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NON-PERTURBATIVE CONTRIBUTIONS TO THE POLYAKOV LOOP ABOVE THE DECONFINEMENT PHASE TRANSITION

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Abstract. The Polyakov loop is an order parameter to the confinement-deconfinement phase transition in pure gluodynamics. Recently a simple phenomenological model has been proposed to describe the available lattice data for the renormalized Polyakov loop in the deconfinement phase [1]. These data exhibit unequivocal inverse power temperature corrections driven by a dimension two gluon condensate. The heavy quark free energy is also studied and compared to lattice data.

Key words: the Polyakov loop, order parameters, phase transmition, deconfinement phase.

Introduction. In pure gluodynamic the confinement-deconfinement transition can be characterized by the breaking of the Z(N_c) discrete symmetry, belonging to the center of the gauge group, SU(N_c). The order parameter is the traced Polyakov loop, defined by

$$L(T) = \left\langle \mathrm{tr}\, \Omega(x) \right\rangle = \left\langle \frac{1}{N_c} \mathrm{tr}\, \mathbf{P} \left(e^{ig\int_0^{1/T} dx_0 A_0(\mathbf{x}, x_0)} \right) \right\rangle, \tag{1}$$

where $\langle\ \rangle$ denotes vacuum expectation value. A_0 is the gluon field in the (Euclidean) time direction. A perturbative evaluation of the Polyakov loop was carried out in [2].

The Polyakov loop correlation functions are related to the change in the free energy arising from the presence of a static quark-antiquark pair in a thermal medium. The colour singlet free energy yields the heavy quark potential at finite temperature,

$$F_1(\mathbf{x}, T)/T = -\log \left\langle \mathrm{Tr}\, L(\mathbf{x}) L^\dagger(0) \right\rangle + c(T). \tag{2}$$

A renormalization procedure for the Polyakov loop based on the computation of singlet and octet correlation functions between Polyakov loops in the limit of large separation has been considered in recent lattice studies [3, 4]. The additive normalization constant, $c(T)$, is ambiguous and can be fixed at short distances by comparison with the zero temperature heavy quark potential, *i.e.* for the singlet free energy: $F_1(r \ll 1/T) \simeq V_{qq}(r)$. With such a prescription the renormalized Polyakov loop is defined as [3, 4]:

$$L^{ren}(T) = \exp\left(-\frac{F_1(r \to \infty, T)}{2T}\right) \equiv \exp\left(-\frac{F_\infty(T)}{2T}\right). \tag{3}$$

In Ref. [1], we have proposed a model to describe the available lattice data for L^{ren}. Here we will show that it also describes consistently the lattice results for the free energy.

Phenomenological model for the Polyakov loop. From now on we work in the Polyakov gauge: $\partial_0 A_0(\mathbf{x}, x_0) = 0$. We assume that, in the deconfinement phase, the field $A_0(\mathbf{x})$ is sufficiently well described by a Gaussian distribution. From (1) one finds[1]

$$L = \exp\left[\frac{-g^2 \left\langle A_{0,a}^2 \right\rangle_T}{4N_c T^2}\right]. \tag{4}$$

To describe the dynamics of the $A_0(\mathbf{x})$ field we use the 3-dimensional reduced effective theory of QCD [5, 6]. Let $D_{00}(\mathbf{k})\delta_{ab}$ denote the 3-dimensional propagator, then

$$\left\langle A_{0,a}^2 \right\rangle_T = (N_c^2 - 1)T \int \frac{d^3k}{(2\pi)^3} D_{00}(\mathbf{k}). \tag{5}$$

In perturbation theory the propagator becomes $D_{00}^P(\mathbf{k}) = 1/(\mathbf{k}^2 + m_D^2)$ ($m_D \sim T$ is the Debye mass), and when inserted in (5) it reproduces the known perturbative result to LO [2]. We will take into account the non-perturbative contributions coming from dimension -2 condensates. So, we consider adding to the propagator new phenomenological pieces driven by positive mass dimension parameters:[2]

$$D_{00}^{NP}(\mathbf{k}) = \frac{m_G^2}{(\mathbf{k}^2 + m_D^2)^2}. \tag{6}$$

Adding the perturbative and non perturbative contributions, we get for the Polyakov loop

[1] The Gaussian ansatz is correct up to $\mathcal{O}(g^5)$ in perturbation theory, and exact in the large N_c limit.

