Romanian Reports in Physics, Vol. 57, No. 4, P. 1011–1019, 2005

A DIFFUSION-REACTION SYSTEM UNDER THE INFLUENCE OF A LOW RADIO-FREQUENCY-FIELD

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(Received June 15, 2005)

Abstract. In this paper we present some theoretical results concerning the influence of a low amplitude *radio-frequency* (RF) field on the dynamics of a diffusion-reaction (D-R) system. The results show that even with very low RF fields one can control the dynamics driving the system in either a chaos state or in a coherent one.

Key words: diffusion-reaction system, RF-field, dynamics control, plasma, Lotka-Voltera.

1. INTRODUCTION

The diffusion-reaction (D-R) systems represent a special class of systems due to the diversity of states in which they can be encountered, beginning with the chemical and biological ones [1, 2] and continuing with lattice systems [3] plasma [4] and even some ecological ones [6]. The dynamics of such systems is mainly controlled by the diffusion processes and the reactions taking place between the components of the system. Studies referring to the self-organization process have revealed the capacity of the D-R system to produce structures based on a spatial-temporal symmetry breaking [1, 2]. Experiments on such cases put into evidence the fact that, following the way in which the components density evolve in time and space, they could be classified in three important types, according to the role played in the process of self-organization:

– activators (a),

– inhibitors (h),

– intermediates (i).

The intermediate component is sometimes also called antagonist, in the sense that it either increases the production rate of the inhibitor or, directly decreases the production rate of the activator. For a D-R system to evolve to a spatial-temporal structure, it is necessary that the diffusion coefficient of the inhibitor be greater than that of the activator. As a result, the frontier of the structure is actually a double layer consisting of an inner activator layer and of an outer inhibitor one.

Once a self-organized structure is achieved, its dynamics depends on some external constraints realized by proper values of the control parameters (external state parameters). Depending on these values the state of the structure could be a chaotic or a coherent one with a desired pattern (temporal or/and spatial). Most of the time, the control parameters are perturbed by some quasi-uncontrollable factors. From a great variety of such factors, the electromagnetic perturbations are by far the most encountered ones.

In this paper we investigate the influence of such a perturbation upon a D-R system. Because of the dominance of the electric force against the magnetic one in the case of non-relativistic velocities (which is the case for the great majority of the D-R systems), we shall refer to the electric component of a RF-field.

2. THEORETICAL MODEL

In what follows we consider a D-R system in which the components are particles, some of which are charged (ions, electrons) and the others are neutral. This system is under the influence of a modulated low amplitude RF-field. Such a system will be described by the following equations:

$$\begin{cases} \dfrac{\partial n_k}{\partial t} = D_k \cdot \nabla^2 n_k - \nabla \cdot \left(n_k \cdot \vec{w}_k \right) + \sum\limits_r R_{kr} & (a) \\ \dfrac{\partial \vec{w}_k}{\partial t} + \left(\vec{w}_k \cdot \nabla \right) \vec{w}_k = \dfrac{q_k}{m_k} \cdot \vec{E} & (b) \\ \nabla \cdot \vec{E} = \dfrac{1}{\varepsilon_0} \cdot \sum q_k \cdot n_k + \dfrac{\tilde{\mu}}{\varepsilon_0} & (c) \end{cases} \qquad (1)$$

where n_k = the particle density of species k; $\vec{w}_k$ = the drift velocity of the k-species particles; q_k and m_k = the charge and the mass of the same particles; $\vec{E}$ = the resultant electric field inside the system; R_{kr} = the reaction term for the k-species particles participating in the r-th reaction (is an element of a matrix R whose rows correspond to the species of the particles and the columns correspond to the reaction order). The index k takes the values "a" for activator, "i" for intermediate and "h" for inhibitor. $\tilde{\mu}$ refers to the external electric RF-field. Let the RF-field be given by $\vec{E}_x(r,t) = \vec{E}_0(r) \cdot \cos(\Omega \cdot t) \cdot \cos(\omega \cdot t)$, where Ω is the modulation pulsation and ω is the high frequency one: $\Omega << \omega$. The control parameter $\tilde{\mu}$ entering in the Poisson equation (1, c) is given by:

