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CONDENSED MATTER

EXCITON CONDENSATION

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Abstract. We present a model for the Bose-Einstein condensation in a low dimensional exciton system. Using the Renormalization Group method and a Φ^4 model, with $z = 2$, we calculate the temperature dependence of the correlation length, magnetic susceptibility, critical density, and the critical temperature. The model can describe the macroscopic coherence observed in GaAs/AlGaAs coupled quantum wells structure.

Key words: exciton condensation, Bose-Einstein condensation, renormalization group.

1. INTRODUCTION

An exciton, in a semiconductor, consist of an electron bound to a hole (with the electron in the conduction band, and the hole in the valence band) [1]. Usually, excitons are created by a light beam on the semiconductor. This creates an equal number of electrons and holes. These, optically generated, composite objects have a very short lifetime because the emission of light. Electrons and holes are fermions, but the composite object, the exciton, is boson. In many-particle physics, bosons can condense into the same microscopic state, predicted by Einstein in 1924. A consequence of this Bose-Einstein condensation (BEC) is that the macroscopic properties depend on a single wavefunction, with its phase coherent over distances much longer than the average separation between particles, and with dramatic physical consequences. Just as Helium-4 atoms and dilute atomic gases, reducing the temperature, excitons can undergo Bose-Einstein condensation [2]. Because the exciton effective mass is small (even smaller than the free electron mass), the exciton BEC can occur at temperatures of several order of magnitude higher than for atoms.

The major obstacle for BEC is the short lifetime of optically generated excitons. The long lifetime can be obtained in the case of “indirect excitons”. They are composite objects created in bilayer quantum well systems. Indirect excitons are bound objects of conduction band electrons in one well and valence band holes in an adjacent well. This separation reduces the recombination rate between electrons and holes and increases their lifetime.

Quantum wells are realized in layered semiconductor structures. These materials allow for the confinement of electrons and holes to two-dimensional systems [3]. Generally, in an insulator, when the binding energy of an exciton exceeds the energy gap, the insulating ground state becomes unstable against the formation of excitons [4] (excitonic insulator. It has been shown that the transition into the excitonic insulator is a second-order phase transition [5]). Excitons are described by Kohn and Sherrington [6] as bosons of type II (an elementary excitation which is a bound complex of equal numbers of fermions and holes. A type I boson is a bound system that consist of an even number of fermions, or an even number of holes), that can Bose condense. The condensed states of type II excitons implies long-range order in coordinate space (or diagonal long-range order). This type of order implies that when excitons condense, the resultant state is not superfluid (this state requires an off-diagonal long-range order [7]).

Using the terminology of Ref. [3], "ideal excitonic superfluidity" is an impossibility. The magnetic field has a strong effect on physical properties of an excitonic insulator [8]. Because the total charge of the system is zero, the excitonic insulator does not display a Meissner effect. Therefore the excitonic insulator in a magnetic filed is described in terms of Landau-level basis functions. The main effects of the magnetic field consist in a replacement of the bandgap by an effective bandgap which is increased by the cyclotron energies of Landau levels in valence and conduction bands, and an increased exciton binding energy with magnetic field. The nature of the phase transition into the excitonic insulator state, under specific conditions, has been analyzed by Guseinov and Keldysh [9].

In a model with the maximum of the valence band located at the center of the Brillouin zone, and the conduction band minima at the edges of the Brillouin zone, and with a narrow energy gap, the system becomes unstable and undergoes a first-order phase transition, if the exciton binding energy is close to the width of the energy gap. The exciton condensation in semiconductor quantum well structures has been analyzed by Zhu *et al.* [10], in order to give a propose in obtaining an exciton fluid at sufficiently high densities and low temperatures to realize the condensed phase, in connection to recent experiments on GaAs quantum wells in magnetic fields. However, the problem of Bose-Einstein condensation in reduced dimensionality systems is a difficult issue [11], because the boson condensation is not favorable in low dimensions. More recently, Chu and Chang [12] showed that fluctuations cannot destroy the exciton condensate, in two dimensions and at finite temperatures. This is attributed to the fact that electrons and holes have opposite charges. The effect of an internal degree of freedom (spin degree of freedom) has been discussed by Fernandez-Rossier and Tejedor [13], in connection to two-dimensional exciton condensation for excitons that have different spin orientations.

