Romanian Reports in Physics, Vol. 57, No. 4, P. 573–583, 2005

QUANTUM TO CLASSICAL TRANSITION IN THE LINDBLAD THEORY OF OPEN QUANTUM SYSTEMS

A. ISAR

Department of Theoretical Physics, Institute of Physics and Nuclear Engineering, Bucharest-Măgurele, Romania
e-mail address: isar@theory.nipne.ro

(Received July 15, 2005)

Abstract. In the framework of the Lindblad theory for open quantum systems we determine the degree of quantum decoherence and classical correlations of a harmonic oscillator interacting with a thermal bath and analyze its transition from quantum to classical behaviour.

Key words: Lindblad theory, open quantum systems, quantum decoherence, classical correlations.

1. INTRODUCTION

The transition from quantum to classical physics and classicality of quantum systems continue to be among the most interesting problems in many fields of physics, for both conceptual and experimental reasons [1, 2, 3]. Two conditions are essential for the classicality of a quantum system [4]: a) quantum decoherence (QD), that means the irreversible, uncontrollable and persistent formation of a quantum correlation (entanglement) of the system with its environment [5], expressed by the damping of the coherences present in the quantum state of the system, when the off-diagonal elements of the density matrix of the system decay below a certain level, so that this density matrix becomes approximately diagonal and b) classical correlations (CC), expressed by the fact that the Wigner function of the quantum system has a peak which follows the classical equations of motion in phase space, that is the quantum state becomes peaked along a classical trajectory.

In the last two decades it has became more and more clear that the classicality is an emergent property of open quantum systems, since both main features of this process – QD and CC – strongly depend on the interaction between the system and its external environment [3, 6, 7]. A remarkable aspect of the current research helping in understanding the nature of the quantum to classical transition is that for the first time there have recently been carried on experiments probing the boundary between the quantum and the classical domains in a controlled way [8, 9].

In most of the literature, the quantum decoherence has been studied for a system coupled to an environment or thermal bath with many degrees of freedom. In this work we study QD and CC for a harmonic oscillator interacting with an environment in the framework of the Lindblad theory for open quantum systems. More concretely we determine the degree of QD and CC and the possibility of simultaneous realization of QD and CC for a system consisting of a harmonic oscillator in a thermal bath. It is found that the system manifests a QD which is more and more significant in time, whereas CC are less and less strong as the system evolves in time.

The organizing of the paper is as follows. In Section 2 we review the Lindblad master equation for the damped harmonic oscillator and in Section 3 we derive the master equation in coordinate representation and the corresponding Fokker-Planck equation in the Wigner representation and determine the density matrix and Wigner function of the considered system. Then in Section 4 we investigate QD and CC and analyze them quantitatively. A summary is given in Section 5.

2. MASTER EQUATION FOR THE HARMONIC OSCILLATOR

The irreversible time evolution of an open system is described by the following general quantum Markovian master equation for the density operator $\rho(t)$ [10, 11, 12]:

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = -\frac{i}{\hbar}[H,\rho(t)] + \frac{1}{2\hbar}\sum_j ([V_j\rho(t),V_j^\dagger] + [V_j,\rho(t)V_j^\dagger]). \quad (1)$$

H is the Hamiltonian of the system and V_j, $V_j^\dagger$ are operators on the Hilbert space of H, which model the environment. In order to obtain, for the damped quantum harmonic oscillator, equations of motion as close as possible to the classical ones, the two possible operators V_1 and V_2 are taken as linear polynomials in coordinate q and momentum p [13, 14, 15] and the harmonic oscillator Hamiltonian H is chosen of the general quadratic form

$$H = H_0 + \frac{\mu}{2}(qp+pq), \qquad H_0 = \frac{1}{2m}p^2 + \frac{m\omega^2}{2}q^2. \quad (2)$$