$$\frac{\tilde{\mu}}{\varepsilon_0} = \nabla \cdot \vec{E}_x = \nabla \cdot \vec{E}_0(r) \cdot \cos(\Omega \cdot t) \cdot \cos(\omega \cdot t) = \frac{\mu}{\varepsilon_0} \cdot \cos(\Omega \cdot t) \cdot \cos(\omega \cdot t) \qquad (2)$$

In what follows we consider only inelastic collisions and the elements R_{kr} will be given using the BGK approximation [7]:

$$R_{kr} = \kappa_{kr} \cdot n_{1(r)}^{\alpha} \cdot n_{2(r)}^{\beta} \quad (3)$$

where κ_{kr} is the rate of the r-th reaction in which a k-species particle can either appear or disappear (in this latter case a minus sign will be in front of κ_{kr}); $n_{1(r)}$ and $n_{2(r)}$ are the densities of those types of particles which collide in the r-th reaction; α, β are some exponents depending on the reaction mechanism. Because the reaction rates κ_{kr} are defined as the mean value of the product between the particle velocity and the cross section of the inelastic collision implied in the r-th reaction, it follows that they will depend on field *i.e.* on the $\tilde{\mu}$ parameter.

• As we consider a low amplitude RF-field, it is to be expected that the perturbations are sufficiently small and, as a consequence, some nonlinear terms of the above equations could be linearized around the equilibrium state specified by:

$$n_k\big|_{eq} = n_{k0}; \quad \vec{E}\Big|_{eq} = 0; \quad \vec{w}_k\big|_{eq} = 0; \quad \tilde{\mu}\big|_{eq} = 0; \quad \sum_r R_{kr}\Big|_{eq} = 0 \quad (4)$$

We shall linearize the second term on right hand side of (1, a) and the second term on left hand side of (1, b). Doing so and eliminating $\vec{w}_k$ and $\vec{E}$ between the equations of system (1) one obtains for every species of particles an equation of the form:

$$\frac{\partial^2 n_k}{\partial t^2} - D_k \cdot \nabla^2 \frac{\partial n_k}{\partial t} - \sum_r \frac{\partial R_{kr}}{\partial t} + \frac{\Omega_k^2}{q_k} \cdot \sum_j q_j \cdot n_j = -\mu \cdot \frac{\Omega_k^2}{q_k} \quad (5)$$

where we have introduced the notation $\frac{n_{k0} \cdot q_k^2}{\varepsilon_0 \cdot m_k} = \Omega_k^2$, which has the dimension of T^{-2}. Taking into account the above considerations concerning κ_{kr} and the amplitude of the external field, we shall specify κ_{kr} as functions of $\tilde{\mu}$ in the form:

$$\kappa_{kr}(\tilde{\mu}) = \kappa_{kr0} + f_{kr}(\tilde{\mu}), \quad f(0) = 0 \quad (6)$$

where κ_{kr0} is the value of κ_{kr} in the absence of the external field, at equilibrium and the function $f_{kr}(\tilde{\mu})$ will be specified for a given D-R system.

