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RADIATION SPECTRUM OF ELECTRONS MOVING IN A MAGNETIC FIELD IN VACUUM*

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Abstract. The main features of the radiation spectrum of two electrons moving in a magnetic field in vacuum are investigated using an improved Lorentz's self-interaction method. Special attention is given to the study of the fine structure of the synchrotron radiation spectral distribution of two electrons moving one by one along a spiral in vacuum in the non-relativistic case. The magnitude of the radiation power for two electrons in dependence of their location along the spiral is obtained.

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Key words: radiation spectrum, electrons, magnetic fields, vacuum self-interaction method, synchrotron radiation Lorentz method.

1. INTRODUCTION

The investigations of the radiation spectrum of electrons moving in magnetic fields in vacuum are important from the point of view of their applications in electronics, plasma physics, etc. [1–4]. The moving high energy electrons in a magnetic field in vacuum give rise to the synchrotron radiation. A question requiring further study is the coherence property of the synchrotron radiation [1, 5–12].

Using the main characteristics of the synchrotron radiation spectrum obtained by an improved Lorentz's self-interaction method for two electrons moving one after another along a spiral in vacuum, the fine structure of the synchrotron radiation spectrum in non-relativistic case was investigated by means of analytical and numerical methods. The magnitude of the radiation power for two electrons in dependence of their location in the spiral is obtained. The influence of the Doppler effect on the peculiarities of the electron's radiation spectrum is investigated, too.

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2. TIME-AVERAGED RADIATION POWER OF ELECTRONS MOVING IN MAGNETIC FIELD

We study two electrons moving one by one in a spiral in vacuum. The law of motion and the velocity of the l^{th} electron are given by the expressions [5–6]

$$\vec{r}_l(t) = r_0 \cos\{\omega_0(t+\Delta t_l)\}\vec{i} + r_0 \sin\{\omega_0(t+\Delta t_l)\}\vec{j} + V_{\parallel}(t+\Delta t_l)\vec{k},$$
$$\vec{V}_l(t) = \frac{\mathrm{d}\vec{r}_l(t)}{\mathrm{d}t}. \tag{1}$$

Here $r_0 = V_{\perp}\omega_0^{-1}$, $\omega_0 = ceB^{ext}\tilde{E}^{-1}$, $\tilde{E} = c\sqrt{p^2 + m_0^2c^2}$, the magnetic induction vector $\vec{B}^{ext} \parallel 0Z$, $V_{\perp}$ and $V_{\parallel}$ are the components of the velocity, $\vec{p}$ and $\tilde{E}$ are the momentum and energy of the electron, e and m_0 are its charge and rest mass, Δt_l is the time shift of l^{th} electron.

The time-averaged radiation power of two electrons is obtained after substituting (2) and (3) into (1). Then we found [5–6]:

$$\overline{P}^{rad} = \int_0^{\infty} W(\omega)\mathrm{d}\omega, \tag{2}$$

$$W(\omega) = \frac{2e^2}{\pi c^2}\int_0^{\infty} \mathrm{d}x\,\omega S_2(\omega)\frac{\sin\left\{\frac{1}{c}\omega\eta(x)\right\}}{\eta(x)}\cos\omega x\left[V_{\perp}^2\cos(\omega_0 x) + V_{\parallel}^2 - c^2\right], \tag{3}$$

where $\eta(x) = \sqrt{V_{\parallel}^2x^2 + 4\frac{V_{\perp}^2}{\omega_0^2}\sin^2\left(\frac{\omega_0}{2}x\right)}$.

The coherence factor $S_2(\omega)$ of two electrons is defined as

$$S_2(\omega) = 2 + 2\cos(\omega\Delta t). \tag{4}$$

Here $\Delta t = \Delta t_2 - \Delta t_1$ is the time shift of the electrons moving along a spiral. The analogous expression for the coherence factor was investigated by Bolotovskii [13].

From the relationships (2) and (3) after some transformations the contributions of separate harmonics to the averaged radiation power can be written as

$$\overline{P}^{rad} = \frac{e^2}{c^3}\sum_{m=-\infty}^{\infty}\int_0^{\infty}\mathrm{d}\omega\,\omega^2\int_0^{\pi}\sin\theta\,\mathrm{d}\theta\,S_2(\omega)\times$$
$$\times\delta\left\{\omega\left(1-\frac{1}{c}V_{\parallel}\cos\theta\right) - m\omega_0\right\}\left\{V_{\perp}^2\left[\frac{m^2}{q^2}J_m^2(q) + J_m'^2(q)\right] + \left(V_{\parallel}^2 - c^2\right)J_m^2(q)\right\}, \tag{5}$$

where $q = \frac{V_\perp}{c}\frac{\omega}{\omega_0}\sin\theta$, $J_m(q)$ and $J'_m(q)$ are the Bessel functions with integer index and their derivatives, respectively.

