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TWO-DIMENSIONAL TIME DEPENDENT CALCULATIONS FOR THE TRAINING REACTOR OF BUDAPEST UNIVERSITY OF TECHNOLOGY AND ECONOMICS

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Abstract. An iterative method was developed for the numerical solution of the coupled two-dimensional time dependent multigroup diffusion equation and delayed precursor equations. Both forward (Explicit) and backward (Implicit) schemes were used. The second scheme was found to be numerically stable, while the first scheme requires that $\Delta t < 10^{-10}$ sec. for stability. An example is given for the second method.

Key words: Training reactor, neutron diffusion, finite difference.

INTRODUCTION

Numerical solutions of the coupled time-dependent transport and precursor equations for reactor kinetics problems of practical interest are prohibitively difficult, so approximate methods are employed [1]. The derivation of the diffusion equation from the continuous energy transport equation is described in detail in a number of references, including Henry [2] and Duderstadt and Hamilton [3]. This paper is concerned with the most common approximation to the time dependent transport equation – the time dependent group diffusion equations. Direct finite difference methods [4] are the most straightforward approach to the accurate solution of the space time kinetics problem, and these methods are characterized by fairly definite error bounds [5].

The primary advantage of finite difference methods is that the complex nonlinear partial differential equations can be expressed in difference form to obtain algebraic expressions. We are concerned in this paper with the forward (explicit) method and the backward (implicit) method.

In the present article, we have developed a two-dimensional space-time dependent neutron diffusion code for simulating the power excursion accidents.

The process involves some thermal-hydraulic feedback through the heat transfer from the fuel to the water used as coolant, condensation and evaporation which take place in the reactor core and in the condenser system. These feedbacks play an important role in the reactivity transients of nuclear power reactors. However, we consider here only the neutron-physical aspects of the transient.

The current work developed two codes for solving the space-time dependent multigroup diffusion equations using the forward (explicit) and the backward (implicit) methods. Both codes were designed for the Training Reactor of the Budapest University of Technology and Economics.

REACTOR DESCRIPTION

In this study the training and research reactor of the Budapest University of Technology and Economics [6] was selected as a reference reactor. The Training Reactor became critical in 1971. Its thermal power is 100 kW. The cylinder-shaped tank is 1.4 m in diameter and filled with desalted water. The coolant level is 5750 mm. The reactor core is made up of 24 fuel assemblies, which together contain 369 pieces of its fuel is EK-10 type fuel rods with an active length of 500 mm. The fuel is 10% enriched uranium dioxide in magnesium matrix. Each fuel element contains 16 fuel rods arranged in a four by four square lattice of 17.5 mm pitch. The reactor core is cooled by the buoyant flow of the coolant below 10 kW power. Over 10 kW the coolant loop is in process at a volumetric flow 5.8 m^3/h. The reflector is graphite [7].

THE MULTIGROUP NEUTRON DIFFUSION EQUATIONS

The multigroup neutron diffusion theory approximation for the space and time dependence of the neutron flux within a nuclear reactor is described by the set of G equations:

$$\frac{1}{v_g}\frac{\partial \phi_g(\mathbf{r},t)}{\partial t} = \operatorname{div}[D_g(\mathbf{r})\nabla\phi_g(\mathbf{r},t)] - [\Sigma_g^a(\mathbf{r}) + \Sigma_g^R(\mathbf{r})]\phi_g(\mathbf{r},t) + \\ + \sum_{g'\neq g}\Sigma_{g'\to g}(\mathbf{r})\phi_{g'}(\mathbf{r},t) + (1\text{-}\beta)\, f_g \sum_{g'=1}^{G}\nu\Sigma_{g'}^{\mathrm{f}}(\mathbf{r})\phi_{g'}(\mathbf{r},t) + \\ + \sum_{i=1}^{M}\lambda_i f_{ig} C_i(\mathbf{r},t), \qquad (g=1,\ldots,G) \tag{1}$$

