HID LAMP DIFFUSE CATHODE MODEL

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Abstract. The paper presents a model able to calculate the temperature distribution inside the cathode of high-pressure mercury and sodium lamps. At the same electric voltage drop and current discharge, two possibilities for cathode electric field exist: higher field \( (E > 10^8 \, \text{V/m}) \) with very high-density current \( (j > 10^8 \, \text{A/m}^2) \), and lower field \( (E < 10^7 \, \text{V/m}) \) with lower current density. These possibilities determine the type of the arc attachment mode on the cathode. The first mode is called “hot-spot”, the second “diffuse mode”. In this paper, the energy balance equation has been analysed for the cathode in diffuse mode of operation in the case of one and two-dimensional models. A comparison between the one and the two dimensional models is carried out. The cathode-plasma interaction was modelled by the non-linear boundary conditions on the electrode surface. The model takes into account the temperature dependence of the thermal conductivity, the total emissivity and the electrical cathode material resistivity.

Key words: high-pressure lamps, diffuse and hot-spot mode.

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1. INTRODUCTION

The electrodes – operating periodically as cathode and anode – are essential parts of the high intensity discharge (HID) and high-pressure sodium (HPS) lamps. The existence of multiple modes of a current transfer from plasma discharge to the cathode has been shown. These different modes are a manifestation of a non-linear thermal balance equation. The non-linear character of the heat transfer equation influences also the different electronic emission mechanism (Schottky-amplified themionic emission, thermo-field emission or field emission). A lot of lamp properties and lamp efficient functioning are in strong relationship with the cathode operation. Many authors have made theoretical models in the various simplifications. Starting from Waymouth [1], Tielemans-Oostvogels [2] and Cram [3], the analysis of the cathode mechanism and phenomena became more complicated and detailed. Theoretical [4–6], experimental [7, 8] and computational [9–12] studies have been elaborated in the last years. Also the correlation between plasma temperature and electrode temperature in metal–halide lamps has been investigated [13]. Lifetime
and the wall blackening are problems in the HID lamps. Both parameters are strongly influenced by the electrode temperature.

The design of the electrodes (in the case of HID and HPS lamps) is a compromise between various, contradictory requirements. First, to reduce the ignition time is preferably to use a very thin rod and thin coil. Secondly, to prolong lamp-life and improve the maintenance of the emissive mixture stored between the coil interstices, the electrodes tail must be large enough. On the other hand, during the electrode operation the temperature distribution in the electrode ensures the maintenance of the barium monolayer without excessive evaporation and sputtering of the emitter material.

For these reasons the electrodes design must be carefully made. Numerical modelling of the electrode is very helpful in the design of performant devices.

In this paper we present the models for a cylindrical cathode of a high-pressure mercury lamp. The discharge is in the stationary regime and the cathode is functioning in the diffuse mode. The cathode operation in the diffuse mode is favored by an electric field smaller than $10^7$ V/m, when we have thermionic cathode emission amplified by the Schottky effect.

2. CATHODE GEOMETRY

Because the electrode reaches such a high temperature operation, it must be made of a refractive material, which has a high melting point. The cathode geometry for mercury or HPS lamp is presented in Fig. 1.

The mercury lamp cathode is made of a tungsten rod over which one or two coil layers of the same material are wrapped with the aim of increasing the radiative surface. Between the coils interstitions the emissive mixture lays down (oxides BaO, CaO and carbonates BaCO$_3$, CaCO$_3$) which, after the reaction,

![Diagram of cathode geometry](image)

**Fig. 1** – Cathode geometry for high-pressure mercury (HID) and sodium (HPS) lamps.
6BaO + W → Ba₃WO₆ + 3Ba, leads to the appearance of barium atoms in excess. These atoms migrate towards the cathode active zone, creating the barium monolayer which has the role to decrease the electron work function from 4.5 eV (case of pure tungsten) to 2.7 eV (case of barium). The role of CaCO₃ is to increase the thermal stability of the emissive mixture.

