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ON THE CLASSICAL APPROXIMATION OF THE INTERACTION BETWEEN ELECTRONS AND VERY INTENSE LASER BEAM

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Abstract. In a previous paper we proved that the solution of the Klein-Gordon equation that describes the behavior of the system electron – very intense laser beam is at the classical limit. We show that this result is in agreement with Feynman's path integral approach to this interaction. This result leads to a simplified model of analysis of the systems composed by atoms and very intense electromagnetic field on the domain where the electrostatic interactions between electrons and nuclei are neglected.

Key words: electromagnetic fields, electrons, laser beams, partial differential equations, quantum theory, relativistic effects, wave functions, Feynman's path integral.

1. INTRODUCTION

It has been known for a long time [1, 2] that the Schrödinger and Klein-Gordon equations for the system charged particle-electromagnetic field have paticular solutions of the so-called Volkov type. In the past two decades, the study of such systems became increasingly important, due to the emergence of the new field of interactions between particles and very high intensity laser fields.

The problem of solving the quantum equation describing the system electron-electromagnetic field, is also relevant for the new field studying the interaction between very intense electromagnetic fields and atoms [3–5]. The solution presented here is valid also in the case of field interaction with atoms, at least for most of the domain of integration, where the electrostatic interactions with nuclei are neglected. For an example of the connection between the two problems, see Faisal's analytical solution of the Klein-Gordon equation describing the interaction between very intense laser fields and atoms [6], which uses the solution of the Klein-Gordon equation for a free particle in a laser field.

Our approach to the Klein-Gordon equation [8] for the system electron–electromagnetic field is similar to those presented in our previous papers for different systems [9–12]. The methods used in these papers are based on a parallel study of a given system using the quantum and classical equations. Our approach has the advantage of simplifying the analysis of electron-laser field systems. In fact we show that the solution of the quantum equation for an electron in very intense laser field is at the classical limit, and hence it can be approximated by a solution of the classical equation describing the behavior of the system.

In recent time there has been an increasing tendency to study the systems of atoms in very intense laser beams, in the frame of Feynman's theory of paths integrals [13–16]. In this paper we show that our model and Feynman's theory lead to similar conclusions, in spite of the fact that they are completely different.

2. BRIEF DESCRIPTION OF OUR MODEL

We begin with a very short description of our model [7]. The equations are written in the international system. We analyze a system composed of an electron interacting with an electromagnetic linear polarized field. In a Cartesian system of coordinates, the vector potential of the field, denoted by $\overline{A}$, is parallel to the *ox* axis and the wave vector, denoted by $\overline{k}$, is parallel to the *oz* axis. The equations are written in the international system.

We consider the following initial hypotheses:

(h1) The electromagnetic field is of the type produced by a very intense laser beam, and the value of the intensity of its electric field is of the order of one atomic unit, namely 5.1423×10^{11} V/m, or greater.

(h2) The interaction between the electron spin and the electromagnetic field is neglected and the behavior of the system is described by the Klein-Gordon equation [17]:

$$\left[c^2\left(-i\hbar\nabla + e\overline{A}\right)^2 - \left(i\hbar\frac{\partial}{\partial t}\right)^2 + \left(mc^2\right)^2\right]\Psi = 0 \tag{1}$$

where $\hbar$, m, c, and Ψ are, respectively, the normalized Planck constant, the electron mass, the velocity of light in vacuum and the wave function. Throughout the paper, we denote by e the absolute value of the electron charge, the sign being written explicitly.

(h3) The dimension of the domain D where the motion of the electron takes place is much smaller than the wavelength of the laser radiation and the following relation is valid

$$\overline{k}\cdot\overline{r} = \frac{2\pi}{\lambda}\frac{\overline{k}}{\left|\overline{k}\right|}\cdot\overline{r} \cong 0 \tag{2}$$

where $\overline{r}$ is the position vector having the origin somewhere in the domain D and λ

is the wavelength of the radiation. Consequently, the intensity of the electric field and the vector potential vary only with respect to time, according to the relations:

$$\overline{E} = \overline{i} E_M \cos \omega t \tag{3}$$

$$\overline{A} = -\overline{i} A_M \sin \omega t = -\overline{i}\, \frac{E_M}{\omega} \sin \omega t \tag{4}$$

In these relations ω is the angular frequency, $\overline{i}$ is the versor of the *ox* axis and the maximum values E_M and A_M are positive.

We rewrite the Klein-Gordon equation using the substitution

$$\Psi = \exp\left(i\sigma/\hbar\right) \tag{5}$$

where σ is a complex valued function of the electron coordinates and time. The Klein-Gordon equation (1) becomes:

$$\left[c^2\left(\nabla\sigma + e\overline{A}\right)^2 - \left(\frac{\partial\sigma}{\partial t}\right)^2 + \left(mc^2\right)^2\right] - i\hbar c^2\left(\Delta\sigma - \frac{\partial^2\sigma}{c^2\partial t^2}\right) = 0 \tag{6}$$

As in our previous papers, [8] and [9], we performed [7] a parallel analysis of the same system using the quantum Klein-Gordon and classical relativistic Hamilton-Jacobin equations. The latter is [18]:

$$c^2\left(\nabla S + e\overline{A}\right)^2 - \left(\frac{\partial S}{\partial t}\right)^2 + \left(mc^2\right)^2 = 0 \tag{7}$$

where S is the classical action, which for the moment has only mathematical significance.