The space and time dependence of the M groups of the delayed neutron precursors are described by:

$$\frac{\partial C_i(\mathbf{r},t)}{\partial t} = \beta_i \sum_{g=1}^{G} \nu\Sigma_g^{\mathrm{f}}(\mathbf{r})\, \phi_g(\mathbf{r},t) - \lambda_i C_i(\mathbf{r},\mathrm{t}), \qquad (i=1,\ldots,M) \tag{2}$$

where,

$\phi_g(\mathbf{r},t)$ = neutron flux in group g, at point $\mathbf{r}$ and time t (n/cm^2 sec)

$C_i(\mathbf{r},t)$ = delayed neutron precursor concentration of type i at point $\mathbf{r}$ and time t (nuclei/cm^3)

$D_g(\mathbf{r})$ = diffusion coefficient in group g (cm)

λ_i = decay constant for delayed neutron precursor type I (sec^{-1})

$\Sigma_{g'\to g}(\mathbf{r})$ = macroscopic slowing down cross-section from group g' to group g (cm^{-1})

$\Sigma_g^{\mathrm{R}}(\mathbf{r})$ = macroscopic removal cross-section in group g (cm^{-1})

f_g = prompt neutron fission spectrum fraction in group g

f_{ig} = fission spectrum fraction for delayed neutron precursor type i

β_i = delayed neutron fraction for precursor type i

Despite the approximations that have gone into the derivation of the previous equations (1–2), we will assume that they adequately describe the time-dependent behavior of the neutron flux in nuclear reactors. The system of equations (1–2) is imposed to the free boundary condition

$$\phi_g(\mathbf{r_B},t) = 0 \qquad \forall\ \mathbf{r_B} \tag{3}$$

and the initial condition

$$\phi_g(\mathbf{r},0) = \phi_o(\mathbf{r}) \tag{4}$$

$$C_i(\mathbf{r},0) = C_{io}(\mathbf{r}) \tag{5}$$

for notational convenience we shall write equations (1–2) in operator notation as

$$\frac{\mathrm{d}\mathbf{F}(\mathbf{r},t)}{\mathrm{d}t} = [\mathbf{V}(\hat{\mathbf{P}} - \hat{\mathbf{D}})(\mathbf{r})]\, \mathbf{F}(\mathbf{r},t) \tag{6}$$

where,

$$\mathbf{V} = \mathrm{diag}\left(v_1,\ldots,v_G\right) \tag{7}$$

is the group velocity matrix

$$\hat{\mathbf{P}}\begin{bmatrix}\mathbf{\Phi}\\ \mathbf{C_i}\end{bmatrix} = (1\text{-}\beta)\, f_g \sum_{g'=1}^{G} \nu\Sigma_{g'}^{f}(\mathbf{r})\phi_{g'}(\mathbf{r},t) \;+\; \sum_{i=1}^{M}\lambda_i f_{ig} C_i(\mathbf{r},t) \tag{8}$$

is the production operator

$$\hat{\mathbf{D}} = \mathrm{div}[D_g(\mathbf{r})\nabla\phi_g(\mathbf{r},t)] \;-[\,\Sigma_g^a(\mathbf{r}) + \Sigma_g^R(\mathbf{r})]\phi_g(\mathbf{r},t) \;+\; \sum_{g'\neq g}\Sigma_{g'\to g}(\mathbf{r})\phi_g(\mathbf{r},t) \tag{9}$$

is the destruction (loss) operator and

$$\mathbf{F} = \mathrm{col}[\phi_1,\ldots,\phi_G] \tag{10}$$

is the flux vector.

$$\mathbf{C}_i = \mathrm{col}[C_1,\ldots,C_M] \tag{11}$$

is the delayed neutron precursor vector.