3. THEORETICAL MODEL

3.1. CALCULATION OF THE CATHODE ACTIVE SURFACE TEMPERATURE

To leave the metal, the electron needs energy to escalate the potential barrier at the metal surface. This energy is called work function. At 0 K the last level occupied by the electron is called Fermi level. By heating the metal, the electrons are lifted to the energy levels higher that Fermi level and their energy is distributed according to the Fermi-Dirac statistic. At higher electrode temperature, more electrons from the tail of the energy distribution function have sufficient energy to escalate the electric potential barrier. This process is called thermionic emission. Electron thermionic emission is enhanced by the existence of a high electric field at the cathode surface and by application of an activator layer (with small work function) on the tungsten electrode.

We suppose that the electron density current is due to the field-enhanced thermionic emission caused by lowering of the potential barrier in the presence of an electric field (Schottky effect) (\( j^{th}_{e} \)) and secondary emission by \( \gamma \)-Townsed process (\( j^{sec}_{e} \)). Using the \( \beta \)-Waymouth coefficient, which gives a relation between ionic (\( j_i \)) and electronic (\( j_e \)) current density in the cathode neighbourhood [1], we have:

\[
\begin{align*}
    j^{th}_{e} &= AT^2 \exp \left( -\frac{e\varphi}{k_B T} \right) \quad \varphi = \varphi_0 - \Delta\varphi; \quad \Delta\varphi = \left( \frac{eE_k}{4\pi \varepsilon_0} \right)^{1/2} \\
    j^{sec}_{e} &= \gamma j_i \\
    \beta &= \frac{j_i}{j_e} = \frac{j_i}{j^{th}_{e} + j^{sec}_{e}}
\end{align*}
\]

Here, \( A = 4\pi e m_e k_B^2 / h^3 \) represents the Richardson-Dushman constant, \( \varphi_0 \) is the electrode material work function, \( \Delta\varphi \) is the Schottky correction of work function. \( e \) and \( m_e \) represent the electron charge, respectively mass, \( k_B \) is the Boltzmann constant, \( h \) is the Planck constant and \( E_k \) is the electric field intensity in
the cathode fall. The cathode field is related to the potential drop over the free fall sheath, $V_k$, by the MacKeon equation \[ E_k^2 \approx \frac{4j_i}{e_0} \left( \frac{m_i V_k}{2e} \right)^{1/2}, \] with $m_i$ the ion mass (mercury or sodium).

If the electric field intensity is inferior to $E < 10^7$ V/m, the cathode operates in the diffuse mode. The diffuse mode functioning of the cathode is also favoured by a high electric discharge current and by a weak electrode cooling. The pressure of the electric discharge plays an important role in the type of electrode mode. The low-pressure favors the diffuse mode functioning.

In the diffuse mode, the plasma discharge covers the all-top part of the electrode. Sometimes, a lateral part of the rod can participate in the electric discharge. In this model, we suppose that just the top area (called the cathode active area) participates through thermionic emission in the electric discharge current. All other electrode surface parts are considered inactive from the electron emission point of view.

So, for a stationary regime of the diffuse cathode mode functioning at the total discharge current intensity $I$, we find the following equation:

\[
1 - \frac{\beta \gamma}{1 + \beta} \frac{h^3}{4\pi^2 m_k k_B T_c^2} \cdot I = T_{act}^2 \exp \left( -\frac{e\phi_0}{k_B T_{act}} \right) \exp \left[ \frac{e^{3/2} E_k^{1/2}}{4\pi^2 e_0} \frac{1}{k_B T_{act}} \right]
\]

which gives the electrode active surface temperature.

### 3.2. THE ONE-DIMENSIONAL CATHODE MODEL

In the one-dimensional model, the transversal dimensions of the cathode are very small in comparison with the longitudinal one ($z$-direction). So, we neglect the radial temperature dependence. We assume that all the electrodes are formed from a tungsten rod with a 0.5 mm radius, or succession of tungsten rods, with different radii and lengths. The influence of the different radii on the longitudinal temperature profile is made through lateral thermal radiation emission.