With these choices the master equation (1) takes the following form [14, 15]:

$$\begin{aligned}\frac{\mathrm{d}\rho}{\mathrm{d}t} = &-\frac{i}{\hbar}[H_0,\rho] - \frac{i}{2\hbar}(\lambda+\mu)[q,\rho p + p\rho] + \frac{i}{2\hbar}(\lambda-\mu)[p,\rho q + q\rho] \\ &-\frac{D_{pp}}{\hbar^2}[q,[q,\rho]] - \frac{D_{qq}}{\hbar^2}[p,[p,\rho]] + \frac{D_{pq}}{\hbar^2}([q,[p,\rho]] + [p,[q,\rho]]).\end{aligned} \quad (3)$$

The quantum diffusion coefficients D_{pp}, D_{qq}, D_{pq} and the dissipation constant λ satisfy the following fundamental constraints [14, 15]: $D_{pp} > 0$, $D_{qq} > 0$ and

$$D_{pp}D_{qq} - D_{pq}^2 \geq \frac{\lambda^2\hbar^2}{4}. \tag{4}$$

In the particular case when the asymptotic state is a Gibbs state $\rho_G(\infty) = = \mathrm{e}^{-\frac{H_0}{kT}} / \mathrm{Tr}\, \mathrm{e}^{-\frac{H_0}{kT}}$, these coefficients become [14, 15]

$$D_{pp} = \frac{\lambda+\mu}{2}\hbar m\omega \coth\frac{\hbar\omega}{2kT}, \quad D_{qq} = \frac{\lambda-\mu}{2}\frac{\hbar}{m\omega}\coth\frac{\hbar\omega}{2kT}, \quad D_{pq} = 0, \tag{5}$$

where T is the temperature of the thermal bath.

From the master equation (3) we can obtain the equations of motion for the expectation values of coordinate and momentum and in the underdamped case $(\omega > \mu)$ considered in this paper, with the notation $\Omega^2 \equiv \omega^2 - \mu^2$, the solutions have the following form [14, 15]:

$$\sigma_q(t) = \mathrm{e}^{-\lambda t}\left(\left(\cos\Omega t + \frac{\mu}{\Omega}\sin\Omega t\right)\sigma_q(0) + \frac{1}{m\Omega}\sin\Omega t\sigma_p(0)\right), \tag{6}$$

$$\sigma_p(t) = \mathrm{e}^{-\lambda t}\left(-\frac{m\omega^2}{\Omega}\sin\Omega t\sigma_q(0) + \left(\cos\Omega t - \frac{\mu}{\Omega}\sin\Omega t\right)\sigma_p(0)\right) \tag{7}$$

and $\sigma_q(\infty) = \sigma_p(\infty) = 0.$

The relation (4) is a necessary condition for the generalized uncertainty inequality

$$\sigma_{qq}(t)\sigma_{pp}(t) - \sigma_{pq}^2(t) \geq \frac{\hbar^2}{4} \tag{8}$$

to be fulfilled, where σ_{qq} and σ_{pp} denote the dispersion (variance) of the coordinate and momentum, respectively, and σ_{pq} denotes the correlation (covariance) of the coordinate and momentum. The equality in relation (8) is realized for a special class of pure states, called correlated coherent states [16] or squeezed coherent states.

The asymptotic values $\sigma_{qq}(\infty)$, $\sigma_{pp}(\infty)$, $\sigma_{pq}(\infty)$ do not depend on the initial values $\sigma_{qq}(0)$, $\sigma_{pp}(0)$, $\sigma_{pq}(0)$ and in the case of a thermal bath with coefficients (5), they reduce to [14, 15]

$$\sigma_{qq}(\infty) = \frac{\hbar}{2m\omega}\coth\frac{\hbar\omega}{2kT}, \quad \sigma_{pp}(\infty) = \frac{\hbar m\omega}{2}\coth\frac{\hbar\omega}{2kT}, \quad \sigma_{pq}(\infty) = 0. \tag{9}$$