SOLVING EQUATION (6)

Forward difference method

In view of equation (6) the forward finite difference algorithm which is the simplest approximate solution to equation (6) takes the form

$$\mathbf{F}_{p+1} = [\mathbf{I} + \Delta t\ \mathbf{V}(\hat{\mathbf{P}} - \hat{\mathbf{D}})]\ \mathbf{F}_p \tag{12}$$

where the argument p denotes the value at time t_p, and $\Delta t = t_{p+1} - t_p$. Let us now write equations (1) and (2) in the form of equation (12) with dropping the spatial dependence for simplicity:

$$\begin{aligned}\phi_{g,p+1} = \phi_{gp} + \mathrm{v}_g \Big\{ &\mathrm{div}\, D_g\nabla\phi_{gp} - [\Sigma_g^a + \Sigma_g^R]\,\phi_{gp} + \\ &+ \sum_{g'\neq g}\Sigma_{g'\to g}\phi_{g'p} + (1\text{-}\beta)\, f_g \sum_{g'=1}^{G}\nu\Sigma_{g'}^{f}\phi_{g'p} + \\ &+ \sum_{i=1}^{M}\lambda_i f_{ig} C_{ip}\Big\}\,\Delta t, \qquad (g = 1,\ldots,G)\end{aligned} \tag{13}$$

Also for the precursor (see equation 11)

$$C_{i,p+1} = C_{ip} + \left[\beta_i \sum_{g=1}^{G}\nu\Sigma_g^f\ \phi_{gp} - \lambda_i C_{ip}\right]\Delta t, \qquad (i = 1,\ldots,M) \tag{14}$$

A code has been developed realizing this technique in two-dimensional geometry, a typical finite difference expressions for all terms of equations (13) and (14) was calculated using the five point scheme and integrating over spatial coordinates. Initial guess for the flux, fission source and precursor concentrations at $t = 0$ is calculated from a static two dimensional neutron diffusion code. If the time dependent code is applied to the critical reactor with this initial values, the solution should be constant in time. This was used as a test of the program. The precursor concentrations C_{ip} are updated at the end of each time step by means of equation (14). Also the fission source is updated after the calculation step of the precursor concentration were done.

The algorithm is simple but unfortunately it suffers from a numerical stability problem, which requires using a small time steps. That is because the magnitude of the highest eigenvalue is too large. Thus $\Delta t < 10^{-10}$ sec is required for stability which is time consuming; consequently, this method is not preferable to use due to this reason.

Iterative process for the backward-difference method

The backward algorithm can eliminate the numerical stability problem associated with the proceeding method, it is simply written instead of equation (12)

$$\mathbf{F}_{p+1} = [\mathbf{I} - \Delta t\ \mathbf{V}(\hat{\mathbf{P}} - \hat{\mathbf{D}})]^{-1}\mathbf{F}_{p+1} \tag{15}$$

The flux at t_{p+1} is obtained by the following iterative formula

$$\mathbf{F}_{p+1} = [\mathbf{I} - \Delta t\ \mathbf{V}(\hat{\mathbf{P}} - \hat{\mathbf{D}})]^{-1}\mathbf{F}_{p} \tag{16}$$

as an approximation of equation (15).

It can be shown that this scheme is stable for any Δt. We have to pay for this stability by computation of the inverse the coefficient matrix for the right-hand side of equation (16) which requires an iteration.

The iteration procedure of equation (16) is basically the same as that used to solve the static source problem. We perform an outer iteration for the source term $\hat{\mathbf{P}}\mathbf{F}$ and an inner iteration for the other terms. An iterative backward difference two dimensional code is developed. Initial guess for the flux, fission source and the delayed neutron precursor from the above mentioned neutron diffusion static code can be made.

The method has been found unconditionally stable. A larger time step can be taken for this implicit method than for the explicit method. But it should be noted that the size of the time step used in the method is usually limited by the effect of the truncation error of the order Δt^2 upon the accuracy of the solution rather than by numerical stability. The method shows a higher number of outer iterations only for the first time steps and a few numbers of outer iterations (from 2 to 7) is needed for the successive outer iterations.

The developed code is a cornerstone step for constructing further developments to cover both three dimensional and thermal hydraulics feedback calculations for different accidental situations.

NUMERICAL RESULTS

Data for both the forward and backward codes

The following Tables [1–3] show the different precursor constants, group constants, and material properties, respectively, which were used in the codes with $G = 4$ and $M = 6$.