#### 3.2.1. Heat transfer equation

The model based on the heat transport equation is made in accordance with the following assumptions:
- the cathode is operating in the “diffuse mode”;
- the radial electrode temperature variation is neglected;
- the heating source inside the electrode is due to the Joule effect;
- the thermal conductivity, electrical resistivity, and thermal emissivity of the electrode material depend on temperature;
– the electrode cooling is made by thermal radiation and by the ambient thermal flow.

So, taking into account all these considerations, the heat transport equation is:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \rho_e(T) \left( \frac{I}{\pi r^2} \right)^2 - \frac{2 \sigma \varepsilon (T)}{r^2} \left( T^4 - T_{amb}^4 \right)$$

(5)

where $\sigma$ is the Stefan-Boltzmann constant, $r$ is the local electrode radius, $T$ is the electrode temperature and $I$ represents the electric discharge current intensity. $\rho$ is the density, $c$ is the heat capacity, $k$ is the thermal conductivity, $\varepsilon$ is the thermal emissivity and $\rho_e = \rho_0 e \left[ 1 + \alpha (T - T_0) \right]$ is the electric resistivity of the electrode. All tungsten material characteristics are given in Table 1.

<table>
<thead>
<tr>
<th>Tungsten materials characteristics</th>
</tr>
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<tbody>
<tr>
<td>$\rho$ [kg/m$^3$]</td>
</tr>
<tr>
<td>$c$ [J/kg/K]</td>
</tr>
<tr>
<td>$k$ [W/mK]</td>
</tr>
<tr>
<td>$\rho_0 e$ [øm]</td>
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<tr>
<td>$\alpha$ [grad$^{-1}$]</td>
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<td>$\varepsilon$</td>
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</tbody>
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The ambient temperature $T_{amb}$ is considered equal to 300 K and represents the temperature far from the electrode. The electrode temperature dependence of thermal conductivity and emissivity are given as functions which fit the literature data [14]. The temporal dependence of equation (5) is introduced in order to assure a quick convergence of the solution. The range of the time is increased until the solution remains unchanged. Also, the supplementary cathode heating by thermal ambient radiation adsorption can be neglected with respect to the cathode cooling by thermal radiation emission ($2 \sigma \varepsilon T^4 / r^2$).

### 3.2.2. Boundary conditions

The temperature at the end of the cathode is experimentally measured and taken as $T_{end} = 600$ K. The active temperature in front of the cathode is $T_{act}$ and is obtained in the section 3.1 by solving equation (4).

### 3.3. THE TWO-DIMENSIONAL MODEL

In this section the heat transfer equation with non-linear boundary condition problem is solved in order to obtain the temperature distribution. The only input
parameters are the cathode geometry, the current discharge and the cathode end temperature of. All the other parameters are calculated as local temperature functions during the algorithm run. The electrode geometry is presented in Fig. 1.

### 3.3.1. Energy balance equation

The temperature distribution inside the cathode is obtained by solving the thermal diffusion equation

\[ \rho c \frac{\partial T}{\partial t} + \text{div} \left\{ -k(T) \cdot \text{grad}[T(r,z)] \right\} = S(r,z) \]  

with given boundary conditions.

For some reasons as in section 3.2.1., the temporal term in equation (6) is introduced to decrease the computational time. In this way, the two-dimensional model computational time is drastically reduced with respect to the stationary equation.

The function \( k(T) \) gives the electrode thermal conductivity temperature dependence in the temperature range \([300–3683] \) K. The source term \( S(r,z) \) in equation (6) represents the dissipated power in the unit volume through the Joule effect:

\[ S(r,z) = \rho_{0e} \left\{ 1 + \alpha \left[ T(r,z) - T_0 \right] \right\} j^2 (r,z) \]  

The constants \( \rho_{0e} \) and \( \alpha \) of the cathode materials, are given in Table 1.