Each harmonic is a set of the frequencies, which are the solution of the equation

$$\omega\left(1 - \frac{V_\parallel}{c}\cos\theta\right) - m\omega_0 = 0. \quad (6)$$

The total radiation power emitted by a single electron is determined according to [14] as

$$P_m^{tot} = \frac{2}{3}\frac{e^2\omega_0^2 V_\perp^2}{c^3}\left(1 - \frac{V^2}{c^2}\right)^{-2}, \quad \omega_0 = \frac{eB^{ext}}{m_0 c}\sqrt{1 - \frac{V^2}{c^2}}. \quad (7)$$

3. FINE STRUCTURE OF RADIATION SPECTRUM OF TWO ELECTRONS MOVING ALONG A SPIRAL IN VACUUM

Our high accuracy numerical calculations of the radiation power spectral distribution of two electrons moving one by one along a spiral were performed at $B^{ext} = 1$ Gs (in vacuum $B^{ext} = H^{ext}$) $c = 2.997925 \cdot 10^{10}$ cm/s. For the velocities components $V_{\perp vac} = 0.2 \cdot 10^{10}$ cm/s = 0.0667·c and $V_{\parallel vac} = 0.2 \cdot 10^{10}$ cm/s = = 0.0667·c, $\omega_{0j} = 0.1751 \cdot 10^8$ rad/s, $r_{oj} = 114.2$ cm (j = 2, …, 6) these spectral distributions in dependence on the location along a spiral in vacuum are shown in Figs. 1–4 (curves 2–6).

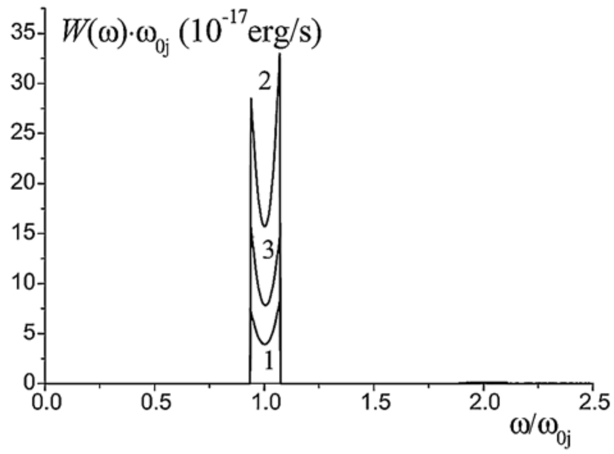

Fig. 1 – Spectral distribution of radiation power for a separate electron (curve 1) and two electrons moving one by one along a spiral at $\Delta t_2 = 0.0001 \cdot \pi / \omega_{02}$, $P_{vac2}^{int} = 0.2888 \cdot 10^{-16}$ erg/s (curve 2) and $\Delta t_3 = 0.5 \cdot \pi / \omega_{03}$, P_{vac3}^{int} = = $0.1411 \cdot 10^{-16}$ erg/s (curve 3).

It is interesting to compare the radiation power spectral distribution for two electrons to that of a separate electron (curve 1 in Figs. 1 to 4). The radiation power of a single electron in vacuum $P_{vac1}^{tot} = 0.713 \cdot 10^{-17}$ erg/s calculated according to the relationship (7) is in good agreement to the power $P_{vac1}^{\text{int}} = 0.717 \cdot 10^{-17}$ erg/s determined after integration of relationships (2) and (3). For the time shift $\Delta t_2 = 0.0001 \cdot \pi / \omega_{02}$ (curve 2 in Figs. 1 to 4) the coherence factor $S_2(\omega) \cong 4$ and the two electrons practicaly radiate as a single charged particle with charge $2e$ and rest mass $2m_0$, *i.e.*, by a factor of four more than a single electron.

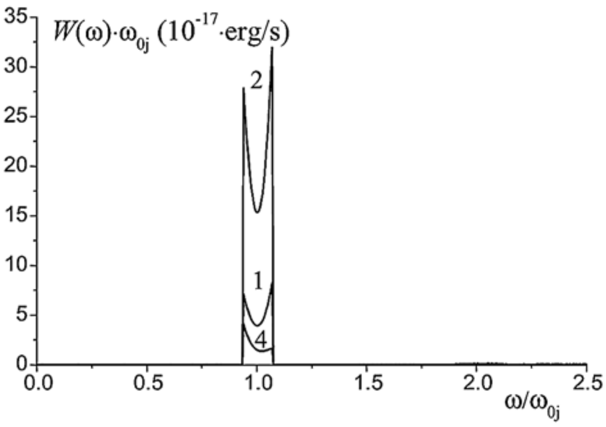

Fig. 2 – Spectral distribution of radiation power for two electrons moving one by one along a spiral. Curve 4: $\Delta t_4 = 0.8 \cdot \pi / \omega_{04}$, $P_{vac4}^{\text{int}} = 0.2938 \cdot 10^{-17}$ erg/s.