3. DENSITY MATRIX AND WIGNER DISTRIBUTION FUNCTION

We consider a harmonic oscillator with an initial Gaussian wave function

$$\Psi(q)=\left(\frac{1}{2\pi\sigma_{qq}(0)}\right)^{\frac{1}{4}}\exp\left[-\frac{1}{4\sigma_{qq}(0)}\left(1-\frac{2i}{\hbar}\sigma_{pq}(0)\right)(q-\sigma_q(0))^2+\frac{i}{\hbar}\sigma_p(0)q\right], \quad (10)$$

where $\sigma_{qq}(0)$ is the initial spread, $\sigma_{pq}(0)$ the initial covariance, and $\sigma_q(0)$ and $\sigma_p(0)$ are the initial averaged position and momentum of the wave packet. The initial state (10) represents a correlated coherent state [16] with the variances and covariance of coordinate and momentum

$$\sigma_{qq}(0)=\frac{\hbar\delta}{2m\omega},\quad \sigma_{pp}(0)=\frac{\hbar m\omega}{2\delta(1-r^2)},\quad \sigma_{pq}(0)=\frac{\hbar r}{2\sqrt{1-r^2}}. \quad (11)$$

Here, δ is the squeezing parameter which measures the spread in the initial Gaussian packet and *r*, with $|r|<1$ is the correlation coefficient at time $t=0.$ The initial values (11) correspond to a minimum uncertainty state, since they fulfil the generalized uncertainty relation

$$\sigma_{qq}(0)\sigma_{pp}(0)-\sigma_{pq}^2(0)=\frac{\hbar^2}{4}. \quad (12)$$

For $\delta=1$ and $r=0$ the correlated coherent state becomes a Glauber coherent state.

From Eq. (3) we derive the evolution equation in coordinate representation:

$$\begin{aligned}\frac{\partial\rho}{\partial t}&=\frac{i\hbar}{2m}\left(\frac{\partial^2}{\partial q^2}-\frac{\partial^2}{\partial q'^2}\right)\rho-\frac{im\omega^2}{2\hbar}(q^2-q'^2)\rho\\&-\frac{1}{2}(\lambda+\mu)(q-q')\left(\frac{\partial}{\partial q}-\frac{\partial}{\partial q'}\right)\rho+\frac{1}{2}(\lambda-\mu)\left[(q+q')\left(\frac{\partial}{\partial q}+\frac{\partial}{\partial q'}\right)+2\right]\rho \quad (13)\\&-\frac{D_{pp}}{\hbar^2}(q-q')^2\rho+D_{qq}\left(\frac{\partial}{\partial q}+\frac{\partial}{\partial q'}\right)^2\rho-2iD_{pq}\hbar(q-q')\left(\frac{\partial}{\partial q}+\frac{\partial}{\partial q'}\right)\rho\end{aligned}$$

and Wigner distribution function $W(q, p, t)$ satisfies the following Fokker-Planck-type equation:

$$\begin{aligned}\frac{\partial W}{\partial t}&=-\frac{p}{m}\frac{\partial W}{\partial q}+m\omega^2 q\frac{\partial W}{\partial p}+(\lambda+\mu)\frac{\partial}{\partial p}(pW)+(\lambda-\mu)\frac{\partial}{\partial q}(qW)\\&+D_{pp}\frac{\partial^2 W}{\partial p^2}+D_{qq}\frac{\partial^2 W}{\partial q^2}+2D_{pq}\frac{\partial^2 W}{\partial p\partial q}.\end{aligned} \quad (14)$$

The first two terms on the right-hand side of both these equations generate a purely unitary evolution. They give the usual Liouvillian evolution. The third and forth terms are the dissipative terms and have a damping effect (exchange of energy with environment). The last three are noise (diffusive) terms and produce fluctuation effects in the evolution of the system. D_{pp} promotes diffusion in momentum and generates decoherence in coordinate q: it reduces the off-diagonal terms, responsible for correlations between spatially separated pieces of the wave packet. Similarly D_{qq} promotes diffusion in coordinate and generates decoherence in momentum p. The D_{pq} term is the so-called "anomalous diffusion" term. It promotes diffusion in the variable $qp+pq,$ just like both the other diffusion terms, but it does not generate decoherence.