The function \( j(r,z) \) represents the current density in a transversal section \((z = \text{constant})\) of the cathode. The current density satisfies the supplementary condition:

\[ 2\pi \int_{0}^{r_0} j(r,z_0) \cdot r \, dr = I \]  

where \( r_0 \) is the cathode radius in the section \( z = z_0 \) and \( I \) represents the discharge current intensity.

### 3.3.2. Boundary conditions

A Neumann condition on the lateral border was imposed like in reference [15]. The physical significance of this condition is the energetic flux continuity between the cathode and the adjacent medium. Therefore:

\[ k_W \left( T_b \right) \left( \frac{\partial T}{\partial n} \right)_b = \bar{n} \left[ \zeta \left( T_b - T_{\text{amb}} \right) + \varepsilon_W \left( T_b \right) \sigma \left( T_b^4 - T_{\text{amb}}^4 \right) \right] \]
where $T_b$ is the cathode temperature on the border, $T_{\text{amb}}$ is the temperature far away from the cathode and $\zeta$ is the convection transport coefficient. The factor $(\frac{\partial T}{\partial n})_b$ represents the temperature derivative in the normal direction ($\vec{n}$) of the cathode surface.

The front surface and the end part of the electrode are characterised by the Dirichlet boundary conditions: $T_{\text{act}} = f(I)$ and $T_{\text{end}} = 600$ K, with $T_{\text{act}}$ given by formula (4).

### 4. RESULTS AND DISCUSSIONS

#### 4.1. RESULTS AND DISCUSSION OF THE ONE-DIMENSIONAL MODEL

In this model the electrode is a tungsten rod with a 0.5 mm radius and 9.5 mm length. In Fig. 2 the solid line represents the temperature distribution in the cathode $z$-direction. This is obtained under the conditions $I = 3$ A and $T_{\text{act}} = 1650$ K.

![Fig. 2 – Longitudinal temperature distribution in the tungsten rod electrode. In solid line: the temperature profile obtained in our model. In dashed line: the temperature profile obtained by Tielemans and Oostvoegels in the assumption of an infinite length of the electrode [2].](image)

In the condition of constant material parameters and stationary case, Tielemans and Oostvoegels have found the analytical solution of the heat transfer equation:

\[
\begin{aligned}
\frac{dT}{dz} &= -\left(\frac{4 \sigma E}{5 rk}\right)^{1/2} T^{5/2} \\
T^{-3/2}(z) &= T_{\text{act}}^{-3/2}(0) - \left(\frac{9 \sigma E}{5 k}\right)^{1/2} \frac{z}{\sqrt{r}}
\end{aligned}
\] (10)
In Fig. 2 the dashed line presents the result of the Tieleman-Oostvoegels model [2]. In their model the temperature is supposed to be equal to zero to infinite. But this assumption is not a realistic one and for this reason a discrepancy exists between our result and their results. Even if we take for the end temperature the value $T_{\text{end}} = 1235$ K given by the Tielemans-Oostvogels model at 9.5 mm distance, an important discrepancy exists. But this is due to the fact that in our model the specific material quantities are function of temperature, while in their model the constant values for tungsten thermal conductivity $k = 93.5 \text{ W/m/K}$ and for tungsten thermal emissivity $\varepsilon = 0.335$ were considered.

It must be noted that the influence of the Joule heating is minor with respect to the cooling through thermal radiation. The ratio between the heating power by the Joule effect over the radiative cooling power is presented in Fig. 3.

![Fig. 3 – The ratio between the Joule heating power over the radiative cooling power of the electrode.](image)

When the temperature decreases and the radiative power loss becomes smaller (the radiative power depends on the quadratic temperature) this ratio becomes bigger at the end of the cathode. On the other hand, the Joule density heating power decreases linearly with the temperature decreasing, and this decrease is less pronounced at the end of the electrode. The cooling by thermal radiative processes is very important if the temperature is higher than 900 K (see Fig. 4).

