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CONSISTENT INTERACTIONS BETWEEN BF AND MASSIVE DIRAC FIELDS. A COHOMOLOGICAL APPROACH

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Abstract. Consistent interactions for a four-dimensional gauge theory, described in the free limit by an abelian BF-type model and one massive Dirac field, are approached in the framework of the deformation theory based on the local BRST cohomology. The deformation procedure leads to an interacting model with modified gauge transformations, whose algebra is open.

Key words: consistent interactions, local BRST cohomology, BF theories.

1. INTRODUCTION

A key point in the development of the BRST formalism was its cohomological understanding, which allowed, among others, a useful investigation of many interesting aspects related to the perturbative renormalization problem [1–4], the anomaly-tracking mechanism [4–8], the simultaneous study of the local and rigid invariances of a given theory [9], as well as to the reformulation of the construction of consistent interactions in gauge theories [10–13] in terms of the deformation theory [14–16], or, actually, in terms of the deformation of the solution to the master equation.

The scope of this paper is to investigate the consistent interactions that can be added to a free, abelian, four-dimensional gauge theory, describing a single massive Dirac field and a BF-type model [17] involving one scalar field, two types of one-forms and one two-form. This work enhances the previous Lagrangian [18] and Hamiltonian [19–20] results on the study of self-interactions in certain classes of BF-type models. The resulting interactions are accurately described by a gauge theory with an open algebra of gauge transformations, which are second-order reducible, like in the case of the free model, but the new reducibility relations take place on-shell. It is also worth noting that the Dirac field gains non-trivial gauge transformations.

Our strategy goes as follows. Initially, we determine in Section 2 the antifield-BRST symmetry of the free model, that splits as the sum between the Koszul-Tate differential and the exterior derivative along the gauge orbits, $s=\delta+\gamma$. Next, in Section 3 we determine the consistent deformations of the solution to the master equation for the free model. The first-order deformation belongs to the local cohomology $H^0(s|d)$, where d is the exterior space-time derivative. The computation of the cohomological space $H^0(s|d)$ proceeds by expanding the co-cycles according to the antighost number, and by further using the cohomological groups $H(\gamma)$ and $H(\delta|d)$. We find that the first-order deformation is parametrized by three kinds of functions depending on the undifferentiated scalar field, which become restricted to fulfill certain equations in order to produce a consistent second-order deformation. We select only those solutions that ensure effective cross-couplings between the BF field spectrum and the Dirac field, and consequently infer that the remaining deformations, of order two and higher, can be taken to vanish. The identification of the interacting model is developed in Section 4, where it turns out that the cross-interactions between the Dirac field and the BF field spectrum are described by a generalized minimal coupling of the type current-vector field in an arbitrary 'background' of the scalar field. The role of the vector field is played by the one-form from the BF theory displaying an abelian $U(1)$ gauge symmetry, while the above mentioned current is associated with the conservation law of the Dirac theory corresponding to a special class of one-parameter rigid symmetries that involve an arbitrary function of the scalar field. Meanwhile, both the gauge transformations corresponding to the coupled model and their algebra are deformed with respect to the initial abelian theory in such a way that the new gauge algebra becomes open and the reducibility relations take place on-shell. Section 5 closes the paper with the main conclusions.

2. FREE MODEL. BRST SYMMETRY

The starting point is represented by the free Lagrangian action

$$\begin{aligned} S_0\left[A^\mu, H^\mu, \varphi, B_{\mu\nu}, \psi^\alpha, \bar{\psi}_\alpha\right] = \int \mathrm{d}^4x \Big(& H^\mu \partial_\mu \varphi + \frac{1}{2} B^{\mu\nu} F_{\mu\nu} + \\ & + \bar{\psi}_\alpha \left(i \left(\gamma^\mu\right)^\alpha{}_\beta \partial_\mu - m \delta^\alpha{}_\beta \right) \psi^\beta \Big), \end{aligned} \tag{1}$$

where we employed the notation $F_{\alpha\beta}=\partial_{[\alpha}A_{\beta]}$ for the field strength of the one-form A_μ. We observe that (1) is written as a sum between the action of a four-dimensional abelian BF theory (involving two one-forms, one scalar field and one two-form) and the action corresponding to a single massive Dirac field. The free action (1) is found invariant under the gauge transformations

$$\delta_\varepsilon A^\mu = \partial^\mu \varepsilon, \quad \delta_\varepsilon H^\mu = 2\partial_\nu \varepsilon^{\mu\nu}, \quad \delta_\varepsilon B^{\mu\nu} = -3\partial_\rho \varepsilon^{\mu\nu\rho}, \tag{2}$$

$$\delta_\varepsilon \varphi = 0, \quad \delta_\varepsilon \psi^\alpha = 0, \quad \delta_\varepsilon \bar{\psi}_\alpha = 0, \tag{3}$$

where the gauge parameters ε, $\varepsilon^{\mu\nu}$ and $\varepsilon^{\mu\nu\rho}$ are bosonic, with $\varepsilon^{\mu\nu}$ and $\varepsilon^{\mu\nu\rho}$ completely antisymmetric. The gauge transformations (2–3), are abelian and off-shell second order reducibile. More precisely, we observe that if in (2) we make the transformations

$$\varepsilon^{\mu\nu} \to \varepsilon^{\mu\nu}_{\theta} = -3\partial_\rho \theta^{\mu\nu\rho}, \tag{4}$$

$$\varepsilon^{\mu\nu\rho} \to \varepsilon^{\mu\nu\rho}_{(\theta)} = 4\partial_\lambda \theta^{\mu\nu\rho\lambda}, \tag{5}$$

with $\theta^{\mu\nu\rho}$ and $\theta^{\mu\nu\rho\lambda}$ arbitrary antisymmetric fields, the gauge transformations of the fields H^μ and $B^{\mu\nu}$ identically vanishes

$$\delta_{\varepsilon_{(\theta)}} H^\mu \equiv 0, \, \delta_{\varepsilon_{(\theta)}} B^{\mu\nu} \equiv 0. \tag{6}$$

Moreover, if in (4) we perform the changes

$$\theta^{\mu\nu\rho} \to \theta^{\mu\nu\rho}_{(\omega)} = 4\partial_\lambda \omega^{\mu\nu\rho\lambda}, \tag{7}$$

with $\omega^{\mu\nu\rho\lambda}$ arbitrary antisymmetric field, the transformed gauge parameters (4) identically vanish

$$\varepsilon^{\mu\nu}_{(\theta_{(\omega)})} \equiv 0. \tag{8}$$

We remark that the BF theory alone is a usual linear gauge theory of Cauchy order equal to four, while the Dirac field is described by a linear, propagating theory, of Cauchy order equal to one, so the overall Cauchy order of the starting model is equal to four.