The density matrix solution of Eq. (13) has the general form of Gaussian density matrices

$$<q|\rho(t)|q'>=\left(\frac{1}{2\pi\sigma_{qq}(t)}\right)^{\frac{1}{2}}\exp\left[-\frac{1}{2\sigma_{qq}(t)}\left(\frac{q+q'}{2}-\sigma_q(t)\right)^2\right.$$
$$\left.-\frac{\sigma(t)}{2\hbar^2\sigma_{qq}(t)}(q-q')^2+\frac{i\sigma_{pq}(t)}{\hbar\sigma_{qq}(t)}\left(\frac{q+q'}{2}-\sigma_q(t)\right)(q-q')+\frac{i}{\hbar}\sigma_p(t)(q-q')\right], \quad (15)$$

where $\sigma(t)\equiv\sigma_{qq}(t)\sigma_{pp}(t)-\sigma_{pq}^2(t)$ is the Schrödinger generalized uncertainty function [17].

For an initial Gaussian Wigner function (corresponding to a correlated coherent state (10)) the solution of Eq. (14) is

$$W(q,p,t)=\frac{1}{2\pi\sqrt{\sigma(t)}}\exp\left\{-\frac{1}{2\sigma(t)}\left[\sigma_{pp}(t)(q-\sigma_q(t))^2+\sigma_{qq}(t)(p-\sigma_p(t))^2\right.\right.$$
$$\left.\left.-2\sigma_{pq}(t)(q-\sigma_q(t))(p-\sigma_p(t))\right]\right\}. \quad (16)$$

In the case of a thermal bath we obtain the following steady state solution for $t\to\infty$ (we denote $\epsilon\equiv\frac{\hbar\omega}{2kT}$):

$$<q|\rho(\infty)|q'>=\left(\frac{m\omega}{\pi\hbar\coth\epsilon}\right)^{\frac{1}{2}}\exp\left\{-\frac{m\omega}{4\hbar}\left[\frac{(q+q')^2}{\coth\epsilon}+(q-q')^2\coth\epsilon\right]\right\}. \quad (17)$$

In the long time limit we have also

$$W_\infty(q,p)=\frac{1}{\pi\hbar\coth\epsilon}\exp\left\{-\frac{1}{\hbar\coth\epsilon}\left[m\omega q^2+\frac{p^2}{m\omega}\right]\right\}. \quad (18)$$

4. QUANTUM DECOHERENCE AND CLASSICAL CORRELATIONS

As we stated in the Introduction, there are two conditions that have to be satisfied in order that a system could be considered as classical. The *first* condition requires that the system should be in one of relatively permanent states (states that are least affected by the interaction of the system with the environment, called by Zurek "preferred states" in the environment induced superselection description [2, 3]) and the interference between different states should be negligible. This implies the destruction of off-diagonal elements representing coherences between quantum states in the density matrix, which is the QD phenomenon. An isolated system has an unitary evolution and the coherence of the state is not lost – pure states evolve in time only to pure states. The loss of coherence can be achieved by introducing an interaction between the system and environment: an initial pure state with a density matrix which contains nonzero off-diagonal terms can non-unitarily evolve into a final mixed state with a diagonal density matrix during the interaction with the environment, like in classical statistical mechanics.

The *second* condition requires that the system should have, with a good approximation, an evolution according to classical laws. This implies that the Wigner distribution function has a peak along a classical trajectory, that means there exist CC between the canonical variables of coordinate and momentum. On the other side, one can have a classical behaviour if the coherences are negligible, without having strong CC (for example, in the case of a classical gas at finite temperature), so that the lack of strong correlations between the coordinate and its canonical momentum does not necessarily mean that the system is quantum. Of course, the correlation between the canonical variables, necessary to obtain a classical limit, should not violate Heisenberg uncertainty principle, *i.e.* the position and momentum should take reasonably sharp values, to a degree in concordance with the uncertainty principle. This is possible, because the density matrix does not diagonalize exactly in position, but with a non-zero width, *i.e.* it is strongly peaked about $q = q'$ and very small for q far from q'.