• If we are interested only in the temporal evolution of the particles densities, we must overcome the spatial dependence contained in the second term on left hand side of (5). This could be performed if we take into account the length over which one encounters a variation of n_k specific to the diffusion process. This is the diffusion length λ_{Dk}. Therefore, the following approximation will be considered:

$$D_k \cdot \nabla^2 \frac{\partial n_k}{\partial t} = D_k \cdot \frac{\partial}{\partial t}\left(\nabla^2 n_k\right) \approx D_k \cdot \frac{\partial}{\partial t}\left(\frac{n_k}{\lambda_{Dk}^2}\right) = \frac{D_k}{\lambda_D^2} \cdot \frac{\partial n_k}{\partial t} = \frac{1}{\tau_{Dk}} \cdot \frac{\partial n_k}{\partial t} \tag{7}$$

where τ_{Dk} is the characteristic diffusion time. So, the equation (5) now reads:

$$\frac{\mathrm{d}^2 n_k}{\mathrm{d}t^2} - \frac{1}{\tau_{Dk}} \cdot \frac{\mathrm{d}n_k}{\mathrm{d}t} - \frac{\mathrm{d}}{\mathrm{d}t}\sum_r R_{kr} + \frac{\Omega_k^2}{q_k} \cdot \sum_j q_j \cdot n_j = -\tilde{\mu} \cdot \frac{\Omega_k^2}{q_k} \tag{8}$$

• Usually, the characteristic lengths of the D-R systems are smaller than the wave length of the RF-field. Due to this, it is not necessary to take into account a possible spatial distribution of the field inside the system.

• We are interested in the study of those phenomena that have time-scales much longer than that of the period $\tau = 2\pi/\omega$. Therefore, if we take the mean (< >) of (8) over a period τ then, the time-derivatives and the functions $n_k(t)$ could be taken as constants. As $\langle \tilde{\mu} \rangle = 0,$ the mean of (8) will reads:

$$\frac{\mathrm{d}^2 n_k}{\mathrm{d}t^2} - \frac{1}{\tau_{Dk}} \cdot \frac{\mathrm{d}n_k}{\mathrm{d}t} - \frac{\mathrm{d}}{\mathrm{d}t}\sum_r \left\langle f_{kr}(\tilde{\mu}) \cdot n_{1(r)}^{\alpha} \cdot n_{2(r)}^{\beta} \right\rangle + \frac{\Omega_k^2}{q_k} \cdot \sum_j q_j \cdot n_j = 0 \tag{9}$$

In (9) we have taken into account (3) and (6) and the property (4) for $\sum_r R_{kr}$. In order to facilitate the computer simulations, the equation (9) will be reduced to a dimensionless form by introducing the following dimensionless variables:

$$N_k = \frac{n_k}{n_0}, \quad \xi = \frac{t}{T}, \quad X_k^2 = \tau^2 \cdot \Omega_k^2, \quad T_{Dk} = \frac{\tau_{Dk}}{\tau}, \quad Q_{jk} = \frac{q_j}{q_k} \tag{10}$$

where n_0 is the equilibrium density for the homogeneous system. Thus, equation (9) becomes:

$$\begin{gathered}\frac{\mathrm{d}^2 N_k}{\mathrm{d}\xi^2} - \frac{1}{T_{Dk}} \cdot \frac{\mathrm{d}N_k}{\mathrm{d}\xi} - n_0 \cdot \tau \cdot \frac{\mathrm{d}}{\mathrm{d}\xi}\sum_r \langle f_{kr}(\tilde{\mu})\rangle \cdot N_{1(r)}^{\alpha} \cdot N_{2(r)}^{\beta} + \\ + X_{ki}^2 \cdot \sum_j Q_{jk} \cdot N_j = 0\end{gathered} \tag{11}$$

It is evident that the product $n_0 \cdot \tau \cdot \frac{\mathrm{d}}{\mathrm{d}\xi}\langle f_{kr}(\tilde{\mu})\rangle$ is also dimensionless.

This will be the equation describing the D-R system under the conditions specified above. In what follows we shall utilize equations (3), (6) and (11) for some particular systems.