In order to construct the BRST symmetry of this "free" theory, we introduce the field/ghost and antifield spectra

$$\Phi^{\alpha_0} = \left(A^\mu, H^\mu, \varphi, B^{\mu\nu}, \psi^\alpha, \bar{\psi}_\alpha\right), \quad \Phi^*_{\alpha_0} = \left(A^*_\mu, H^*_\mu, \varphi^*, B^*_{\mu\nu}, \psi^*_\alpha, \bar{\psi}^{*\alpha}\right), \tag{9}$$

$$\eta^{\alpha_1} = \left(\eta, C^{\mu\nu}, \eta^{\mu\nu\rho}\right), \quad \eta^*_{\alpha_1} = \left(\eta^*, C^*_{\mu\nu}, \eta^*_{\mu\nu\rho}\right), \tag{10}$$

$$\eta^{\alpha_2} = \left(C^{\mu\nu\rho}, \eta^{\mu\nu\rho\lambda}\right), \quad \eta^*_{\alpha_2} = \left(C^*_{\mu\nu\rho}, \eta^*_{\mu\nu\rho\lambda}\right), \tag{11}$$

$$\eta^{\alpha_3} = C^{\mu\nu\rho\lambda}, \quad \eta^*_{\alpha_3} = C^*_{\mu\nu\rho\lambda}. \tag{12}$$

The fermionic ghosts η^{α_1} respectively correspond to the bosonic gauge parameters $\varepsilon^{\alpha_1} = (\varepsilon, \varepsilon^{\mu\nu}, \varepsilon^{\mu\nu\rho\lambda})$, the bosonic ghosts for ghosts η^{α_2} are due to the first-order reducibility relations (4–5), while the fermionic ghosts for ghosts for ghosts η^{α_3} are required by the second-order reducibility relations (7). The star variables represent the antifields of the corresponding fields/ghosts. Their Grassmann parities are obtained *via* the usual rule

$$\varepsilon\left(\chi^*\right) = \left(\varepsilon\left(\chi\right) + 1\right) \bmod 2, \tag{13}$$

where we employed the notations

$$\chi = \left(\Phi^{\alpha_0}, \eta^{\alpha_1}, \eta^{\alpha_2}, \eta^{\alpha_3}\right), \quad \chi^* = \left(\Phi^*_{\alpha_0}, \eta^*_{\alpha_1}, \eta^*_{\alpha_2}, \eta^*_{\alpha_3}\right). \tag{14}$$

Since both the gauge generators and the reducibility functions are field-independent, it follows that the BRST differential reduces to

$$s = \delta + \gamma, \tag{15}$$

where δ is the Koszul-Tate differential, and γ means the exterior longitudinal derivative. The Koszul-Tate differential is graded in terms of the antighost number $(\mathrm{agh}, \mathrm{agh}(\delta) = -1,\ \mathrm{agh}(\gamma) = 0)$ and enforces a resolution of the algebra of smooth functions defined on the stationary surface of field equations for the action (1), $C^\infty(\Sigma)$, $\Sigma : \delta S_0 / \delta \Phi^{\alpha_0} = 0$. The exterior longitudinal derivative is graded in terms of the pure ghost number $(\mathrm{pgh},\ \mathrm{pgh}(\gamma) = 1,\ \mathrm{pgh}(\delta) = 0)$ and is correlated with the gauge symmetry *via* its cohomology at pure ghost number zero computed in $C^\infty(\Sigma)$, which is isomorphic to the algebra of physical observables for the free model. The two degrees of the generators from the BRST complex are valued as

$$\mathrm{pgh}\left(\Phi^{\alpha_0}\right) = 0,\ \ \mathrm{pgh}\left(\eta^{\alpha_1}\right) = 1,\ \ \mathrm{pgh}\left(\eta^{\alpha_2}\right) = 2,\ \ \mathrm{pgh}\left(\eta^{\alpha_3}\right) = 3, \tag{16}$$

$$\mathrm{pgh}\left(\Phi^*_{\alpha_0}\right) = \mathrm{pgh}\left(\eta^*_{\alpha_1}\right) = \mathrm{pgh}\left(\eta^*_{\alpha_2}\right) = \mathrm{pgh}\left(\eta^*_{\alpha_3}\right) = 0, \tag{17}$$

$$\mathrm{agh}\left(\Phi^{\alpha_0}\right) = \mathrm{agh}\left(\eta^{\alpha_1}\right) = \mathrm{agh}\left(\eta^{\alpha_2}\right) = \mathrm{agh}\left(\eta^{\alpha_3}\right) = 0, \tag{18}$$

$$\mathrm{agh}\left(\Phi^*_{\alpha_0}\right) = 1,\ \ \mathrm{agh}\left(\eta^*_{\alpha_1}\right) = 2,\ \ \mathrm{agh}\left(\eta^*_{\alpha_2}\right) = 3, \mathrm{agh}\left(\eta^*_{\alpha_3}\right) = 4, \tag{19}$$

while the actions of δ and γ on them read as

$$\delta\Phi^{\alpha_0} = \delta\eta^{\alpha_1} = \delta\eta^{\alpha_2} = \delta\eta^{\alpha_3} = 0, \tag{20}$$

$$\delta A^*_\mu = \partial^\nu B_{\nu\mu},\ \ \delta H^*_\mu = -\partial_\mu \varphi,\ \ \delta\varphi^* = \partial_\mu H^\mu,\ \ \delta B^*_{\mu\nu} = -\frac{1}{2} F_{\mu\nu}, \tag{21}$$

$$\delta\psi^{*}_{\alpha} = -\left(i\left(\gamma^{\mu}\right)^{\beta}_{\ \alpha}\partial_{\mu} + m\delta^{\beta}_{\alpha}\right)\bar{\psi}_{\beta},\quad \delta\bar{\psi}^{*\alpha} = -\left(i\left(\gamma^{\mu}\right)^{\alpha}_{\ \beta} - m\delta^{\alpha}_{\beta}\right)\psi^{\alpha}, \tag{22}$$

$$\delta\eta^{*} = -\partial^{\mu}A^{*}_{\mu},\ \delta C^{*}_{\mu\nu} = \partial_{[\mu}H^{*}_{\nu]},\ \delta\eta^{*}_{\mu\nu\rho} = \partial_{[\mu}B^{*}_{\nu\rho]}, \tag{23}$$

$$\delta C^{*}_{\mu\nu\rho} = -\partial_{[\mu}C^{*}_{\nu\rho]},\ \delta\eta^{*}_{\mu\nu\rho\lambda} = -\partial_{[\mu}\eta^{*}_{\nu\rho\lambda]},\ \delta C^{*}_{\mu\nu\rho\lambda} = \partial_{[\mu}C^{*}_{\nu\rho\lambda]}, \tag{24}$$

$$\gamma\Phi^{*}_{\alpha_0} = \gamma\eta^{*}_{\alpha_1} = \gamma\eta^{*}_{\alpha_2} = \gamma\eta^{*}_{\alpha_3} = 0, \tag{25}$$

$$\gamma A^{\mu} = \partial^{\mu}\eta,\ \gamma H^{\mu} = 2\partial^{\nu}C_{\mu\nu},\ \gamma B^{\mu\nu} = -3\partial_{\rho}\eta^{\mu\nu\rho},\ \gamma\varphi = 0, \tag{26}$$

$$\gamma\psi^{\alpha} = \gamma\bar{\psi}_{\alpha} = 0,\ \gamma\eta = 0, \gamma C^{\mu\nu} = -3\partial_{\rho}C^{\mu\nu\rho},\ \gamma\eta^{\mu\nu\rho} = 4\partial_{\lambda}\eta^{\mu\nu\rho\lambda}, \tag{27}$$

$$\gamma C^{\mu\nu\rho} = 4\partial_{\lambda}C^{\mu\nu\rho\lambda},\ \gamma\eta^{\mu\nu\rho\lambda} = \gamma C^{\mu\nu\rho\lambda} = 0. \tag{28}$$