Using new variables $\Sigma = (q + q')/2$ and $\Delta = q - q'$, the density matrix (15) can be rewritten as

$$\rho(\Sigma, \Delta, t) = \\ = \sqrt{\frac{\alpha}{\pi}} \exp\left[-\alpha\Sigma^2 - \gamma\Delta^2 + i\beta\Sigma\Delta + 2\alpha\sigma_q(t)\Sigma + i\left(\frac{\sigma_p(t)}{\hbar} - \beta\sigma_q(t) \right)\Delta - \alpha\sigma_q^2(t) \right], \tag{19}$$

with the abbreviations

$$\alpha = \frac{1}{2\sigma_{qq}(t)}, \quad \gamma = \frac{\sigma(t)}{2\hbar^2\sigma_{qq}(t)}, \quad \beta = \frac{\sigma_{pq}(t)}{\hbar\sigma_{qq}(t)} \tag{20}$$

and the Wigner transform of the density matrix (19) is

$$W(q,p,t)=\frac{1}{2\pi\hbar}\sqrt{\frac{\alpha}{\gamma}}\exp\left\{-\frac{[\hbar\beta(q-\sigma_q(t))-(p-\sigma_p(t))]^2}{4\hbar^2\gamma}-\alpha(q-\sigma_q(t))^2\right\}. \tag{21}$$

a) *Degree of quantum decoherence (QD)*

The representation-independent measure of the degree of QD [4] is given by the ratio of the dispersion $1/\sqrt{2\gamma}$ of the off-diagonal element $\rho(0,\Delta,t)$ to the dispersion $\sqrt{2/\alpha}$ of the diagonal element $\rho(\Sigma,0,t)$:

$$\delta_{QD}=\frac{1}{2}\sqrt{\frac{\alpha}{\gamma}}, \tag{22}$$

which in our case gives

$$\delta_{QD}(t)=\frac{\hbar}{2\sqrt{\sigma(t)}}. \tag{23}$$

The finite temperature Schrödinger generalized uncertainty function, calculated in Ref. [17], has the expression

$$\begin{aligned}\sigma(t)=\frac{\hbar^2}{4}\Bigg\{&\mathrm{e}^{-4\lambda t}\left[1-\left(\delta+\frac{1}{\delta(1-r^2)}\right)\coth\epsilon+\coth^2\epsilon\right]\\&+\mathrm{e}^{-2\lambda t}\coth\epsilon\Bigg[\left(\delta+\frac{1}{\delta(1-r^2)}-2\coth\epsilon\right)\frac{\omega^2-\mu^2\cos(2\Omega t)}{\Omega^2}\\&+\left(\delta-\frac{1}{\delta(1-r^2)}\right)\frac{\mu\sin(2\Omega t)}{\Omega}+\frac{2r\mu\omega(1-\cos(2\Omega t))}{\Omega^2\sqrt{1-r^2}}\Bigg]+\coth^2\epsilon\Bigg\}.\end{aligned} \tag{24}$$

In the limit of long times Eq. (24) yields

$$\sigma(\infty)=\frac{\hbar^2}{4}\coth^2\epsilon, \tag{25}$$

so that we obtain

$$\delta_{QD}(\infty)=\tanh\frac{\hbar\omega}{2kT}, \tag{26}$$

which for high T becomes

$$\delta_{QD}(\infty)=\frac{\hbar\omega}{2kT}. \tag{27}$$

We see that δ_{QD} decreases, and therefore QD increases, with temperature, *i.e.* the density matrix becomes more and more diagonal at higher T and the contributions of the off-diagonal elements get smaller and smaller. At the same time the degree of purity decreases and the degree of mixedness increases with T.

For $T = 0$ the asymptotic (final) state is pure and δ_{QD} reaches its initial maximum value 1. A pure state undergoing unitary evolution is highly coherent: it does not lose its coherence, *i.e.*, off-diagonal coherences never vanish. $\delta_{QD} = 0$ when the quantum coherence is completely lost. So, when $\delta_{QD} = 1$ there is no QD and only if $\delta_{QD} < 1$, there is a significant degree of QD, when the magnitude of the elements of the density matrix in the position basis are peaked preferentially along the diagonal $q = q'$. When $\delta_{QD} \ll 1$, we have a strong QD.

b) *Degree of classical correlations (CC)*

In defining the degree of CC, the form of the Wigner function is essential, but not its position around $\sigma_q(t)$ and $\sigma_p(t)$. Consequently, for simplicity we consider zero values for the initial expectations values of the coordinate and momentum and the expression (21) of the Wigner function becomes

$$W(q, p, t) = \frac{1}{2\pi\hbar}\sqrt{\frac{\alpha}{\gamma}}\exp\left[-\frac{(\hbar\beta q - p)^2}{4\hbar^2\gamma} - \alpha q^2\right]. \quad (28)$$