[2] Such ansatz parallels those made at zero temperature in the presence of condensates [7].

$$-\log L = -\frac{N_c^2-1}{4N_c}\frac{g^2 m_D}{4\pi T} + \frac{g^2 \left\langle A_{0,a}^2 \right\rangle_T^{NP}}{4N_c T^2}. \tag{7}$$

Comparison to lattice data. The perturbative contribution to the Polyakov loop has logarithmic corrections in *T*. We will assume that $\left\langle A_{0,a}^2 \right\rangle_T^{NP}$ is constant in *T*, or at most logarithmic. This means that $\log L$ has a temperature power correction.

The lattice data of the renormalized Polyakov loop of Ref. [3] for $N_f = 0$ flavors and Ref. [4] for $N_f = 2$ flavors have been displayed in Fig. 1. These data suggest a linear fit of the form $-2\log L = a + b(T_c/T)^2$ [1, 8], which yields

$$a = \begin{cases} -0.23(1), & N_f = 0 \\ -0.26(1), & N_f = 2 \end{cases}, \qquad g^2 \left\langle A_{0,a}^2 \right\rangle_T^{NP} = \begin{cases} (0.87 \pm 0.02\,\text{GeV})^2, & N_f = 0 \\ (0.71 \pm 0.01\,\text{GeV})^2, & N_f = 2 \end{cases}, \tag{8}$$

with $\chi^2/DOF = 0.45,\ 6.14,$ for $N_f = 0,2$ respectively. The perturbative results for the highest temperature $6T_c$ are in qualitative agreement with the fitted values for a, Eq. (8).[3]

Finite temperature results for the pressure in pure gluodynamic [9] yield for the gluon condensate (0.93(7) GeV)^2 (in the temperature region used in our fits and in Landau gauge). Our result is also in reasonable agreement with existing zero temperature values [10].

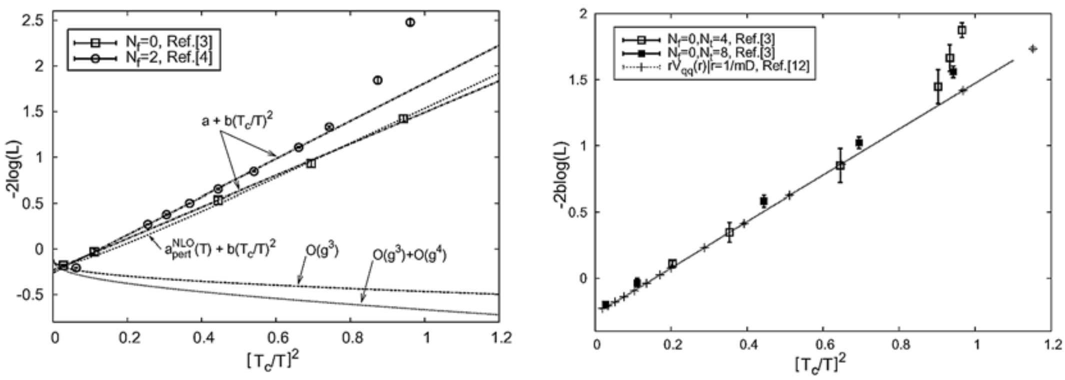

Fig. 1 – The logarithmic dependence of the renormalized Polyakov loop versus the inverse temperature squared in units of the critical temperature. Lattice data from [3, 4]. At the left we plot fits with *a* adjustable constant or predicted by NLO perturbation theory [2]. Purely perturbative LO and NLO results for $N_f = 0$ are shown for comparison. The right figure illustrates the duality between the Polyakov loop and the quark-antiquark potential at zero temperature. The line represents $rV_{qq}(r)$, takes from lattice data [12], and modified with the change $r = 1/m_D$.