The overall degree of the BRST complex is named ghost number (gh) and is defined as the difference between the pure ghost number and the antighost number, such that $\mathrm{gh}(s) = 1$. The BRST symmetry admits a canonical action in a structure named antibracket, $s\cdot = \left(\cdot, \bar{S}\right)$, where its canonical generator, $\bar{S}$, is a bosonic functional of ghost number zero, that satisfies the classical master equation $\left(\bar{S}, \bar{S}\right) = 0.$ The notation (,) signifies the antibracket, which is defined by decreeing the fields/ghosts conjugated to the corresponding antifields. In the case of the free theory under discussion, the solution to the master equation takes the form

$$\begin{aligned}\bar{S} = S_0 + \int \mathrm{d}^4x\big(&A^{*}_{\mu}\partial^{\mu}\eta + 2H^{*}_{\mu}\partial_{\nu}C^{\mu\nu} - 3B^{*}_{\mu\nu}\partial_{\rho}\eta^{\mu\nu\rho} - 3C^{*}_{\mu\nu}\partial_{\rho}C^{\mu\nu\rho} + \\ &+4\eta^{*}_{\mu\nu\rho}\partial_{\lambda}\eta^{\mu\nu\rho\lambda} + 4C^{*}_{\mu\nu\rho}\partial_{\lambda}C^{\mu\nu\rho\lambda}\big),\end{aligned} \tag{29}$$

and we observe that it contains pieces with the antighost number ranging from zero to three.

3. BRST DEFORMATION PROCEDURE

3.1. GENERAL SETTING

A consistent deformation of the free action (1) and of its gauge invariances (2–3) defines a deformation of the corresponding solution to the master equation that preserves both the master equation and the field/antifield spectra. So, if

$$S_0\left[A^{\mu}, H^{\mu}, \varphi, B^{\mu\nu}, \psi^{\alpha}, \bar{\psi}_{\alpha}\right] + g\int \mathrm{d}^4x a_0 + O\left(g^2\right),$$

stands for a consistent deformation of the free action, with deformed gauge transformations $\bar{\delta}_\varepsilon A_\mu = \partial_\mu \varepsilon + g\beta_\mu + O(g^2)$, $\bar{\delta}_\varepsilon H_\mu = 2\partial^\nu \varepsilon_{\mu\nu} + \rho_\mu + O(g^2)$, $\bar{\delta}_\varepsilon \varphi = g\beta + O(g^2)$, $\bar{\delta}_\varepsilon B^{\mu\nu} = -3\partial_\rho \varepsilon^{\mu\nu\rho} + g\beta^{\mu\nu} + O(g^2)$, $\bar{\delta}_\varepsilon \bar{\psi}_\alpha = g\sigma_\alpha + O(g^2)$ and $\bar{\delta}_\varepsilon \psi^\alpha = g\lambda^\alpha + O(g^2)$, then the deformed solution

$$S = \bar{S} + g\int \mathrm{d}^4 x a + O(g^2), \tag{30}$$

satisfies the master equation $(S,S)=0$, where the non-integrated density of the first-order deformation starts like

$$a = a_0 + A^{*\mu}\bar{\beta}_\mu + H^{*\mu}\bar{\rho}_\mu + \varphi^*\bar{\beta} + B^*_{\mu\nu}\bar{\beta}^{\mu\nu} + \bar{\psi}^{*\alpha}\bar{\sigma}_\alpha + \psi^*_\alpha\bar{\lambda}^\alpha + \text{“more”}.$$

The terms $\bar{\beta}_\mu$, $\bar{\rho}_\mu$, $\bar{\beta}$, $\bar{\beta}^{\mu\nu}$, $\bar{\sigma}_\alpha$ and $\bar{\lambda}^\alpha$ are obtained by replacing the gauge parameters ε, $\varepsilon^{\mu\nu}$ and $\varepsilon^{\mu\nu\rho}$ respectively with the fermionic ghosts η, $C^{\mu\nu}$ and $\eta^{\mu\nu\rho}$ into the functions β_μ, ρ_μ, β, $\beta^{\mu\nu}$, σ_α and λ^α.

The master equation $(S,S)=0$ holds to order g if and only if

$$sa = \partial_\mu j^\mu, \tag{31}$$

for some local j^μ. In order to solve this equation, we develop a according to the antighost number

$$a = a_0 + a_1 + \cdots + a_I,\ \ \mathrm{agh}(a_K) = K,\ \ \mathrm{gh}(a_K) = \varepsilon(a_K) = 0. \tag{32}$$

The number of the terms in the expansion (31) is finite and it can be shown that we can take the last term in a to be annihilated by γ

$$\gamma a_I = 0. \tag{33}$$

Consequently, we need to compute the cohomology of γ, $H(\gamma)$, in order to determine the component of the highest antighost number in a. From the definitions (25–28) it is simple to see that $H(\gamma)$ is spanned by $F_{\mu\nu}$, $\partial_\mu H^\mu$, φ, $\partial_\mu B^{\mu\nu}$, $\bar{\psi}_\alpha$, ψ^α and the antifields $\chi^* = (\Phi^*_{\alpha_0}, \eta^*_{\alpha_1}, \eta^*_{\alpha_2}, \eta^*_{\alpha_3})$, by their space-time derivatives, as well as by the undifferentiated ghosts $\eta^{A_1} = (\eta, \eta^{\mu\nu\rho\lambda}, C^{\mu\nu\rho\lambda})$. (The derivatives of the ghosts η^{A_1} are removed from $H(\gamma)$ since they are γ-exact, as can be seen from the first formula in (26), the last definition in (27), and respectively the first relation in (28).) If we denote by $e^M(\eta^{A_1})$ the elements with pure ghost number M of a basis in the space of the polynomials in the ghosts η^{A_1}, it follows that the general solution to the equation (33) takes the form

$$a_I = \mu_I\left(\left[F_{\mu\nu}\right],\left[\partial_\mu H^\mu\right],[\varphi],\left[\partial_\mu B^{\mu\nu}\right],[\bar{\psi}_\alpha],\left[\psi^\alpha\right],\left[\chi^*\right]\right)e^I\left(\eta^{A_1}\right), \tag{34}$$

where $\mathrm{agh}(\mu_I)=I$ and $\mathrm{pgh}\left(e^I\right)=I$. The notation $f\left([q]\right)$ means that f depends on q and its space-time derivatives up to a finite order. The equation (31) projected on antighost number $(I-1)$ becomes

$$\delta a_I + \gamma a_{I-1} = \partial^\mu \overset{(I-1)}{m}_\mu . \tag{35}$$

Replacing (34) in (35), it follows that the last equation possesses solutions with respect to a_{I-1} if the coefficients μ_I pertain to the homological space $H_I\left(\delta|d\right)$, *i.e.*, $\delta\mu_I = \partial_\mu l^\mu_{I-1}$. In the meantime, since our free model is linear and of Cauchy order equal to four, according to the general results from [21–22] we get that $H_J\left(\delta|d\right)$ vanishes for $J>4$, so we can assume the first-order deformation stops at the antighost number four ($I=4$)

$$a = a_0 + a_1 + a_2 + a_3 + a_4, \tag{36}$$

where a_4 is of the form (34) with μ_4 from $H_4\left(\delta|d\right)$.