A ridge of the Wigner function (28) in phase space is at $p = \hbar\beta q$, showing the correlation between q and p. As a measure of the degree of CC we take the relative sharpness of this peak in the phase space determined from the dispersion $\hbar\sqrt{2\gamma}$ in p in Eq. (28) and the magnitude of the average of p $(p_0 = \hbar\beta q)$ [4]:

$$\delta_{CC} = \frac{2\sqrt{\alpha\gamma}}{|\beta|}, \quad (29)$$

where we identified q as the dispersion $1/\sqrt{2\alpha}$ of q. δ_{CC} is a good measure of the "squeezing" of the Wigner function in phase space [4]: in the state (28), more "squeezed" is the Wigner function, more strongly established are CC.

For our case, we obtain

$$\delta_{CC}(t) = \frac{\sqrt{\sigma(t)}}{|\sigma_{pq}(t)|}, \quad (30)$$

where $\sigma(t)$ is given by Eq. (24) and $\sigma_{pq}(t)$ can be calculated using formulas given in Refs. [14, 15]:

$$\sigma_{pq}(t) = \frac{\hbar}{4\Omega^2}e^{-2\lambda t}\left\{\left[\mu\omega(2\coth\epsilon - \delta - \frac{1}{\delta(1-r^2)}) - \frac{2\omega^2 r}{\sqrt{1-r^2}}\right]\cos(2\Omega t)\right.$$
$$\left. + \omega\Omega\left(\delta - \frac{1}{\delta(1-r^2)}\right)\sin(2\Omega t) + \mu\omega\left(\delta + \frac{1}{\delta(1-r^2)} - 2\coth\epsilon\right) + \frac{2\mu^2 r}{\sqrt{1-r^2}}\right\}. \quad (31)$$

When δ_{CC} is of order of unity, we have a significant degree of classical correlations. The condition of strong CC is $\delta_{CC} \ll 1,$ which assures a very sharp peak in phase space. Since $\sigma_{pq}(\infty) = 0,$ in the case of an asymptotic Gibbs state, we get $\delta_{CC}(\infty) \to \infty,$ so that our expression shows no CC at $t = \infty.$

c) *Discussion with Gaussian density matrix and Wigner function*

We have seen that if the initial wave function is Gaussian, then the density matrix (15) and the Wigner function (16) remain Gaussian for all times (with time-dependent parameters which determine their amplitude and spread) and centered along the trajectory given by Eqs. (6) and (7), which are the solutions $\sigma_q(t)$ and $\sigma_p(t)$ of the dissipative equations of motion. This trajectory is exactly classical for $\lambda = \mu$ and only approximately classical for not large $\lambda - \mu.$

The degree of QD has an evolution which shows that in general QD increases with time and temperature. The degree of CC has a more complicated evolution, but the general tendency is that CC are less and less strong with increasing time and temperature. $\delta_{QD} < 1$ and δ_{CC} is of the order of unity for long enough time, so that we can say that the considered system interacting with the thermal bath manifests both QD and CC and a true quantum to classical transition takes place. Dissipation promotes quantum coherences, whereas fluctuation (diffusion) reduces coherences and promotes QD. The balance of dissipation and fluctuation determines the final equilibrium value of δ_{QD}. The quantum system starts as a pure state, with a Wigner function well localized in phase space (Gaussian form). This state evolves approximately following the classical trajectory (Liouville flow) in phase space and becomes a quantum mixed state during the irreversible process of QD.