3. EXAMPLES

A) We start with plasma consisting of neutral atoms (n), ions, electrons and excited atoms. These types of particles, as has been shown in [4], could be identified with the specific types of D-R system particles as follows:

Ions $\rightarrow$ activators (a)

Electrons $\rightarrow$ inhibitors (h)

Excited atoms $\rightarrow$ intermediates (i)

We consider the following reactions:

1) Excitations: $h + n \rightarrow h + i$

2) Ionizations: $h + i \rightarrow a + h + h$

3) Recombinations: $a + h \rightarrow i$

4) Spontaneous de-excitations: $i \rightarrow n + (h\nu)$

The neutral atoms density is n_0 and it remains practically constant. For this reason it will be considered in the dimensionless variables N_k. The reaction matrix R reads (the rows order is a, h, i and the columns order corresponds to the above reactions):

$$R = \begin{pmatrix} 0 & \kappa_{a2} \cdot n_h \cdot n_i & -\kappa_{a3} \cdot n_a \cdot n_h & 0 \\ 0 & \kappa_{h2} \cdot n_h \cdot n_i & -\kappa_{h3} \cdot n_a \cdot n_h & 0 \\ \kappa_{i1} \cdot n_h & -\kappa_{i2} \cdot n_h \cdot n_i & \kappa_{i3} \cdot n_a \cdot n_h & -\kappa_{i4} \cdot n_i \end{pmatrix} \tag{12}$$

The following functions $f_{kr}(\tilde{\mu})$ will be considered:

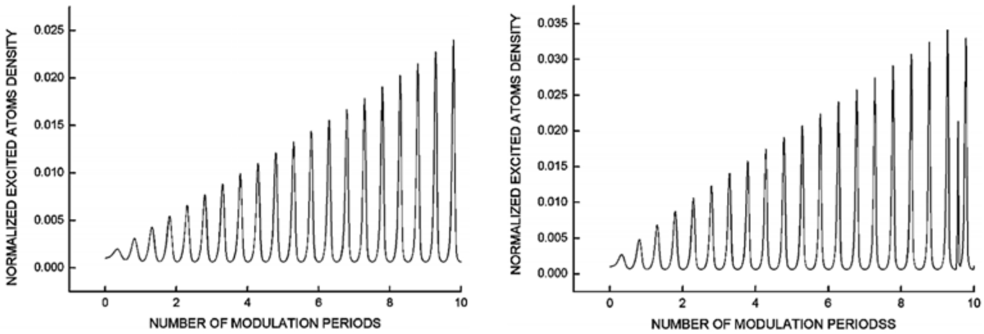
$$\begin{cases} f_{a2}(\tilde{\mu}) = f_{h2}(\tilde{\mu}) = a \cdot \tilde{\mu}^2 ; \quad f_{a3}(\tilde{\mu}) = f_{h3}(\tilde{\mu}) = -b \cdot \tilde{\mu}^2 \\ f_{i1}(\tilde{\mu}) = c \cdot \tilde{\mu}^2; \quad f_{i2}(\tilde{\mu}) = -a \cdot \tilde{\mu}^2; \quad f_{i3}(\tilde{\mu}) = b \cdot \tilde{\mu}^2; \quad f_{i4}(\tilde{\mu}) = -d \cdot \tilde{\mu}^2 \end{cases} \tag{13}$$

Note: Generally, one could take for functions $f_{kr}(\tilde{\mu})$ a polynomial expression but, with $\tilde{\mu}(t)$ given by (2), it is evident that $\langle \tilde{\mu} \rangle = \langle \tilde{\mu}^3 \rangle = 0.$ So, we retained in $f_{kr}(\tilde{\mu})$ only a term in $\tilde{\mu}^2$, whose mean is different from zero. In order to solve equation (11) for this example we have taken the following values for the parameters entering in this equation:

$$N_0 = 10^9, \quad \tau_{Da} = \tau_{Di} = 1, \quad \tau_{Dh} = 10^{-1}, \quad a = c = 10^{-4}, \quad b = d = 10^{-3}$$

