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INTERACTING FIELDS. A FIRST ORDER SOLUTION*

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Abstract. The aim of this paper is to study the SO(3,1) U(1) gauge minimally coupled charged spinless field to a spherically symmetric curved space-time. The first order analytical approximation solution for the system of Klein-Gordon-Maxwell-Einstein equations is derived. Using these solutions, the electric current components and further the boson system electric charge are evaluated. The chosen metric tensor is of a static conformal metric tensor type. The anterior results are developed and new aspects are highlighted.

Key words: gauge fields SO(3,1) U(1) interacting fields, curved space-time, analytical solution, Klein-Gordon-Maxwell-Einstein equations.

1. INTRODUCTION

The study of boson stars (BS) can be traced back to the work of

Kaup [1] and Ruffini and Bonazzola [2] more than 30 years ago. They found asymptotic solutions to the Einstein-Klein-Gordon equations for spherical symmetric equilibrium.

This result originates from the following specific feature of

boson stars: the boson star is protected from gravitational collapse by the Heisenberg uncertainty principle, instead of the Pauli exclusion principle that applies to fermionic stars.

Jetzer [3] and van der Bij extended the BS model to include the coupling with U(1) gauge group macroscopic stable boson stars, since they have been considered to provide a considerable fraction of the non-baryonic part of dark matter. These configurations are "macroscopic quantum states" and are only prevented from collapsing gravitationally by the Heisenberg uncertainty principle.

Boson stars in the presence of a dilaton or an axidilaton have also been studied, as well as boson-fermion stars. All these models have demonstrated the

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same characteristic: new interactions tend to increase the critical values of the mass and particle number, although the particular values are very model dependent. The stability against perturbations around the equilibrium state has been discussed also by a number of authors [2, 3, 4, 5].

2. KLEIN-GORDON-MAXWELL EQUATIONS ON CURVED SPACE-TIME

For a curved space-time, the line element can be expressed using the metric tensor $g_{\mu\nu}$ as

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu$$

In this point, a pseudo-orthonormal tetradic frame $\{e_a\}_{a=1,4}$ could be introduced, with the correspondent metric tensor η^{ab} of minkowskian kind

$$\eta^{ab} = diag\begin{bmatrix}1 & 1 & 1 & -1\end{bmatrix} \tag{1}$$

In this configuration, considering a charged boson of mass m_0 coupled to the electromagnetic field, the system is described by the $SO(3,1)\times U(1)$ gauge invariance Lagrangean density [3, 4 and 6]:

$$L = \eta^{ab}\bar{\Phi}_{,a}\Phi_{,b} + m_0^2\bar{\Phi}\Phi + \frac{1}{4}F^{ab}F_{ab} \tag{2}$$

where $(.)_{|a} = e_a(\ )$ and

$$\Phi_{,a} = \Phi_{|a} - ieA_a\Phi, \quad \text{and} \quad \bar{\Phi}_{,a} = \bar{\Phi}_{|a} + ieA_a\bar{\Phi} \tag{3}$$

The Maxwell tensor

$$F_{ab} = A_{b:a} - A_{a:b} \tag{4}$$

is expressed in the terms of the Levi-Civita covariant derivative of the four-potential $\{A_a\}_{a=\overline{1,4}}$, *i.e.*,

$$A_{a;b} = A_{a|b} - A_c\Gamma^c{}_{ab} \tag{5}$$

where the Christoffel symbols are computed as

$$\Gamma^c{}_{ab} = g^{cd}\,\frac{1}{2}\left(\frac{\partial g_{da}}{\partial x^b} + \frac{\partial g_{db}}{\partial x^a} - \frac{\partial g_{ba}}{\partial x^d}\right) \tag{6}$$

For the scalar field, the Klein-Gordon equation can be derived using (2) by varying with respect to different fields,

$$\Box\,\Phi - m_0^2\Phi = 2ieA^c\Phi_{|c} + e^2A^cA_c\Phi \tag{7}$$

and it is Hermitic conjugated.