From expressions (22) and (29) we notice that the key parameter which describes QD and CC is γ. This coefficient determines the spread of the Wigner function (21) around the path in phase space and measures the contribution of non-diagonal terms in the density matrix (19). Therefore, when decoherence increases, the correlations between the canonical variables of coordinate and momentum decrease. The extreme limit of QD ($\gamma \to \infty$) is incompatible with CC and that of CC ($\gamma \to 0$) is incompatible with QD. Their simultaneous realization is not a trivial task: QD requires interaction with an environment, which inevitably suppresses CC and produces fluctuations in the evolution of the system, whereas classical predictability requires these fluctuations to be small. Therefore the existence of the environment is crucial for the quantum to classical transition and, consequently, classicality is an emergent property of an open quantum system. We can say that a relative competition appears between QD and existence of CC, since decoherence (diagonalizing or the decreasing of the width of the density matrix) implies a spreading of the Wigner distribution function (which is the Fourier transform of the density matrix) along the trajectory in phase space, whereas CC require the existence of sharp peaks in the Wigner function. Although there exists this competition, there is a broad compromise regime in which QD and CC can hold well simultaneously.

5. SUMMARY

We have studied QD and CC with the Markovian equation of Lindblad in order to understand the transition from quantum to classical mechanics for a system consisting of an one-dimensional harmonic oscillator in interaction with a thermal bath in the framework of the theory of open quantum systems based on quantum dynamical semigroups. Depending on the relative magnitude between the measures of QD and CC, the system recovers classicality in a significant measure. The classicality is conditioned by the CC, expressed by the fact that the Wigner function has a peak which follows (exactly for $\lambda = \mu$ and approximately for $\lambda \neq \mu$) the classical trajectory in phase space and also by QD, expressed by the loss of quantum coherence in the case of a thermal bath at finite temperature. For an initial Gaussian quantum state, which is a correlated coherent state, the Wigner function is positive for all times, so that it represents a true classical probability distribution in phase space and it describes CC.

The study of classicality using QD and CC leads to a deeper understanding of the quantum origins of the classical world. As a result of the progress made in the last two decades, the quantum to classical transition has become a subject of experimental investigations, while previously it was mostly a domain of theory [2, 3]. The issue of quantum to classical transition points to the necessity of a better understanding of open quantum systems and the Lindblad theory provides a selfconsistent treatment of damping as a general extension of quantum mechanics to open systems.

REFERENCES

1. E. Joos, H. D. Zeh, C. Kiefer, D. Giulini, J. Kupsch and I. O. Stamatescu, *Decoherence and the Appearance of a Classical World in Quantum Theory,* 2nd Edn (Springer, Berlin, 2003).
2. J. P. Paz and W. H. Zurek, in *Coherent Atomic Matter Waves, Les Houches Session LXXII,* ed. by R. Kaiser, C. Westbrook and F. David (Springer, Berlin, 2001), p. 533.
3. W. H. Zurek, Rev. Mod. Phys., **75**, 715 (2003).
4. M. Morikawa, Phys. Rev., D **42**, 2929 (1990).
5. R. Alicki, Open Sys. and Information Dyn., **11**, 53 (2004).
6. J. P. Paz, S. Habib and W. H. Zurek, Phys. Rev., D **47**, 488 (1993).
7. W. H. Zurek, Phys. Today **44**, No. 10, 36 (1991); **46**, No. 12, 81 (1993); Prog. Theor. Phys., **89**, 281 (1993).
8. M. Brune, E. Hagley, J. Dreyer, X. Maître, A. Maali, C. Wunderlich, J.-M. Raimond and S. Haroche, Phys. Rev. Lett., **77**, 4887 (1996).

9. D. A. Kokorowski, A. D. Cronin, T. D. Roberts, D. E. Pritchard, Phys. Rev. Lett., **86**, 2191 (2001).
10. E. B. Davies, *Quantum Theory of Open Systems* (Academic Press, New York, 1976).
11. G. Lindblad, Commun. Math. Phys., **48**, 119 (1976).
12. H. Spohn, Rev. Mod. Phys., **52**, 569 (1980).
13. G. Lindblad, Rep. Math. Phys., **10**, 393 (1976).
14. A. Sandulescu and H. Scutaru, Ann. Phys. (N.Y.) **173**, 277 (1987).
15. A. Isar, A. Sandulescu, H. Scutaru, E. Stefanescu and W. Scheid, Int. J. Mod. Phys., E **3**, 635 (1994).
16. V. V. Dodonov, E. V. Kurmyshev and V. I. Man'ko, Phys. Lett., A **79**, 150 (1980).
17. A. Isar and W. Scheid, Phys. Rev., A **66**, 042117 (2002).