$$\tau = 7.8 \cdot 10^{-2}, \quad \mu = 3 \cdot 10^{-2}$$

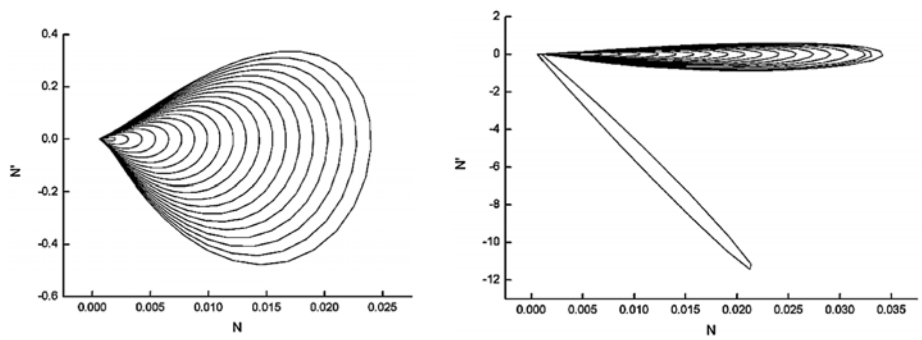
In Fig. 1 the evolution of the excited atoms density $N_i(\xi)$ over a number of 10 periods τ is shown, for two different values of the modulation amplitude. This evolution could also be watched in an experiment by monitoring the light emitted from plasma. This has been done in [5] and the results are similar to the theoretical ones presented here.



(a) (b)

Fig. 1 – The evolution of the excited atoms normalized density over 10 periods τ, for two different values of the amplitude μ: a) $\mu = 2\cdot10^{-2}$ and b) $\mu = 3\cdot10^{-2}$.

In Fig. 2 the two dimensional phase space section $\left(N_i(\xi), N_i'(\xi)\right)$, corresponding to the two cases in Fig. 1, is shown. Here one can better observe the change in the dynamics of the D-R system.



(a) (b)

Fig. 2 – The two dimensional phase-space $\left(N_i(\xi), N_i'(\xi)\right)$, for the two cases in Fig. 1.

The above results show that this D-R system is very sensitive to the amplitude of the RF-field. Such a sensitivity is also put in evidence for the values of the modulation period τ.

B) In this second example we consider a D-R system in which the reactions are of Lotka-Volters type:

1) $S + a \rightarrow a + a$

2) $A + h \rightarrow h + h$

3) $h \rightarrow E$

where S stands for "substrate" and E stands for "extinction" of the species h. It is known that in a prey–predator model a is the "prey" and h is the "predator". The corresponding reaction matrix R is given bellow:

$$R = \begin{pmatrix} \kappa_{a1} \cdot n_S \cdot n_a & -\kappa_{a2} \cdot n_a \cdot n_h & 0 \\ 0 & \kappa_{a2} \cdot n_a n_h & -\kappa_{h3} \cdot n_h \end{pmatrix} \tag{14}$$

For $f_{kr}(\tilde{\mu})$ functions we consider similar expressions to those in example **A):**

$$f_{a1}(\tilde{\mu}) = a \cdot \tilde{\mu}^2, \quad f_{a2}(\tilde{\mu}) = -b \cdot \tilde{\mu}^2, \; f_{h3}(\tilde{\mu}) = -c \cdot \tilde{\mu}^2 \tag{15}$$

As in the Lotka-Voltera model one considers chargeless components, it results that