The Maxwell system equations can be written as

$$F^{ab}{}_{:c} = -ie\eta^{ab}\left[\bar{\Phi}\left(\Phi_{|b} - ieA_b\Phi\right) - \left(\bar{\Phi}_{|b} - ieA_b\bar{\Phi}\right)\Phi\right] \tag{8}$$

Building up the energy-momentum tensor

$$T_{ab} = \bar{\Phi}_{:a}\Phi_{:b} + \bar{\Phi}_{:b}\Phi_{:a} + F_{ac}F_b^c - \eta_{ab}L \tag{9}$$

the Einstein equation it can be derived

$$G_{ab} = kT_{ab} \tag{10}$$

The aim of this paper is to compute the first order equations system solutions and using them to compute the electric charge of the boson system in a different space-time type [3, 4, 5, 7].

2. FIELDS SYSTEM EQUATIONS ON STATIC CONFORMAL SPACE-TIME

Let us consider a spherically symmetric configuration described by a static conformal metric tensor type, expressed in Schwarzchild coordinates as

$$\begin{aligned} ds^2 &= e^{2(\Xi t+\Sigma)}a(r)^2\,dr^2 + \\ &+ e^{2(\Xi t+\Sigma)}\left(r^2d\theta^2 + r^2\sin(\theta)\,d\varphi^2\right) - e^{2(\Xi t+\Sigma)}b(r)^2\,dt^2 \end{aligned} \tag{11}$$

The pseudo-orthonormal tetradic frame $\{e_a\}_{a=1,4}$ with the corresponding dual orthonormal base is

$$\begin{aligned} &\omega^1 = e^{(\Xi t+\Sigma)}a(r)dr && \omega^2 = e^{(\Xi t+\Sigma)}rd\theta \\ &\omega^3 = e^{(\Xi t+\Sigma)}a(r)r\sin\theta d\varphi && \omega^4 = e^{(\Xi t+\Sigma)}b(r)dt \end{aligned} \tag{12}$$

The Christoffel symbols derived in this frame are

$$\Gamma^1_{22} = -\Gamma^2_{12} = -\frac{1}{ra(r)}e^{-(\Xi t+\Sigma)}$$

$$\Gamma^1_{33} = -\Gamma^3_{13} = -\frac{1}{ra(r)}e^{-(\Xi t+\Sigma)}$$

$$\Gamma^1_{41} = \Gamma^4_{11} = \frac{\Xi}{b(r)}e^{-(\Xi t+\Sigma)}$$

$$\Gamma^1_{44} = \Gamma^4_{14} = \frac{b'(r)}{b(r)a(r)} e^{-(\Xi t+\Sigma)} \tag{13}$$

$$\Gamma^2_{33} = -\Gamma^3_{23} = -\frac{\cot\theta}{r} e^{-(\Xi t+\Sigma)}$$

$$\Gamma^2_{42} = \Gamma^4_{22} = \frac{\Xi}{b(r)} e^{-(\Xi t+\Sigma)}$$

$$\Gamma^3_{43} = \Gamma^4_{33} = \frac{\Xi}{b(r)} e^{-(\Xi t+\Sigma)}$$

where we used

$$b'(r) = \frac{db(r)}{dr}$$

Further, the Einstein tensor G_{ab} can be computed and the non-vanishing components in this frame are

$$\begin{aligned}
G_{11} &= -\frac{e^{-2(\Xi t+\Sigma)}}{a(r)^2 b(r)^2 r^2}\left[a(r)^2\left(b(r)^2 + r^2\Xi^2\right) - b(r)^2 - 2rb'(r)b(r)\right] \\
G_{14} &= 2\frac{e^{-2(\Xi t+\Sigma)}}{a(r)b(r)^2}\left[\Xi\, b'(r)\right] \\
G_{22} &= -\frac{e^{-2(\Xi t+\Sigma)}}{ra(r)^3 b(r)^2}\Big[a(r)^3 r\Xi^2 - a(r)\left(b'(r)b(r) + rb(r)b''(r)\right) + \\
&\quad + a'(r)\left(b(r)^2 + rb(r)b'(r)\right)\Big] \\
G_{33} &= G_{22} \\
G_{44} &= \frac{e^{-2(\Xi t+\Sigma)}}{r^2 a(r)^3 b(r)^2}\left[a(r)^3\left(3r^2\Xi^2 + b(r)^2\right) + b(r)^2\left(2ra'(r) - a(r)\right)\right]
\end{aligned} \tag{14}$$