3.2. FIRST-ORDER DEFORMATION

By direct computation, we infer that the most general representative of $H_4\left(\delta|d\right)$ can be taken of the type

$$\begin{aligned}(\mu_4)_{\mu\nu\rho\lambda} = {} & \frac{\delta W}{\delta\varphi}C^*_{\mu\nu\rho\lambda} + \frac{\delta^2 W}{\delta\varphi^2}\left(H^*_{[\mu}C^*_{\nu\rho\lambda]} + C^*_{[\mu\nu}C^*_{\rho\lambda]}\right) + \\ & + \frac{\delta^3 W}{\delta\varphi^3}H^*_{[\mu}H^*_{\nu}C^*_{\rho\lambda]} + \frac{\delta^4 W}{\delta\varphi^4}H^*_{\mu}H^*_{\nu}H^*_{\rho}H^*_{\lambda},\end{aligned} \tag{37}$$

with $W(\varphi)$ an arbitrary function depending on the undifferentiated scalar field. On the other hand, the elements of pure ghost number equal to four of the basis in the space of polynomials in the ghosts η^{A_1} are

$$\eta C^{\mu\nu\rho\lambda},\ \ \eta^{\alpha\beta\gamma\delta}\eta^{\mu\nu\rho\lambda}. \tag{38}$$

In order to couple (37) to the second element in (38), we need some completely antisymmetric constants $k_{\alpha\beta\gamma\delta}$ that, by covariance arguments, can only be proportional to the completely antisymmetric four-dimensional symbol $\varepsilon_{\alpha\beta\gamma\delta}$. So, the most general form of the last representative from the expansion (36) is

$$
\begin{aligned}
a_4 = & \left(\frac{\delta W}{\delta\varphi} C^*_{\mu\nu\rho\lambda} + \frac{\delta^2 W}{\delta\varphi^2} \left(H^*_{[\mu} C^*_{\nu\rho\lambda]} + C^*_{[\mu\nu} C^*_{\rho\lambda]} \right) + \right. \\
& \left. + \frac{\delta^3 W}{\delta\varphi^3} H^*_{[\mu} H^*_{\nu} C^*_{\rho\lambda]} + \frac{\delta^4 W}{\delta\varphi^4} H^*_{\mu} H^*_{\nu} H^*_{\rho} H^*_{\lambda} \right) \eta C^{\mu\nu\rho\lambda} + \\
& + \frac{1}{2} \left(\frac{\delta M}{\delta\varphi} C^*_{\mu\nu\rho\lambda} + \frac{\delta^2 M}{\delta\varphi^2} \left(H^*_{[\mu} C^*_{\nu\rho\lambda]} + C^*_{[\mu\nu} C^*_{\rho\lambda]} \right) + \right. \\
& \left. + \frac{\delta^3 M}{\delta\varphi^3} H^*_{[\mu} H^*_{\nu} C^*_{\rho\lambda]} + \frac{\delta^4 M}{\delta\varphi^4} H^*_{\mu} H^*_{\nu} H^*_{\rho} H^*_{\lambda} \right) \eta^{\mu\nu\rho\lambda} \varepsilon_{\alpha\beta\gamma\delta} \eta^{\alpha\beta\gamma\delta},
\end{aligned}
\tag{39}
$$

where the numerical factor ½ in the second term was taken for convenience, and the functions W and M are arbitrary functions of the undifferentiated scalar field. By computing the action of δ on a_4 and by taking into account the relations (25–28) it follows that the solution to the equation (35) for $I = 4$ is precisely given by

$$
\begin{aligned}
a_3 = & -\left(\frac{\delta W}{\delta\varphi} C^*_{\nu\rho\lambda} + \frac{\delta^2 W}{\delta\varphi^2} H^*_{[\nu} C^*_{\rho\lambda]} + \frac{\delta^3 W}{\delta\varphi^3} H^*_{\nu} H^*_{\rho} H^*_{\lambda} \right) \times \left(4 A_\mu C^{\mu\nu\rho\lambda} + \eta C^{\nu\rho\lambda} \right) + \\
& + 2 \left(W \eta^*_{\mu\nu\rho\lambda} + 4 \frac{\delta W}{\delta\varphi} H^*_{\mu} \eta^*_{\nu\rho\lambda} + 6 \left(\frac{\delta W}{\delta\varphi} C^*_{\mu\nu} + \frac{\delta^2 W}{\delta\varphi^2} H^*_{\mu} H^*_{\nu} \right) B^*_{\rho\lambda} \right) C^{\mu\nu\rho\lambda} - \\
& - \left(\frac{\delta M}{\delta\varphi} C^*_{\mu\nu\rho} + \frac{\delta^2 M}{\delta\varphi^2} H^*_{[\mu} C^*_{\nu\rho]} + \frac{\delta^3 M}{\delta\varphi^3} H^*_{\mu} H^*_{\nu} H^*_{\rho} \right) \eta^{\mu\nu\rho} \varepsilon_{\alpha\beta\gamma\delta} \eta^{\alpha\beta\gamma\delta}.
\end{aligned}
\tag{40}
$$

By means of the equation (33) projected on the antighost number two

$$
\delta a_3 + \gamma a_2 = \partial^\mu \overset{(2)}{m}_\mu ,
\tag{41}
$$

the solution (40) and the definitions (25–28) lead to

$$
\begin{aligned}
a_2 = & \left(\frac{\delta W}{\delta\varphi} C^*_{\mu\nu} + \frac{\delta^2 W}{\delta\varphi^2} H^*_{\mu} H^*_{\nu} \right) \left(-3 A_\rho C^{\mu\nu\rho} + \eta C^{\mu\nu} \right) - \\
& - 2 \left(W \eta^*_{\mu\nu\rho} + \frac{\delta W}{\delta\varphi} H^*_{[\mu} B^*_{\nu\rho]} \right) C^{\mu\nu\rho} + \left(\left(\frac{\delta M}{\delta\varphi} C^*_{\mu\nu} + \frac{\delta^2 M}{\delta\varphi^2} H^*_{\mu} H^*_{\nu} \right) B^{\mu\nu} + \right. \\
& \left. + 2 \left(\frac{\delta M}{\delta\varphi} H^*_{\mu} A^{*\mu} - M \eta^* \right) \right) \varepsilon_{\alpha\beta\gamma\delta} \eta^{\alpha\beta\gamma\delta} - \\
& - \frac{9}{4} \left(\frac{\delta M}{\delta\varphi} C^*_{\mu\nu} + \frac{\delta^2 M}{\delta\varphi^2} H^*_{\mu} H^*_{\nu} \right) \varepsilon_{\alpha\beta\gamma\delta} \eta^{\mu\alpha\beta} \eta^{\nu\gamma\delta}.
\end{aligned}
\tag{42}
$$