At the electrode active surface this ratio takes the value

$$\frac{\rho_c I^2 \left[ 1 + \alpha \left( T_{\text{act}} - T_0 \right) \right]}{2\pi^2 \sigma T_{\text{act}}^4 r^3} \approx 0.0313.$$ 

A significant value (when the Joule heating becomes comparable with the radiative loss) is obtained for the electrode radii inferior to 0.16 mm. In conclusion, in the one-dimensional modelling, the Joule term can be neglected if the electrode radius is bigger than 0.16 mm. Such transversal dimension of the electrode is an unrealistic one for HID lamps. So, the role of the Joule term in the one-dimensional model is minor.
It is important to note that the most part of the radiative cooling is produced in the vicinity of the active area. In this zone, the heating is produced by ionic bombardment and the cooling by radiation and electron emission (Nottingham effect).

Benilov has shown (for a simple cylindrical cathode) that if the longitudinal dimension of the cathode decreases, the thermal conduction loss increases while radiation losses decrease \[6\]. But, for all lamp categories the transversal dimensions of the electrode are smaller than the longitudinal one.

4.2. RESULTS AND DISCUSSION OF THE TWO-DIMENSIONAL MODEL

In the case of the two-dimensional model, the results are obtained by solving equation (5) by the finite element method. This is carried out by using FEMLAB package. Equation (5) was solved in the case of the tungsten cylindrical electrode with radii and length \( r_1 = 0.5 \text{ mm}, r_2 = 1.5 \text{ mm}, r_3 = 1 \text{ mm}, \) respectively \( d_1 = 1.5 \text{ mm}, d_2 = 3 \text{ mm} \) and \( d_3 = 5 \text{ mm} \). Some results are presented below.

In Fig. 5 is presented the temperature distribution in the HID lamps electrode obtained for an active temperature surface \( T_{act} = 1650 \text{ K} \) which corresponds to a 3 A current intensity discharge.

It could be observed that in the case of the diffuse mode, the temperature distribution is symmetrical. The emissive layer consumption in the \( z \)-direction is the same for all the points of the emissive mixture deposition area. Meanwhile, the barium atoms migration from the cold toward the hot areas presents a cylindrical symmetry. The cathode continues to work until the end of the emissive layer. The discharge burns quietly. In addition, in this cathode mode functioning the consumption of the emissive layer by sputtering is reduced.

A comparison between the axis temperature distribution in the HID tungsten electrode (represented in Fig. 6), and the temperature distribution obtained in the
one-dimensional model (see Fig. 2) shows a difference. The cooling by the radiant emission of the tungsten wrapped coil is obvious. In the zone with large radii the temperature decreases quickly.

In the electrode design it is important to know the temperature distribution in the bulk cathode. An analysis of the electrode radiator influence on the temperature distribution can be made. So, in order to favour the emissive layer formation and to diminish the barium atoms loss by vaporisation, we can choose the optimal value of electrode dimensions (see Fig. 1).
Also, the plasma temperature in the electrode vicinity is lower, and the Hg atom density is higher. Knowledge of the temperature distribution at the cathode surface allows the evaluation of the $W - \text{Hg}$ and $\text{Ba} - \text{Hg}$ mixture formation with a negative role in the thermionic emission and lifetime of the lamp.

5. CONCLUSIONS

One and two-dimensional high-pressure mercury cathode models were elaborated. The models allow the determination of the active surface temperature as function of the discharge current intensity and the temperature distribution in the bulk cathode region. This distribution is obtained by solving the heat transfer equation by the finite element method. The interaction between cathode and plasma discharge is modelled through non-linear boundary conditions. The model can be used in the electrode lamp design in order to increase the HID lamp efficiency and to increase the lifetime.

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REFERENCES