Working in the minimally symmetric ansatz $A_1 = A_1(r,t)$, $A_4 = A_4(r,t)$, $\Phi = \Phi(r,t)$, the single non-vanishing Maxwell tensor component is

$$F_{14} = -F_{41} = -\frac{e^{-(\Xi t+\Sigma)}}{a(r)b(r)}\left[a(r)A_{1,t} - b'(r)A_4 - b(r)A_{4,r} + a(r)\Xi A_1\right] \tag{15}$$

The Klein-Gordon equation (7) turns into

$$\begin{aligned}
&\frac{e^{-2(\Xi t+\Sigma)}}{a(r)^2 b(r)^2}\left[\Phi_{,r}\left(\frac{2b(r)^2}{r} - \frac{a'(r)b(r)^2}{a(r)} + b'(r)b(r)\right) - \right. \\
&\left. -2\Phi_{,t}a(r)^2\Xi + b(r)^2\Phi_{,rr} - a(r)^2\Phi_{,tt}\right] - m_0{}^2\Phi = \\
&\quad = 2iee^{-(\Xi t+\Sigma)}\left\{\frac{1}{a(r)}A_1\Phi_{,r} - \frac{1}{b(r)}A_4\Phi_{,t}\right\} + e^2\Phi\left[\left(A_1\right)^2 - \left(A_4\right)^2\right]
\end{aligned} \tag{16}$$

and it is Hermitic conjugated.

The Maxwell system equations (8) can be written as

$$\frac{e^{-2(\Xi t+\Sigma)}}{a(r)^2 b(r)^2}\Big[2a(r)^2\,\Xi A_{1,t} + a(r)b'(r)\big(-\Xi A_4 - A_{4,t}\big) + +a(r)b(r)\big(-\Xi A_{4,r} - A_{4,rt}\big)\Big] + \frac{e^{-2(\Xi t+\Sigma)}}{b(r)^2}\big(\Xi^2 A_1 + A_{1,tt}\big) = = -ie\frac{e^{-(\Xi t+\Sigma)}}{a(r)}\big(\bar{\Phi}\Phi_{,r} - \bar{\Phi}_{,r}\Phi\big) - 2e^2\bar{\Phi}\Phi A_1 \tag{17}$$

and, respectively,

$$\frac{e^{-2(\Xi t+\Sigma)}}{a(r)^2 b(r)^2}\Bigg[-\frac{a(r)'b'(r)b(r)}{a(r)}A_4 - \frac{a'(r)}{a(r)}b(r)^2 A_{4,r} + +a(r)b'(r)\big(\Xi A_1 + A_{1,t}\big)b(r) - b'(r)^2 A_4 - 2\frac{a(r)b(r)}{r}A_{1,t} + +2\frac{b'(r)b(r)}{r}A_4 + b(r)b'(r)A_{4,r} + b(r)^2 A_{4,rr} - 2\frac{a(r)b(r)\Xi}{r}A_1\Bigg] + + \frac{e^{-2(\Xi t+\Sigma)}}{a(r)^2 b(r)^2}\Bigg[2\frac{b(r)^2}{r}A_{4,r} - a(r)b(r)\Xi A_{1,r} + b(r)b''(r)A_4\Big] - -a(r)b(r)A_{1,rt}\Big] = ie\frac{e^{-(\Xi t+\Sigma)}}{b(r)}\big(\bar{\Phi}\Phi_{,t} - \bar{\Phi}_{,t}\Phi\big) + 2e^2\bar{\Phi}\Phi A_4 \tag{18}$$

The necessary Lorentz condition can be read as

$$\frac{e^{-(\Xi t+\Sigma)}}{a(r)b(r)}\Bigg[b(r)A_{1,r} - 3a(r)\Xi A_4 + \frac{2b(r)}{r}A_1 - a(r)A_{4,t} + b'(r)A_1\Bigg] = 0 \tag{19}$$