Next, we investigate the equation (33) projected on the antighost number one

$$\delta a_2 + \gamma a_1 = \partial^\mu \overset{(1)}{m}_\mu, \tag{43}$$

which, combined with (42), further yields

$$\begin{aligned} a_1 = {} & \frac{\delta W}{\delta\varphi} H_\mu^* \left(2A_\nu C^{\mu\nu} - H^\mu \eta\right) + W\left(2B_{\mu\nu}^* C^{\mu\nu} + \varphi^* \eta\right) + \\ & +2\left(\frac{\delta M}{\delta\varphi} H_\rho^* B^{\rho\alpha} - M A^{*\alpha}\right) \varepsilon_{\alpha\beta\gamma\delta} \eta^{\beta\gamma\delta} + \bar{a}_1, \end{aligned} \tag{44}$$

where $\bar{a}_1$ is the general solution to the "homogeneous" equation

$$\gamma \bar{a}_1 = 0. \tag{45}$$

Using the definitions (21–22), we easily find that

$$\bar{a}_1 = iU\left(\bar{\psi}^{*\alpha} \bar{\psi}_\alpha - \psi_\alpha^* \psi^\alpha\right)\eta - \frac{\delta U}{\delta\varphi} H_\mu^* \bar{\psi}_\alpha \left(\gamma^\mu\right)^\alpha{}_\beta \psi^\beta \eta, \tag{46}$$

where U is an arbitrary function of the scalar field. The equation (33) projected on the antighost number zero reads as

$$\delta a_1 + \gamma a_0 = \partial^\mu \overset{(0)}{m}_\mu. \tag{47}$$

Introducing (46) in (44), we obtain the solution for (47) to be of the type

$$a_0 = -A_\mu \left(U(\varphi)\bar{\psi}_\alpha \left(\gamma^\mu\right)^\alpha{}_\beta \psi^\beta + W(\varphi) H^\mu\right) + \frac{1}{2}\varepsilon_{\alpha\beta\gamma\delta} M(\varphi) B^{\alpha\beta} B^{\gamma\delta}. \tag{48}$$

Combining the formulas (39), (40), (42), (44) and (48), the first-order deformation of the solution to the master equation for the model under study can be written in the form

$$\begin{aligned} S_1 = \int d^4x \Bigg(& -A_\mu \left(U(\varphi)\bar{\psi}_\alpha \left(\gamma^\mu\right)^\alpha{}_\beta \psi^\beta + W(\varphi) H^\mu\right) + \frac{1}{2} M(\varphi) \varepsilon_{\alpha\beta\gamma\delta} B^{\alpha\beta} B^{\gamma\delta} + \\ & + H_\mu^* \left(\frac{\delta W}{\delta\varphi}\left(2A_\nu C^{\mu\nu} - H^\mu \eta\right) - \frac{\delta U}{\delta\varphi} \bar{\psi}_\alpha \left(\gamma^\mu\right)^\alpha{}_\beta \psi^\beta \eta + 2\frac{\delta M}{\delta\varphi} B^{\mu\alpha} \varepsilon_{\alpha\beta\gamma\delta} \eta^{\beta\gamma\delta}\right) + \\ & + iU(\varphi)\left(\bar{\psi}^{*\alpha}\bar{\psi}_\alpha - \psi_\alpha^* \psi^\alpha\right)\eta - 2\varepsilon_{\alpha\beta\gamma\delta} M(\varphi) A^{*\alpha} \eta^{\beta\gamma\delta} + \\ & + W(\varphi)\left(2B_{\mu\nu}^* C^{\mu\nu} + \varphi^* \eta\right) + \left(\frac{\delta W}{\delta\varphi} C_{\mu\nu}^* + \frac{\delta^2 W}{\delta\varphi^2} H_\mu^* H_\nu^*\right)\left(-3A_\rho C^{\mu\nu\rho} + \eta C^{\mu\nu}\right) - \\ & - 2\left(\frac{\delta W}{\delta\varphi} H_{[\mu}^* B_{\nu\rho]}^* + W(\varphi)\eta_{\mu\nu\rho}^*\right) C^{\mu\nu\rho} + \left(\left(\frac{\delta M}{\delta\varphi} C_{\mu\nu}^* + \frac{\delta^2 M}{\delta\varphi^2} H_\mu^* H_\nu^*\right) B^{\mu\nu} + \right. \end{aligned}$$

$$+2\left(\frac{\delta M}{\delta\varphi}H^{*}_{\mu}A^{*\mu}-M(\varphi)\eta^{*}\right)\Bigg)\varepsilon_{\alpha\beta\gamma\delta}\eta^{\alpha\beta\gamma\delta}-$$

$$-\frac{9}{4}\left(\frac{\delta M}{\delta\varphi}C^{*}_{\mu\nu}+\frac{\delta^{2}M}{\delta\varphi^{2}}H^{*}_{\mu}H^{*}_{\nu}\right)\varepsilon_{\alpha\beta\gamma\delta}\eta^{\mu\alpha\beta}\eta^{\nu\gamma\delta}-\left(\frac{\delta W}{\delta\varphi}C^{*}_{\nu\rho\lambda}+\frac{\delta^{2}W}{\delta\varphi^{2}}H^{*}_{[\nu}C^{*}_{\rho\lambda]}+\right.$$

$$\left.+\frac{\delta^{3}W}{\delta\varphi^{3}}H^{*}_{\nu}H^{*}_{\rho}H^{*}_{\lambda}\right)\left(4A_{\mu}C^{\mu\nu\rho\lambda}+\eta C^{\nu\rho\lambda}\right)+2W(\varphi)\eta^{*}_{\mu\nu\rho\lambda}C^{\mu\nu\rho\lambda}+$$

$$+4\left(2\frac{\delta W}{\delta\varphi}H^{*}_{\mu}\eta^{*}_{\nu\rho\lambda}+3\left(\frac{\delta W}{\delta\varphi}C^{*}_{\mu\nu}+\frac{\delta^{2}W}{\delta\varphi^{2}}H^{*}_{\mu}H^{*}_{\nu}\right)B^{*}_{\rho\lambda}\right)C^{\mu\nu\rho\lambda}-$$