By comparing (6) with (7), we observe that the term in the last parenthesis of (6) can be interpreted as describing the quantum behavior of the system. Our method from [7] is based on showing that the two equations (6) and (7) have solutions σ and, respectively, S, which are equal with a very good approximation for a system satisfying the hypotheses (h1)–(h3). This implies that the system is at the classical limit, because in this case the term in the last parenthesis of (6) is negligible. We proved [7] that

$$\sigma = S + \alpha + i\beta \tag{8}$$

where α and β are real valued functions having the dimension of action. Our evaluations were performed [7] for an electric field whose intensity is equal to one atomic unit, namely 5.1423×10^{11} V/m and for the angular frequency of a YAG:Nd laser, which is 1.777×10^{15} s^{-1}. For these numerical data we proved [7] that $|\alpha|/|S| = 3.775\times10^{-14}$ and $|\beta|/|S| = 1.650\times10^{-8}$, resulting that the solution of the equation (1) is at the classical limit for the system in discussion.

3. ANALYSIS OF OUR RESULT IN THE LIGHT OF THE FEYNMAN'S PATH INTEGRAL APPROACH

In 1948 Richard Feynman proposed an alternative formulation of the quantum mechanics. Feynman proved [13] that if we are given the value of the wave function at some point $\overline{r}_1$, in the space of the coordinates, at the time t_1, the Schrödinger equation enables us to find the value of the wave function at a point $\overline{r}_2$, at a time t_2. It results a new function, having the significance of the amplitude of probability to go from a point $\overline{r}_1$, at the time t_1, to the point $\overline{r}_2$, at a time t_2. The Feynman's function is

$$K_{r_1,t_1\to r_2,t_2} = \sum_n a_n \exp\left\{iS\left[r_n(t)\right]/\hbar\right\} \tag{9}$$

where $S\left[r_n(t)\right]$ is the action, which corresponds to an evolution on a classical path, denoted by n, while a_n is the amplitude to evolve on that path. The summing in (9) is made on an infinite number of paths.

The alternative formulation of the quantum mechanics developed by Feynman is based on the study of the functions given by (9). It builds on the familiar Lagrangian concept of the action of an orbit in space and time and appears to be closer to classical concepts.

The analysis of the interaction between very intense laser beams and atoms is based on the same hypotheses, like (h1)–(h3), namely the spin interactions are neglected, $\overline{A}$ is the classical vector potential in the interaction terms and the relation (2) is valid. Like in our approach [7], we limit to the interaction of the atoms with the linear polarized electromagnetic field.

With the above hypotheses, the analysis in the frame of the Feynman's model leads to the following results [14–16].

1) By the action of the laser field, an electron may be ionized at some time t_i and return to its parent ion at the later time $t_f = t_i + \tau$, where it may recombine with the ion emitting a photon of high energy, or rescatter and move out of the laser focus.

2) The most important result from [15] is that the sum in (9) reduces over a few quantum orbits, numbered by the subscript n. This is the case corresponding to the classical limit in the Feynman's theory.

3) The total action is the sum of three terms, given by the following relation

$$S\left[r_n(t)\right] = S_{bound,n}(t_i) + S_{free,n}\left(t_{fn}, t_{in}\right) + S_{final,n}\left(t_{fn}\right) \tag{10}$$

where $S_{bound,n}(t_i)$ is the action corresponding to the time interval when the electron is bound, $S_{free,n}\left(t_{fn}, t_{in}\right)$ is the action on the time interval when the electron is free and $S_{final,n}\left(t_{fn}\right)$ corresponds to the final time interval, when the electron is rescattered. We denote, respectively, by I, II and III, the domains corresponding to the three components of the action.

4) In the case of a linear polarized field, the electrons are generated with negligible velocity in the domain II [14, 15].

It is easy to show that when the electron enters with very small velocity in the domain II, according to the above result 4), then the basic equations of the Hamilton-Jacobi formalism, which are

$$S = \int_{t_0}^{t} \left(-mc^2 \sqrt{1 - \frac{v^2}{c^2}} - \overline{v} \cdot e\overline{A} \right) \mathrm{d}t \tag{11}$$

$$\nabla S = \overline{p} - e\overline{A} \tag{12}$$

$$\overline{p} = \frac{m\overline{v}}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{13}$$

$$\frac{\mathrm{d}\,\overline{p}}{\mathrm{d}t} = e\frac{\partial \overline{A}}{\partial t} = -e\overline{E} \tag{14}$$

lead to trajectories which are always parallel to the vectors $\overline{A}$ and $\overline{E}$ [7]. Consequently, the system is in the limit case, when the number of the possible classical trajectories is roughly equal to one, corresponding to the classical limit in Feynman's theory of integral paths. We observe that this result agrees with the above result 2). This result is validated by experiment, because in [19] it is shown that the electrons are preferentially ejected along the direction of the laser's linear polarization and the distributions become narrower with increasing number of absorbed photons.

We point out that our solution from [7] is valid for the domain II, where the electrostatic interactions with nuclei are neglected. It is easy to observe that, though our solution and the approach based on the Feynman's theory are completely different, they lead to similar conclusions: the behavior of the domain II can be approximated classically.

CONCLUSIONS

The analysis of the interaction between very intense laser beam and atoms, on the domain where the electrostatic interactions with nuclei are neglected can be performed with both, our model from [7] and Feynman's path integrals theory. In spite of the fact that these approaches are completely different, they lead to the conclusion, that the solutions of the quantum equations are at the classical limit on the domain where the electrostatic interactions are neglected.

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