Table 1

Precursor constants

Family index	1	2	3	4	5	6
$\beta_i \cdot 10^5$	21	142	128	257	75	27
$T_{1/2}$ (sec)	55.7	22.7	6.2	2.3	0.615	0.23

Table 2

Group constants

Group number	1	2	3	4
$1/v_g$	5.3×10^{-10}	2.7×10^{-9}	1.8×10^{-7}	4.5×10^{-6}
f_g	0.7655	0.2345	0	0

Table 3

Material properties

Material 1 (Reflector)				
	Group 1	Group 2	Group 3	Group 4
D	2.24921	0.94578	0.590007	0.146567
Σ_a	5.56486×10^{-4}	1.14107×10^{-5}	1.15532×10^{-3}	1.85842×10^{-2}
Σ_R	0.11082	0.16831	0.151362	0
$\nu\Sigma_f$	0	0	0	0
Material 2 (Core)				
	Group 1	Group 2	Group 3	Group 4
D	1.84809	0.967891	0.657195	0.260954
Σ_a	4.00512×10^{-3}	2.45935×10^{-3}	2.65997×10^{-2}	1.20460×10^{-1}
Σ_R	0.0783021	0.0786561	0.0681291	0
$\nu\Sigma_f$	8.84515×10^{-3}	1.09286×10^{-3}	0.0154743	0.207364

Example

Configuration of the example

Material 1	Material 2	Material 1
0 (cm)	60	81.6	151.6

A 32 × 33 mesh interval was used with different mesh spacing. The program was first tested for the critical state, the solution was constant in time as expected. We show an example which simulates a slowly diverging super critical reactor, the reactor was critical for $\Delta t < 0$. In the moment $\Delta t = 0$, the first group absorption cross section in the region 2 was perturbed uniformly and decreased by 6% and had a reactivity about 20 cents. The Backward code was used with $\Delta t = 10^{-4}$, the computed thermal fluxes of a core mesh line 11 are given in Fig. 1. In the same transient, the flux of the 1st group of a certain core node was drawn *versus* time for a total operating time of 50 sec (see Fig. 2). The doubling time of the flux is 12 sec which agrees with the theoretical calculations.

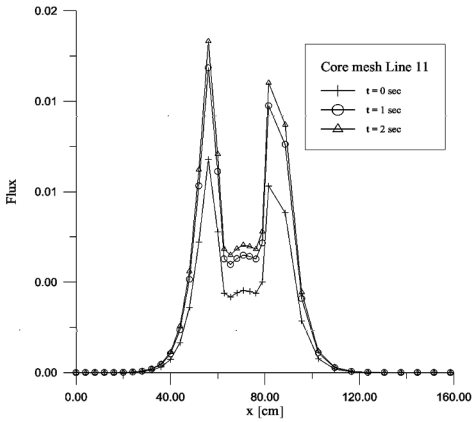

Fig. 1 – Variation of the thermal group neutron fluxes with space.

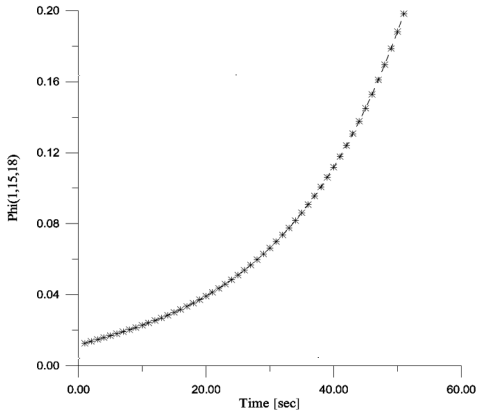

Fig. 2 – The flux for a certain core mesh $\phi(1, 15, 18)$ *versus* time.

CONCLUSION

After some preliminary tests we have seen that only the backward difference scheme is applicable for handling the time dependence of this kind of process. The backward method has been found unconditionally stable. A larger time step can be taken for this implicit method than for the explicit method. As to the spatial dependence, we have chosen the usual finite difference approximation. The code

can be applied to simulate fast transients such as those resulting from the ejection of the control rode.

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