$$-\left(\frac{\delta M}{\delta\varphi}C^{*}_{\mu\nu\rho}+\frac{\delta^{2}M}{\delta\varphi^{2}}H^{*}_{[\mu}C^{*}_{\nu\rho]}+\frac{\delta^{3}M}{\delta\varphi^{3}}H^{*}_{\mu}H^{*}_{\nu}H^{*}_{\rho}\right)\eta^{\mu\nu\rho}\varepsilon_{\alpha\beta\gamma\delta}\eta^{\alpha\beta\gamma\delta}+$$

$$+\left(\frac{\delta W}{\delta\varphi}C^{*}_{\mu\nu\rho\lambda}+\frac{\delta^{2}W}{\delta\varphi^{2}}\left(H^{*}_{[\mu}C^{*}_{\nu\rho\lambda]}+C^{*}_{[\mu\nu}C^{*}_{\rho\lambda]}\right)+\frac{\delta^{3}W}{\delta\varphi^{3}}H^{*}_{[\mu}H^{*}_{\nu}C^{*}_{\rho\lambda]}+\right.$$

$$\left.+\frac{\delta^{4}W}{\delta\varphi^{4}}H^{*}_{\mu}H^{*}_{\nu}H^{*}_{\rho}H^{*}_{\lambda}\right)\eta C^{\mu\nu\rho\lambda}+\frac{1}{2}\left(\frac{\delta M}{\delta\varphi}C^{*}_{\mu\nu\rho\lambda}+\frac{\delta^{3}M}{\delta\varphi^{3}}H^{*}_{[\mu}H^{*}_{\nu}C^{*}_{\rho\lambda]}+\right. \tag{49}$$

$$\left.+\frac{\delta^{2}M}{\delta\varphi^{2}}\left(H^{*}_{[\mu}C^{*}_{\nu\rho\lambda]}+C^{*}_{[\mu\nu}C^{*}_{\rho\lambda]}\right)+\frac{\delta^{4}M}{\delta\varphi^{4}}H^{*}_{\mu}H^{*}_{\nu}H^{*}_{\rho}H^{*}_{\lambda}\right)\eta^{\mu\nu\rho\lambda}\varepsilon_{\alpha\beta\gamma\delta}\eta^{\alpha\beta\gamma\delta}\Bigg).$$

It is by construction an s-co-cycle of the ghost number zero, such that S_0+gS_1 is the solution to the master equation to order g.

3.3. HIGHER-ORDER DEFORMATIONS

Next, we investigate the equations that control the higher-order deformations. The equation that controls the consistency of the first-order deformation reads as

$$\frac{1}{2}(S_1,S_1)+sS_2=0, \tag{50}$$

where S_2 denotes the second-order deformation of the solution to the master equation for the deformed theory

$$(S,S)=0,\quad S=\bar{S}+gS_1+g^2S_2+\ldots. \tag{51}$$

Using (49), by direct computation, we obtain that

$$\frac{1}{2}(S_1,S_1)=\varepsilon_{\mu\nu\rho\lambda}\int d^4x\left(\sum_{a=0}^{4}\left(T^{\mu\nu\rho\lambda}_{a}\frac{\delta^{a}X}{\delta\varphi^{a}}+U^{\mu\nu\rho\lambda}_{a}\frac{\delta^{a}Y}{\delta\varphi^{a}}\right)+\right.$$

$$+2\left(\frac{\delta(MU)}{\delta\varphi}H^*_\sigma\bar{\psi}_\alpha\left(\gamma^\sigma\right)^\alpha{}_\beta\psi^\beta - iMU\left(\bar{\psi}_\alpha\bar{\psi}^{*\alpha}-\psi^*_\alpha\psi^\alpha\right)\right)\eta^{\mu\nu\rho\lambda} - \\ -2M(\varphi)U(\varphi)\bar{\psi}_\alpha\left(\gamma^\mu\right)^\alpha{}_\beta\psi^\beta\eta^{\nu\rho\lambda}\Big), \tag{52}$$

where

$$T_0^{\mu\nu\rho\lambda} = 4A^{*\mu}C^{\nu\rho\lambda} + B^{\mu\nu}C^{\rho\lambda} + H^\mu\eta^{\nu\rho\lambda} - 2\eta^*C^{\mu\nu\rho\lambda} - \varphi^*\eta^{\mu\nu\rho\lambda}, \tag{53}$$

$$T_1^{\mu\nu\rho\lambda} = \left(H^*_\alpha H^\alpha + C^*_{\alpha\beta}C^{\alpha\beta} + C^*_{\alpha\beta\gamma}C^{\alpha\beta\gamma} + C^*_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}\right)\eta^{\mu\nu\rho\lambda} + \\ +\left(2H^*_\alpha A^{*\alpha} + C^*_{\alpha\beta}B^{\alpha\beta} - C^*_{\alpha\beta\gamma}\eta^{\alpha\beta\gamma}\right)C^{\mu\nu\rho\lambda} + \\ +\left(3C^*_{\alpha\beta}C^{\alpha\beta\mu} - 2H^*_\alpha C^{\alpha\mu}\right)\eta^{\nu\rho\lambda} + 3H^*_\alpha C^{\alpha\mu\nu}B^{\rho\lambda}, \tag{54}$$

$$T_2^{\mu\nu\rho\lambda} = H^*_\alpha\left(\left(H^*_\beta B^{\alpha\beta} - 3C^*_{\beta\gamma}\eta^{\alpha\beta\gamma}\right)C^{\mu\nu\rho\lambda} + 3H^*_\beta C^{\alpha\beta\mu}\eta^{\nu\rho\lambda}\right) + \\ +\left(H^*_\alpha\left(3C^*_{\beta\gamma}C^{\alpha\beta\gamma} + H^*_\beta C^{\alpha\beta} + C^*_{\beta\gamma\delta}C^{\alpha\beta\gamma\delta}\right) + 3C^*_{\alpha\beta}C^*_{\gamma\delta}C^{\alpha\beta\gamma\delta}\right)\eta^{\mu\nu\rho\lambda}, \tag{55}$$

$$T_3^{\mu\nu\rho\lambda} = H^*_\alpha H^*_\beta\left(\left(H^*_\gamma C^{\alpha\beta\gamma} + 3C^*_{\gamma\delta}C^{\alpha\beta\gamma\delta}\right)\eta^{\mu\nu\rho\lambda} - H^*_\gamma\eta^{\alpha\beta\gamma}C^{\mu\nu\rho\lambda}\right), \tag{56}$$

$$T_4^{\mu\nu\rho\lambda} = H^*_\alpha H^*_\beta H^*_\gamma H^*_\delta C^{\alpha\beta\gamma\delta}\eta^{\mu\nu\rho\lambda}, \tag{57}$$