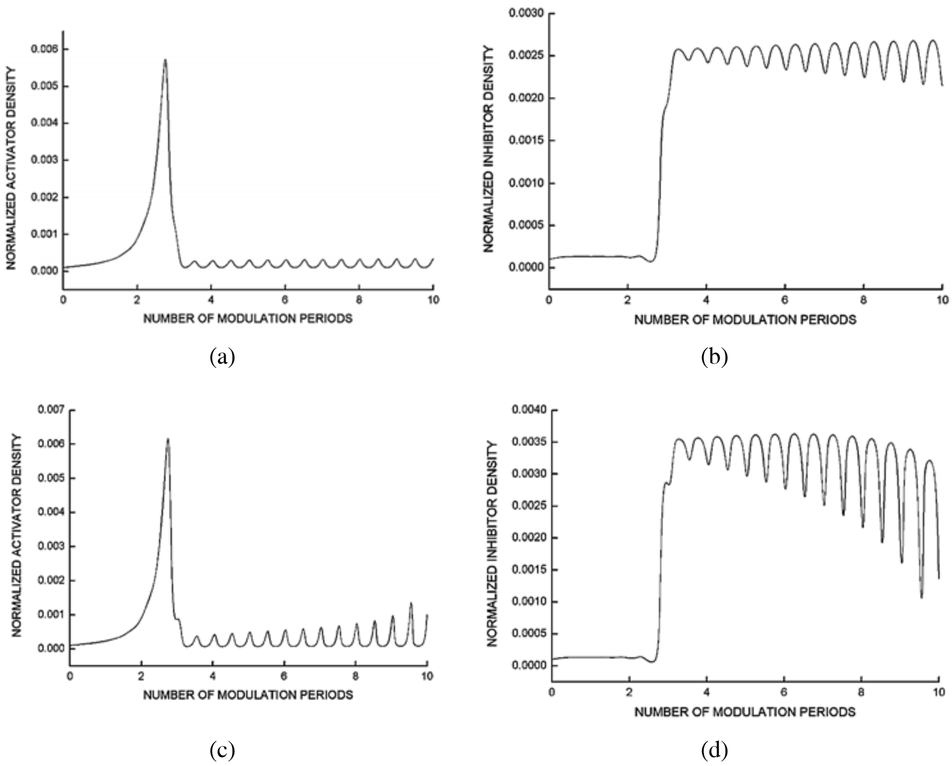


Fig. 3 – The evolution of the two components (a and h) normalized densities over 10 periods τ, for two different values of the amplitude μ: a) and b) for $\mu_1 = 1 \cdot 10^{-2}$, c) and d) for $\mu_2 = 1.17 \cdot 10^{-2}$.

the last term in eq. (11) will become zero. Taking the substrate density n_S as the normalizing density, the equations for the densities N_a and N_h read:

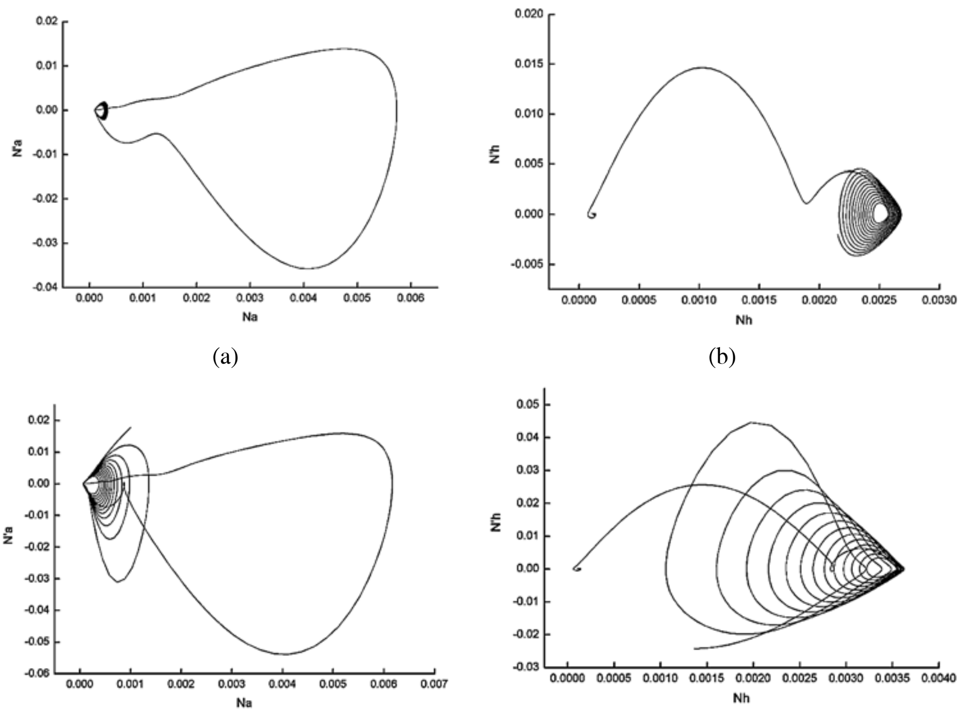
$$\begin{cases} \dfrac{\mathrm{d}^2 N_a}{\mathrm{d}\xi^2} - \dfrac{1}{T_{Da}} \cdot \dfrac{\mathrm{d}N_a}{\mathrm{d}\xi} - n_S \cdot \tau \cdot \dfrac{\mathrm{d}}{\mathrm{d}\xi}\Big[N_a \left(\kappa_{a10} - \kappa_{a20} \cdot N_h \right) + \left\langle \tilde{\mu}^2 \cdot N_a \left(a + b \cdot N_h \right) \right\rangle \Big] = 0 \\ \dfrac{\mathrm{d}^2 N_h}{\mathrm{d}\xi^2} - \dfrac{1}{T_{Dh}} \cdot \dfrac{\mathrm{d}N_h}{\mathrm{d}\xi} - n_S \cdot \tau \cdot \dfrac{\mathrm{d}}{\mathrm{d}\xi}\Big[N_h \left(\kappa_{a20} \cdot N_a - \kappa_{h30} \right) + \left\langle \tilde{\mu}^2 \cdot N_h \left(c - b \cdot N_a \right) \right\rangle \Big] = 0 \end{cases} \quad (16)$$

For this example we have taken the following values for the parameters entering in this equation:

$$n_S = 10^6, \quad T_{Da} = 10, \quad T_{Dh} = 10^2, \quad a = 10^{-3}, \quad b = 10^2, \quad c = 10^{-2}$$

$$\tau = 1.5 \cdot 10^{-2}, \quad \mu = 1.17 \cdot 10^{-2}, \quad \kappa_{a10} = 10^{-4}, \quad \kappa_{a20} = 10^{-1}, \quad \kappa_{h30} = 10^{-3}$$

In Fig. 3 the evolution of the two components (a, h) is shown for two different values of the amplitude μ ($\mu_1 = 1 \cdot 10^{-2}$ and $\mu_2 = 1.17 \cdot 10^{-2}$) and in Fig. 4 a



(a) (b) (c) (d)

Fig. 4 – The two dimensional section of the phase space corresponding to each of two components (a and h) at different values of the amplitude μ: a) and b) for $\mu_1 = 1 \cdot 10^{-2}$; c) and d) for $\mu_2 = 1.17 \cdot 10^{-2}$.

two dimension section of the phase space for both components is presented also for the above two values of the amplitude. These results reveal a high sensitivity of the dynamics of this kind of D-R system on the amplitude of the external field. A similar sensitivity on the modulation period τ is also found, but the results are not presented here.

4. CONCLUSIONS

There is, of course, a huge variety of D-R systems types. In this paper we have investigated the possibility to control the dynamics of only two such systems: plasma and a diffusion system with reactions similar to Lotka-Voltera ones. The control was supposed to be performed using a RF-field with modulated low amplitude. The both examples give similar results concerning the sensitivity of the dynamics to the amplitude of the external field, though this is relatively low. One observes that increasing the amplitude of the field the dynamics tends to become chaotic. On the other hand this tendency seems to be balanced by the periodical variation of the amplitude. It is to be expected that a suitable pair of values for amplitude and period of the modulation could impose a desired dynamics.

The method presented in this paper could be used for other types of systems too. We suppose that especially the systems modeling the neurons and other types of living cells are suitable to be investigated with this method (equations (8) and 11).

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