\beta\beta$ decay mode. Such a procedure became very employed after the recent claim for positive evidence of the $0\nu\beta\beta$ decay mode observed in Heidelberg-Moscow experiment [37].

Before doing this we remaind the main results reported recently by the neutrino oscillation experiments and from cosmological experiments.

¿From atmospheric ν-oscillation experiments the recent results of SuperKamiokande (SK) concerning the existence of an up-down asymmetry between muon events coming from the reactions [38]:

$$\pi \rightarrow \mu + \nu_{\mu}; \quad \mu \rightarrow \nu_{\mu} + \nu_{e} \tag{4.5}$$

strengthen the previous results of the same experiment (1996-2000) in favor of ν oscillations in the channel $\mu \rightarrow \tau$. This result was confirmed by the K2K reactor experiment [39]. The best fit for the ν oscillating parameters are found:

$$\Delta m^2 \sim 2.6 \times 10^{-3}; sin^2 2\theta \sim 1.0 \tag{4.6}$$

Also, solar neutrino experiments provide us with interesting results. The SNO experiment [41] has performed a carefull analysis of solar neutrino through the Charge-Charge, Neutral-Charge and Elastic-Scattering neutrino reactions.

The results obtained confirmed the SK results [40] and, a combined analysis of both experiments, allows to eliminate several possible scenarious of solar ν oscillations addmitted so far. Moreover, the results from the KamLand reactor experiment impose the LMA solution as the only valid, finding the best fit parameters [42]:

$$\Delta m^2 \sim 2.6 \times 10^{-5}; sin^2 2\theta \sim 1.0 \tag{4.7}$$

Thus, there are now compeling evidence that at least one flavor of neutrino has mass.

In parallel there was an impressive progress in cosmological data. Experiments aiming at measuring the Cosmic Microwave Background (CMB) gave, by combining their data (WMAP, 2dFGRS, CMI, etc.) very stringent limits for neutrino masses, namely [43]:

$$\Sigma_i m_i = 0.71 eV \tag{4.8}$$

To deduce the $< m_\nu >$ from all these recent data one proceeds as follows: i) write down the neutrino mixing matrix (U) in a convenient form; ii) choose a hierachy neutrino mass spectrum; iii) make use of ν oscillation data to fix as many parameters as possible from the U matrix; iv) determine $< m_\nu >$ as function od the remaining parameters and determine the range of its values such that to fit the present data.

According to the present data, doing that one obtains the following lower limits of the $< m_\nu >$:

i) Normal hierachy ($m_1 << m_2 < m_3$): $< m_\nu > \quad >$ several meV

ii) Inverse hierachy ($m_1 >> m_2 \sim m_3$): $< m_\nu > \quad >$ several tens of meV

Looking to this results and to the limits that future direct experiments propose to reach for $0\nu\beta\beta$ decay half-lives one can conclude the following: single beta decay experiments (the most ambitious beeing KATRIN) will be not able to reach such sensibilities. However the most ambitious $\beta\beta$ decay experiments will reach such values. This is an exciting results for these experiments which put them in a key position for determining the scale of the neutrino mass. In addition such experiments are the only able to say something about the nature of neutrinos: Dirac or Majorana?

Now, with these lower and upper limits for $< m_\nu >$ and using our calculated NME we also estimated time-scales for neutrinoless half-lives that experiments should reach to measure neutrino masses of about 0.1 eV. These results are presented in Table 2 toghether with the present limits reached by the corresponding experiments.

Nucleus	$T^{2\nu}$ $[yr]$	$T^{0\nu}$ $[yr]$	$T^{0\nu}_{0.1eV}$ $[yr]$
^{76}Ge	1.55×10^{21} [50]	$> 1.9 \times 10^{25}$ [50]	$2.83 - 4 \times 10^{26}$
^{82}Se	8.3×10^{19} [49]	$> 9.5 \times 10^{21}$ [49]	$5.6 - 7.6 \times 10^{25}$
^{96}Zr	2.1×10^{19} [49]	$> 1.0 \times 10^{21}$ [49]	$0.63 - 1.02 \times 10^{26}$
^{100}Mo	0.95×10^{19} [45]	$> 5.2 \times 10^{22}$ [45]	$1.14 - 1.29 \times 10^{25}$
^{116}Cd	2.6×10^{19} [51]	$> 0.7 \times 10^{23}$ [51]	$0.78 - 1.06 \times 10^{26}$
^{128}Te	7.7×10^{24} [44]	$> 7.7 \times 10^{24}$ [44]	$1.38 - 1.90 \times 10^{27}$
^{130}Te	2.6×10^{21} [47]	$> 5.6 \times 10^{22}$ [47]	$0.98 - 1.08 \times 10^{26}$
^{136}Xe	$> 3.6 \times 10^{20}$ [46]	$> 4.4 \times 10^{23}$ [46]	$5.63 - 6.22 \times 10^{26}$

Table 2: The integrated phase space factors $F_1^{2\nu}$ and $F_1^{0\nu}$ [25], the recent experimental 2ν- and 0ν- $\beta\beta$ decay half-lives and the estimated by the SQRPA $0\nu\beta\beta$ half-lives for $m_\nu = 0.1$ eV The experimental half-lives are taken from the references indicated in parenthesis.

5 Conclusions

We reviewed the recent results obtained from $\beta\beta$ decay researches in connection to the neutrino properties. First, we showed the broader potential of this process to provide us with information about beyond SM physics. Then, I presented the most employed nuclear structure methods for the computation of the NME relevant for $\beta\beta$ decay. These are the QRPA-based methods and their extensions beyond the QBA. The error in calculation the NME within these methods was estimated to $\sim 50\%$. Further, using our calculated values for the NME, we derived upper limits for the neutrino masses from experimental $0\nu\beta\beta$ decay half-lives. Alternatively, using data from ν-related experiments (ν-oscillations, cosmological data) and also using the calculated NME, one can predict lower and upper limits for the observability of $0\nu\beta\beta$ decay mode. It results that some planned future $\beta\beta$ decay experiments will be able to reach the range these predictions. This places the $\beta\beta$ decay experiments in a key position among the other ν-related experiments in determining fundamental properties of the neutrino like its nature (Dirac or Majorana) and its absolute mass.

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