$$U_0^{\mu\nu\rho\lambda} = \left(\frac{1}{2}\eta^*_{\alpha\beta\gamma\delta}\eta^{\alpha\beta\gamma\delta} + \eta^*_{\alpha\beta\gamma}\eta^{\alpha\beta\gamma} + B^*_{\alpha\beta}B^{\alpha\beta} - 6A^*_\alpha A^\alpha\right)\eta^{\mu\nu\rho\lambda} + \\ +\left(A^{*\mu}\eta + \frac{3}{2}B^*_{\alpha\beta}\eta^{\alpha\beta\mu} - A_\alpha B^{\alpha\mu}\right)\eta^{\nu\rho\lambda} + \frac{1}{2}B^{\mu\nu}B^{\rho\lambda}\eta, \tag{58}$$

$$U_1^{\mu\nu\rho\lambda} = \left(\frac{1}{4}\eta C^{*\mu\nu\rho\lambda} - A^\mu C^{*\nu\rho\lambda} + 3B^{*\mu\nu}C^{*\rho\lambda} - 2\eta^{*\mu\nu\rho}H^{*\lambda}\right)\eta_{\alpha\beta\gamma\delta}\eta^{\alpha\beta\gamma\delta} \\ +\left(\left(\frac{1}{2}C^*_{\alpha\beta\gamma}\eta + \frac{3}{2}C^*_{\alpha\beta}A_\gamma - 3B^*_{\alpha\beta}H^*_\gamma\right)\eta^{\alpha\beta\gamma} + \left(\frac{1}{2}C^*_{\alpha\beta}B^{\alpha\beta} + A^{*\alpha}H^*_\alpha\right)\eta\right)\eta^{\mu\nu\rho\lambda} + \\ +H^*_\alpha\left(A_\beta\left(B^{\alpha\beta}\eta^{\mu\nu\rho\lambda} + \frac{3}{2}\eta^{\alpha\beta\mu}\eta^{\nu\rho\lambda}\right) - \eta B^{\alpha\mu}\eta^{\nu\rho\lambda}\right) + \frac{3}{4}\eta C^*_{\alpha\beta}\eta^{\alpha\beta\mu}\eta^{\nu\rho\lambda}, \tag{59}$$

$$U_2^{\mu\nu\rho\lambda} = H^*_\alpha\left(\frac{3}{2}\left(C^*_{\beta\gamma}\eta + H^*_\beta A_\gamma\right)\eta^{\alpha\beta\gamma} + H^*_\beta B^{\alpha\beta}\eta\right)\eta^{\mu\nu\rho\lambda} + \\ +\frac{3}{4}H^*_\alpha H^*_\beta\eta\eta^{\alpha\beta\mu}\eta^{\nu\rho\lambda} + \left(H^{*\mu}\left(C^{*\nu\rho\lambda}\eta + 3H^{*\nu}B^{*\rho\lambda}\right) + \\ +3\left(\frac{1}{4}C^{*\mu\nu}\eta + H^{*\mu}A^{*\nu}\right)C^{*\rho\lambda}\right)\eta_{\alpha\beta\gamma\delta}\eta^{\alpha\beta\gamma\delta}, \tag{60}$$

$$U_3^{\mu\nu\rho\lambda} = \frac{1}{2} H^{*\mu} H^{*\nu} \left(3C^{*\rho\lambda}\eta + 2H^{*\rho} A^{\lambda} \right) \eta_{\alpha\beta\gamma\delta} \eta^{\alpha\beta\gamma\delta} + \\ + \frac{1}{2} H^{*}_{\alpha} H^{*}_{\beta} H^{*}_{\gamma} \eta\eta^{\alpha\beta\gamma} \eta^{\mu\nu\rho\lambda}, \tag{61}$$

$$U_4^{\mu\nu\rho\lambda} = \frac{1}{2} \eta H^{*\mu} H^{*\nu} H^{*\rho} H^{*\lambda} \eta_{\alpha\beta\gamma\delta} \eta^{\alpha\beta\gamma\delta}, \tag{62}$$

and

$$X(\varphi) = W(\varphi) M(\varphi), \quad Y(\varphi) = W(\varphi) \frac{\delta M(\varphi)}{\delta\varphi}. \tag{63}$$

Because none of the terms in (52) are BRST-exact, the consistency of the first-order deformation (49) demands that $W(\varphi)$, $M(\varphi)$ and $U(\varphi)$ verify the equations

$$W(\varphi) M(\varphi) = 0, \quad W(\varphi) \frac{\delta M(\varphi)}{\delta\varphi} = 0, \quad M(\varphi) U(\varphi) = 0, \tag{64}$$

such that we can conclude that the second-order deformation of the solution to the master equation can be chosen to vanish. So, the deformed solution to the master equation stops at order one in the parameter of the deformation, *i.e.*,

$$S = \bar{S} + gS_1. \tag{65}$$

However, in order to obtain effective interactions among the BF and Dirac fields we must take

$$M(\varphi) \equiv 0, \tag{66}$$

such that the equations (64) are satisfied for $W(\varphi)$ and $U(\varphi)$ arbitrary functions of the scalar field.

4. IDENTIFICATION OF THE INTERACTING THEORY

Replacing the solution (66) of the equations (64) into the first-order deformation (49), the fully deformed solution to the master equation (65), consistent to all orders in the coupling constant, becomes

$$S = \int d^4x \left(\bar{\psi}_{\alpha} \left(\left(i\gamma^{\mu} \right)^{\alpha}{}_{\beta} \left(\partial_{\mu} + igU(\varphi) A_{\mu} \right) - m\delta^{\alpha}{}_{\beta} \right) \psi^{\beta} + H^{\mu} \left(\partial_{\mu}\varphi - gW(\varphi) A_{\mu} \right) + \right. \\ + \frac{1}{2} B^{\mu\nu} F_{\mu\nu} + H^{*}_{\mu} \left(2 \left(\partial_{\nu} C^{\mu\nu} + g \frac{\delta W}{\delta\varphi} A_{\nu} C^{\mu\nu} \right) - g \frac{\delta W}{\delta\varphi} H^{\mu} \eta - \right. \\ \left. - g \frac{\delta U}{\delta\varphi} \bar{\psi}_{\alpha} \left(\gamma^{\mu} \right)^{\alpha}{}_{\beta} \psi^{\beta} \eta \right) + B^{*}_{\mu\nu} \left(-3\partial_{\rho} \eta^{\mu\nu\rho} + 2gW(\varphi) C^{\mu\nu} \right) +$$