The energy and the chemical potential of the system depend strongly on the spin polarization, and when electrons and holes are located in different planes the condensate can be totally spin polarized or spin unpolarized. The interest in superfluidity of condensed excitons has growing in the last decade, mainly due to the experimental investigations of exciton system in CuO_2. Here, a supersonic ballistic exciton propagation was found [14], attributed either to a superfluid state or to the fact that excitons are dragged by a flow of ballistic phonons. Other models for superfluidity of excitons, but in type-II semiconductor quantum wells (such as GaAs/AlAs structure) were proposed in Ref. [15]. In this paper we will study using the renormalization group (RNG), the possibility of a BEC of excitons in systems with a reduced dimensionality. In such systems, which usually are considered to be quasi-2D systems, the BEC is known as a quasicondensation because it does not appear in the thermodynamic limit. The model we explore is similar to the one applied to a 2D bosonic system [16], concerning the physical properties of the condensed state in He.

2. RENORMALIZATION GROUP METHOD

The bosonic system formed by excitons is described by the following action:

$$S = \frac{1}{2}\int \mathrm{d}\tau\, \mathrm{d}^d r \left[\left(\partial_\tau - \nabla^2 - \mu \right) |\Phi|^2 + \frac{t_0}{8} |\Phi|^4 \right] \tag{1}$$

where $\Phi \equiv \Phi(\mathbf{r}, \tau)$ is the bosonic field describing the electron-hole density fluctuations, t_0 the bare interaction between the bosons and μ the effective chemical potential. In the case of an excitonic system formed in a semiconductor, the effective chemical potential is given by:

$$\mu = \epsilon_g - \epsilon_0 \tag{2}$$

where ϵ_g is the semiconducting band gap and ϵ_0 is the exciton binding energy. The radius of the exciton is $r_0 \sim \epsilon_0^{1/2}$. The existence of a low density parameter is determined by the range of the interaction, and the mean separation between constituent particles. In this case:

$$r_0 \sqrt{\mu} \ll 1 \tag{3}$$

With this condition we can describe the condensation of the excitons in a large temperature interval, including the critical region. Concerning the phase transition in the exciton gas we will use the method given by Popov [17]. We will introduce an intermediate momentum p_i, with $\mu < \varepsilon_i < \varepsilon_0$, and $\varepsilon_i = p_i^2/2m$. The effective interaction between particles, u_0, in the t-matrix approximation is:

$$u_0 = \frac{t_0}{1 + t_0 \Pi} \tag{4}$$

with the polarization operator Π:

$$\Pi = \int_{1/r_0}^{p_i} \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{1}{k^2} \tag{5}$$

In the two-dimensional case the effective interaction is approximated as:

$$u_0 \simeq \frac{2}{\pi \ln(\varepsilon_0/\varepsilon_i)} \tag{6}$$

and the resulting effective interaction is repulsive and small. This allows us to use the RNG method to describe the Bose system formed by excitons. The standard RNG differential equations corresponding to the renormalized chemical potential, interaction, and temperature are:

$$\frac{\mathrm{d}\mu(l)}{\mathrm{d}l} = 2\mu(l) + K_2 F_1(T) u(l) \tag{7}$$

$$\frac{\mathrm{d}u(l)}{\mathrm{d}l} = -\frac{K_2}{4} u^2(l) \tag{8}$$

$$\frac{\mathrm{d}T(l)}{\mathrm{d}l} = 2T(l) \tag{9}$$

with $K_2 = 1/2\pi$ and:

$$F_1(T) = \frac{1}{2} \coth \frac{1}{2T(l)} \tag{10}$$

The set of the RNG differential equations has the following solutions:

$$T(l) = T \exp(2l) \tag{11}$$

$$u(l) = \frac{1}{C(l + l_0)} \tag{12}$$

where $C = K_2/4$ and $l_0 = 8\pi/u_0$.