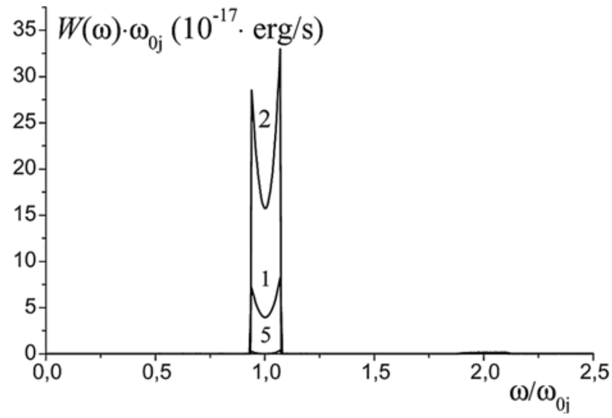

Fig. 3 – Spectral distribution of radiation power for two electrons moving one by one along a spiral. Curve 5: $\Delta t_5 = \pi / \omega_{05}$, $P_{vac5}^{\text{int}} = 0.502 \cdot 10^{-18}$ erg/s.

For time shift $\Delta t_2 = 0.5 \cdot \pi / \omega_{02}$ the radiation power of two electrons is $P_{vac2}^{\text{int}} = 0.1489 \cdot 10^{-16}$ erg/s.

For the time shift $\Delta t_4 = 0.8 \cdot \pi / \omega_{04}$ (curve 4 in Fig. 2) the backward radiation power of two electrons is higher than the forward one.

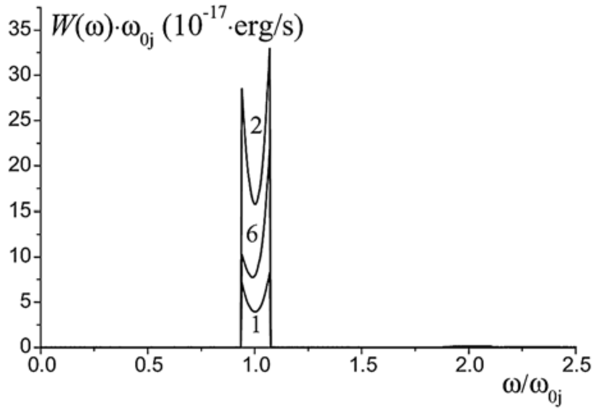

Fig. 4 – Spectral distribution of radiation power for two electrons moving one by one along a spiral. Curve 6: $\Delta t_6 = 1.5 \cdot \pi / \omega_{06}$, $P^{\text{int}}_{vac6} = 0.1467 \cdot 10^{-16}$ erg/s.

For the time shift $\Delta t_5 = \pi / \omega_{05}$ (curve 5 in Fig. 3) the radiation power of two electrons is by an order of magnitude lower than that of a single electron.

In the range from $\Delta t = \pi / \omega_0$ to $\Delta t = 2\pi / \omega_0$ the two electrons moving along a spiral prefer to radiate forwards.

The magnitude of the radiation power for two electrons moving one by one along a spiral in dependence of their location is presented in Fig. 5. With increasing Δt the radiation power of the system of two electrons tends asymptotically to twice of the radiation power of a single electron.

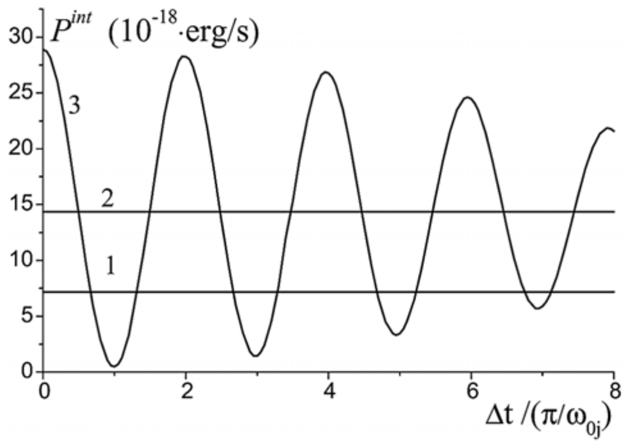

Fig. 5 – Radiation power of the single electron (curve 1). Radiation power of two separate electrons (curve 2). Radiation power of two electrons moving one by one along a spiral (curve 3).

4. CONCLUSIONS

For small time shifts the coherence factor $S_2(\omega) \cong 4$ and the two electrons practically radiate as a single charged particle with charge $2e$ and rest mass $2m_0$, *i.e.*, by a factor of four more than a separate electron. With increasing time shift the radiation power of the system of the two charges tends to double the amount of radiation power of a single charge.

The coherence factor leads to essential changes in the radiation power spectral distribution of the two electrons in dependence of their position along the spiral. The Doppler effect establishes the boundaries between the bands of separate harmonics in the radiation spectrum of charged particles.

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