$$
\begin{aligned}
&+igU(\varphi)\left(\bar{\psi}^{*\alpha}\bar{\psi}_{\alpha}-\psi^{*}_{\alpha}\psi^{\alpha}\right)\eta+gW(\varphi)\varphi^{*}\eta+A^{*}_{\mu}\partial^{\mu}\eta-\\
&-3C^{*}_{\mu\nu}\left(\partial_{\rho}C^{\mu\nu\rho}+g\frac{\delta W}{\delta\varphi}A_{\rho}C^{\mu\nu\rho}\right)+\eta^{*}_{\mu\nu\rho}\left(4\partial_{\lambda}\eta^{\mu\nu\rho\lambda}-2gW(\varphi)C^{\mu\nu\rho}\right)+\\
&+g\left(\frac{\delta W}{\delta\varphi}C^{*}_{\mu\nu}\eta C^{\mu\nu}+\frac{\delta^{2}W}{\delta\varphi^{2}}H^{*}_{\mu}H^{*}_{\nu}\left(-3A_{\rho}C^{\mu\nu\rho}+\eta C^{\mu\nu}\right)\right)-\\
&-2g\left(\frac{\delta W}{\delta\varphi}H^{*}_{[\mu}B^{*}_{\nu\rho]}C^{\mu\nu\rho}-W(\varphi)\eta^{*}_{\mu\nu\rho\lambda}C^{\mu\nu\rho\lambda}\right)+\\
&+4C^{*}_{\mu\nu\rho}\left(\partial_{\lambda}C^{\mu\nu\rho\lambda}+g\frac{\delta W}{\delta\varphi}A_{\lambda}C^{\mu\nu\rho\lambda}\right)+\\
&+4g\left(\frac{\delta^{2}W}{\delta\varphi^{2}}H^{*}_{[\mu}C^{*}_{\nu\rho]}+\frac{\delta^{3}W}{\delta\varphi^{3}}H^{*}_{\mu}H^{*}_{\nu}H^{*}_{\rho}\right)A_{\lambda}C^{\mu\nu\rho\lambda}-\\
&-g\left(\frac{\delta W}{\delta\varphi}C^{*}_{\mu\nu\rho}+\frac{\delta^{2}W}{\delta\varphi^{2}}H^{*}_{[\mu}C^{*}_{\nu\rho]}+\frac{\delta^{3}W}{\delta\varphi^{3}}H^{*}_{\mu}H^{*}_{\nu}H^{*}_{\rho}\right)\eta C^{\mu\nu\rho}+ \qquad (67)\\
&+4g\left(2\frac{\delta W}{\delta\varphi}H^{*}_{\mu}\eta^{*}_{\nu\rho\lambda}+3\left(\frac{\delta W}{\delta\varphi}C^{*}_{\mu\nu}+\frac{\delta^{2}W}{\delta\varphi^{2}}H^{*}_{\mu}H^{*}_{\nu}\right)B^{*}_{\rho\lambda}\right)C^{\mu\nu\rho\lambda}+\\
&+g\left(\frac{\delta W}{\delta\varphi}C^{*}_{\mu\nu\rho\lambda}+\frac{\delta^{2}W}{\delta\varphi^{2}}\left(H^{*}_{[\mu}C^{*}_{\nu\rho\lambda]}+C^{*}_{[\mu\nu}C^{*}_{\rho\lambda]}\right)+\frac{\delta^{3}W}{\delta\varphi^{3}}H^{*}_{[\mu}H^{*}_{\nu}C^{*}_{\rho\lambda]}+\right.\\
&\left.+\frac{\delta^{4}W}{\delta\varphi^{4}}H^{*}_{\mu}H^{*}_{\nu}H^{*}_{\rho}H^{*}_{\lambda}\right)\eta C^{\mu\nu\rho\lambda}.
\end{aligned}
$$

From the deformed solution to the master equation (67), we identify the entire gauge structure of the interacting gauge theory. The pieces of the antighost number zero of (67) represent the Lagrangian action of the deformed theory

$$
\begin{aligned}
S^{L}\left[A^{\mu},H^{\mu},\varphi,B_{\mu\nu},\psi^{\alpha},\bar{\psi}_{\alpha}\right]=\int \mathrm{d}^{4}x\Big(&\frac{1}{2}B^{\mu\nu}F_{\mu\nu}+H^{\mu}\left(\partial_{\mu}\varphi-gW(\varphi)A_{\mu}\right)+\\
&+\bar{\psi}_{\alpha}\left(\left(i\gamma^{\mu}\right)^{\alpha}{}_{\beta}\left(\partial_{\mu}+igU(\varphi)A_{\mu}\right)-m\delta^{\alpha}{}_{\beta}\right)\psi^{\beta}\Big).
\end{aligned} \qquad (68)
$$

The terms of the antighost number one from (67) give us the deformed gauge transformations

$$\bar{\delta}_{\varepsilon}A_{\mu}=\partial_{\mu}\varepsilon,\quad \bar{\delta}_{\varepsilon}B^{\mu\nu}=-3\partial_{\rho}\varepsilon^{\mu\nu\rho}+2gW(\varphi)\varepsilon^{\mu\nu}, \qquad (69)$$

$$\bar{\delta}_{\varepsilon}\varphi=gW(\varphi)\varepsilon,\quad \bar{\delta}_{\varepsilon}\psi^{\alpha}=-iU(\varphi)\psi^{\alpha}\varepsilon,\quad \bar{\delta}_{\varepsilon}\bar{\psi}_{\alpha}=iU(\varphi)\bar{\psi}_{\alpha}\varepsilon, \qquad (70)$$

$$\bar{\delta}_{\varepsilon}H^{\mu}=2\left(\partial_{\nu}\varepsilon^{\mu\nu}+g\frac{\delta W}{\delta\varphi}A_{\nu}\varepsilon^{\mu\nu}\right)-g\varepsilon\left(\frac{\delta W}{\delta\varphi}H^{\mu}+\frac{\delta U}{\delta\varphi}\bar{\psi}_{\alpha}\left(\gamma^{\mu}\right)^{\alpha}{}_{\beta}\psi^{\beta}\right). \qquad (71)$$

We observe that the Dirac field becomes endowed with non-trivial gauge transformations, which can be regarded as the gauge version of a one-parameter rigid symmetry of the Dirac theory multiplied by an arbitrary function of the scalar field. The cross-interactions between the BF field spectrum and the Dirac field are expressed by a generalized minimal coupling with the one-form A_μ with abelian-$U(1)$ gauge transformations *via* the conserved current corresponding to the previously mentioned rigid symmetry in a 'background' of the scalar field.

We notice that there appear two types of pieces with the antighost number two in (67). Ones are quadratic in the pure ghost number one fields, while the others are linear in the ghosts of ghosts. Analyzing the structure of the former kind of terms, we conclude that the deformed gauge algebra is open, while from the latter kind of terms we find that the first-order reducibility functions are modified with respect to the initial model and the first-order reducibility relations hold on-shell. The presence of the terms with antighost numbers higher than two reveals, on the one hand, the higher-order gauge structure corresponding to the deformed gauge algebra and, on the one hand, the second-stage reducibility relations, which also take place on-shell.

5. CONCLUSION

To conclude, in this paper we have investigated the consistent Lagrangian interactions that can be introduced between an abelian BF-type theory and one massive Dirac field with the help of the deformation of the solution to the master equation combined with cohomological techniques. Starting with the BRST differential of the free theory, $s = \delta + \gamma$, we fully compute the first-order deformation and show that we can take all the deformations of orders two and higher to vanish. The cross-interactions between the BF field spectrum and the Dirac field are described by a generalized minimal coupling. Our deformation procedure modifies the gauge transformations, as well as their algebra, which becomes open. Meanwhile, the reducibility relations take place on-shell, by contrast to the free model.

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