$$\mu(l) = \exp[\Lambda(l)] \left[\mu - \mu_c + \frac{1}{2\pi} \int_0^l \mathrm{d}l' \frac{e^{-2l'} u(l')}{e^{1/T(l')} - 1} \right] - \frac{u(l)}{8\pi} \tag{13}$$

Here $\mu_c = u_0/8\pi$ and $\Lambda(l)$ has the following expression:

$$\Lambda(l) = 2l \left[1 - \frac{1}{l} \ln\left(1 + \frac{l}{l_0} \right) \right] \tag{14}$$

The occurrence of a phase transition in the exciton system can be studied with the help of a new parameter $t_\mu(l)$, introduced as:

$$t_\mu(l) = \mu(l) + \frac{u(l)}{8\pi} \tag{15}$$

With Eqs. (12) and (13) this new parameter becomes:

$$t_\mu(l) = \exp[\Lambda(l)]\left[t_\mu(0) + \frac{u_0 T}{4\pi}\int_a^\infty \frac{dx}{e^x - 1}\right] \tag{16}$$

where $a = u_0/4\pi$. The renormalization procedure will be stopped at $l = l^*$, where l^* is the solution of the equation:

$$t_\mu(l^*) = 1 \tag{17}$$

and is given by:

$$\exp(-2l^*) \simeq \frac{T}{\ln(1/T)} \tag{18}$$

With the help of this relation, the RNG method allow for the evaluation of the thermodynamic properties of the system in the critical region [18].

The correlation length, ξ, will be calculated as:

$$\xi \simeq \xi_0 \exp(l^*) \tag{19}$$

and its temperature dependent formula is:

$$\xi(T) \simeq \xi_0 \frac{|\ln(1/T)|^{1/2}}{T^{1/2}} \tag{20}$$

The magnetic susceptibility, χ, is given by:

$$\chi \simeq \chi_0 \exp(2l^*) \tag{21}$$

and as a function of temperature:

$$\chi(T) \simeq \chi_0 \frac{|\ln(1/T)|}{T} \tag{22}$$

Here ξ_0 and χ_0 are constants.

Another important parameter is the critical temperature for the BEC in the excitonic system. This will be calculated using the critical density, $n(T)$, expressed with the help of the Bose-Einstein function in $l = l^*$:

$$n(T) = \exp(-2l^*)\int_0^\infty \frac{d^2k}{(2\pi)^2}\left\{\exp\left[\frac{k^2+\mu}{T(l^*)}\right] - 1\right\}^{-1} \tag{23}$$

In two dimensions Eq. (23) leads to the following result:

$$n(T) \simeq T \ln|\ln(1/T)| \tag{24}$$

This equation can be inverted to give the critical temperature:

$$T_c(n) \simeq \frac{n}{\ln\left|\ln(1/n)\right|} \tag{25}$$

Other important thermodynamic parameters (*e.g.*, specific heat) can also be calculated using the RNG method (see Refs. [16, 18, 19].

3. CONCLUSION

We developed a formalism based on the RNG method for bosonic systems in order to describe the physical properties of an exciton gas. We considered the case of a two dimensional system, a configuration which is very close to the quasi two dimensional geometries of GaAs quantum wells, where an exciton condensation can be observed. Our results show that in the case of a two dimensional system the critical temperature for the BEC in the exciton system has a double logarithmical dependence on density. We proposed a microscopic description of the exciton condensation in a two dimensional system in terms of an effective action similar to that for the interacting Bose liquid with repulsive short-range interaction. For this model the effective coupling constant between the excitons was evaluated using the t-matrix approximation, and based on the characteristic energy scale we showed that it is small. Accordingly, the RNG method in the one-loop approximation can give relevant results. The critical density presents a non-linear temperature dependence, which can describe the behavior of the experimental results in a relevant temperature interval. However, even this simplified model showed the importance of the quantum effects in the condensation of the excitons.

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