With these results, the Einstein equations explicitly become

$$-\frac{e^{-2(\Xi t+\Sigma)}}{a(r)^2 b(r)^2 r^2}\Big[a(r)^2\big(b(r)^2 + r^2\Xi^2\big) - b(r)^2 - 2rb'(r)b(r)\Big] = = \kappa\Big[\big(\bar{\Phi}_{;1}\Phi_{;1} + \bar{\Phi}_{;4}\Phi_{;4}\big) - m_0^2\bar{\Phi}\Phi - \frac{1}{2}\big(F_{14}\big)^2\Big] \tag{20}$$

and

$$-\frac{e^{-2(\Xi t+\Sigma)}}{ra(r)^3 b(r)^2}\Big[a(r)^3 r\Xi^2 - a(r)\big(b'(r)b(r) - rb(r)b''(r)\big) + + a'(r)\Big(b(r)^2 + rb(r)b'(r)\Big)\Big] = = -\kappa\Big[\big(\bar{\Phi}_{;1}\Phi_{;1} - \bar{\Phi}_{;4}\Phi_{;4}\big) + m_0^2\bar{\Phi}\Phi - \frac{1}{2}\big(F_{14}\big)^2\Big] \tag{21}$$

respectively,

$$\begin{aligned}&\frac{e^{-2(\Xi t+\Sigma)}}{r^2 a(r)^3 b(r)^2}\Big[a(r)^3\left(3r^2\Xi^2+b(r)^2\right)+b(r)^2\left(2ra'(r)-a(r)\right)\Big]=\\&=\kappa\left[\left(\bar{\Phi}_{;1}\Phi_{;1}+\bar{\Phi}_{;4}\Phi_{;4}\right)+m_0^2\bar{\Phi}\Phi+\frac{1}{2}\left(F_{14}\right)^2\right]\end{aligned}\tag{22}$$

where the energy-momentum tensor T_{ab} has the explicit form

$$\begin{aligned}T_{11}&=-m_0^2\bar{\Phi}\Phi+e^2\bar{\Phi}\Phi\left(A_1^2+A_4^2\right)+\\&+e^{-(\Xi t+\Sigma)}\left[\frac{A_1}{a(r)}\left(\bar{\Phi}\Phi_{,r}-\bar{\Phi}_{,r}\Phi\right)+\frac{A_4}{b(r)}\left(\bar{\Phi}\Phi_{,t}-\bar{\Phi}_{,t}\Phi\right)\right]+\\&+e^{-2(\Xi t+\Sigma)}\left[\frac{1}{a(r)^2}\bar{\Phi}_{,r}\Phi_{,r}+\frac{1}{b(r)^2}\bar{\Phi}_{,t}\Phi_{,t}\right]+\\&-\frac{1}{2}\frac{1}{a(r)^2 b(r)^2}e^{-2(\Xi t+\Sigma)}\left[a(r)A_{1,t}-b'(r)A_4-b(r)A_{4,r}+a(r)\Xi A_1\right]^2\end{aligned}$$

$$\begin{aligned}T_{22}2&=m_0^2\bar{\Phi}\Phi+e^2\bar{\Phi}\Phi\left(A_1^2-A_4^2\right)-\\&-e^{-(\Xi t+\Sigma)}\left[\frac{A_1}{a(r)}\left(\bar{\Phi}\Phi_{,r}-\bar{\Phi}_{,r}\Phi\right)-\frac{A_4}{b(r)}\left(\bar{\Phi}\Phi_{,t}-\bar{\Phi}_{,t}\Phi\right)\right]+\\&+e^{-2(\Xi t+\Sigma)}\left[\frac{1}{a(r)^2}\bar{\Phi}_{,r}\Phi_{,r}-\frac{1}{b(r)^2}\bar{\Phi}_{,t}\Phi_{,t}\right]+\\&-\frac{1}{2}\frac{1}{a(r)^2 b(r)^2}e^{-2(\Xi t+\Sigma)}\left[a(r)A_{1,t}-b'(r)A_4-b(r)A_{4,r}+a(r)\Xi A_1\right]^2\end{aligned}\tag{23}$$

$$T_{33}=T_{22}$$

$$\begin{aligned}T_{44}&=m_0^2\bar{\Phi}\Phi+e^2\bar{\Phi}\Phi\left(A_1^2+A_4^2\right)+\\&+e^{-(\Xi t+\Sigma)}\left[\frac{A_1}{a(r)}\left(\bar{\Phi}\Phi_{,r}-\bar{\Phi}_{,r}\Phi\right)+\frac{A_4}{b(r)}\left(\bar{\Phi}\Phi_{,t}-\bar{\Phi}_{,t}\Phi\right)\right]+\\&+e^{-2(\Xi t+\Sigma)}\left[\frac{1}{a(r)^2}\bar{\Phi}_{,r}\Phi_{,r}+\frac{1}{b(r)^2}\bar{\Phi}_{,t}\Phi_{,t}\right]+\\&+\frac{1}{2}\frac{1}{a(r)^2 b(r)^2}e^{-2(\Xi t+\Sigma)}\left[a(r)A_{1,t}-b'(r)A_4-b(r)A_{4,r}+a(r)\Xi A_1\right]^2\end{aligned}$$