[3] We consider the LO and NLO perturbative result of Ref. [2]

Heavy quark free energy. The quark-antiquark potential can be related to the scattering amplitude corresponding to one gluon exchange. In the non-relativistic limit

$$F_1(\mathbf{x},T) = -\frac{4}{3}g^2\int\frac{d^3k}{(2\pi)^2}e^{i\mathbf{k}\cdot\mathbf{x}}D_{00}(\mathbf{k}). \quad (9)$$

We can consider non perturbative contributions for the free energy by adding the non perturbative term (6) to the perturbative propagator, *i.e.* $D_{00}(\mathbf{k}) = D_{00}^{P}(\mathbf{k}) + D_{00}^{NP}(\mathbf{k})$. At LO, $\mathcal{O}(g^2)$, and NLO, $\mathcal{O}(g^3)$, the singlet free energy has the form

$$F_1(\mathbf{x},T) = -\frac{N_c^2-1}{2N_c}\left(\frac{g^2}{4\pi r}+\frac{1}{N_c^2-1}\frac{g^2\left\langle A_{0,a}^2\right\rangle_T^{NP}}{T}\right)e^{-m_D r} - -\frac{N_c^2-1}{2N_c}\frac{g^2 m_D}{4\pi}+\frac{g^2\left\langle A_{0,a}^2\right\rangle_T^{NP}}{2N_c T}. \quad (10)$$

Taking the value obtained in (8) for $g^2\left\langle A_{0,a}^2\right\rangle_T^{NP}$, we can match the free energy to lattice data [11]. This way, one obtains the r and T dependence of the running coupling α_s. This is displayed in Fig. 2. One can see a smooth behaviour, and it confirms the better fit of the Polyakov loop with a = constant. The values are relatively small, which is in contrast with existing analysis of α_s at finite T [3, 11].

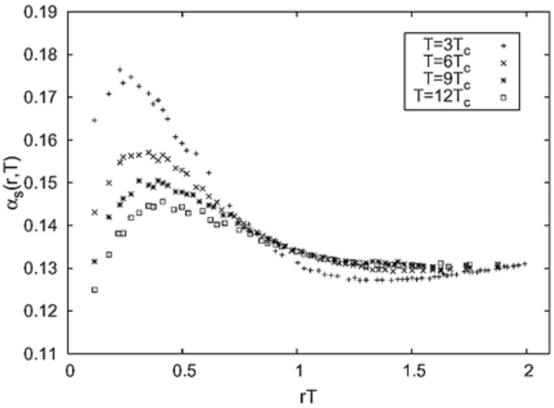

Fig. 2 – The r dependence of the running coupling in pure gluodynamic for different values of temperature. It is obtained by matching Eq. (10) to lattice data for the free energy from Ref. [11].

If we consider the limit of large separation in the free energy, Eq. (10), one re-obtains $F_\infty(T) = -2T\log L$. In the zero temperature limit (we take $m_D r \to 0$ in (10))

$$F_1(\mathbf{x},T) \overset{T\to 0}{\sim} -\frac{N_c^2-1}{2N_c}\frac{g^2}{4\pi r}+\frac{g^3\left\langle A_{0,a}^2\right\rangle_{T=0}^{NP}}{2N_c}r \equiv V_{qq}(r), \quad (11)$$

which is just the quark-antiquark potential at $T = 0$ [12]. The Coulomb term is the standard perturbative result at LO. The second one is a non perturbative linear contribution, and we get for the string tension $\sigma = g^3 \left\langle A_{0,a}^2 \right\rangle_{T=0}^{NP} /2N_c$. From Eqs. (7) and (11) we deduce a property:

$$F_\infty(T) = V_{qq}(r)\,|_{r=1/m_D}\,, \tag{12}$$

which is valid if we assume that $\alpha_s(r, T = 0)\left\langle A_{0,a}^2 \right\rangle_{T=0}^{NP} = \alpha_s(r \to \infty, T)\left\langle A_{0,a}^2 \right\rangle_{T}^{NP}$. This duality between $F_\infty(T)$ and $V_{qq}(r)$ is only valid at LO in perturbation theory.[4]

To check numerically Eq. (12), we plot in Fig. 1 the lattice data for $-2b\log L$ versus $(T_c/T)^2$. We can see a remarkable agreement. This duality looks deeply into the analogy between the quark-antiquark potential at zero temperature and the Polyakov loop.

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[4] The duality is formal, *i.e.* we are assuming $g = \text{constant} \neq g(r,T)$. If we take into account the different assymptotic behaviours of α_s : $\alpha_s(r) \equiv \alpha_s(r, T = 0)$ and $\alpha_s(T) \equiv \alpha_s(r \to \infty, T)$; Eq. (12) writes $bF_\infty(T) = V_{qq}(r)\,|\,r = \gamma/T$, with $b = (\alpha_s(r)/\alpha_s(T))^{3/4}$ and $\gamma = (\alpha_s(r)/\alpha_s(T)^3)^{1/4}/\sqrt{4\pi}$. (13)