$$\begin{aligned}T_{14}&=\left(\bar{\Phi}_{;1}\Phi_{;4}+\bar{\Phi}_{;4}\Phi_{;1}\right)=e^{-(\Xi t+\Sigma)}\left[\frac{A_1}{b(r)}\left(\bar{\Phi}\Phi_{,t}-\bar{\Phi}_{,t}\Phi\right)+\frac{A_4}{a(r)}\left(\bar{\Phi}\Phi_{,r}-\bar{\Phi}_{,r}\Phi\right)\right]+\\&+2e^2\bar{\Phi}\Phi A_1A_4+\frac{e^{-2(\Xi t+\Sigma)}}{a(r)b(r)}\left[\bar{\Phi}_{,t}\Phi_{,r}+\bar{\Phi}_{,r}\Phi_{,t}\right]\end{aligned}$$

To solve, we start with the physical assumptions that the feedback of gravity can be neglected in the first order approximation [2, 4, 6, 7 and 8]. The equation for potential Φ (imposing $\Xi = 0$, $a(r) = b(r) = 1$ and $\Sigma = 0$) can be written as [9]:

$$\Phi_{,rr} - \frac{2}{r}\Phi_{,t} - \Phi_{,tt} - {m_0}^2\Phi = 0 \tag{24}$$

and it is Hermitic conjugated.

The considered solutions have the spherical symmetric form

$$\Phi = \frac{N}{r}e^{i(\omega t - kr)} \quad \text{and} \quad \overline{\Phi} = \frac{N}{r}e^{-i(\omega t - kr)} \tag{25}$$

Using the same conditions, from Maxwell equations one can read, from (11) and (12)

$$A_{1,rr} + \frac{1}{2r}A_{1,r} - \frac{2}{r^2}A_1 - A_{1,tt} = -2ek\frac{|N|^2}{r^2} \tag{26}$$

and, respectively,

$$A_{4,rr} + \frac{2}{r}A_{4,r} - A_{4,tt} = 2e\omega\frac{|N|^2}{r^2} \tag{27}$$

Considering the particular solutions

$$A_1 = ek|N|^2 \tag{28}$$

from Maxwell equation (27) it can be found that

$$A_4(r,t) = 2e\omega|N|^2 \log\left(\frac{r}{r_o}\right) + 2ek\frac{|N|^2}{r}t \tag{29}$$

Introducing this first order perturbative solution in the Einstein equation, one can read, in the hypothesis $k = 0$, $b = b(r)$ and $a = a(r)$:

$$\begin{aligned} &-\frac{1}{a(r)^2 b(r)^2 r^2}\left[a(r)^2 b(r)^2 - b(r)^2 - 2rb'(r)b(r)\right] = \\ &= \kappa\left[\frac{N^2}{r^4} + 2m_0^2\frac{N^4}{r^2} + 2e^2 m_0^2\frac{N^4}{r^2} + 4e^2 m_0^2\frac{N^4}{r^2}\log\left(\frac{r}{r_0}\right)\right] \end{aligned} \tag{30}$$

and

$$\begin{aligned} &\frac{\left[a(r)\left(b'(r)b(r) + rb(r)b''(r)\right) - a'(r)\left(b(r)^2 - rb(r)b'(r)\right)\right]}{ra(r)^3 b(r)^2} = \\ &= k\left[\frac{|N|^2}{r^4} - 2e^2{m_0}^2\frac{|N|^4}{r^2} + 4e^2{m_0}^2\frac{|N|^4}{r^2}\log\left(\frac{r}{r_0}\right)\right] \end{aligned} \tag{31}$$

respectively,

$$\frac{1}{r^2 a(r)^3 b(r)^2}\left[a(r)^3 b(r)^2 + b(r)^2\left(2ra'(r) - a(r)\right)\right] = \\ = -k\left[-\frac{|N|^2}{r^4} + 2e^2 {m_0}^2 \frac{|N|^4}{r^2} + 4e^2 {m_0}^2 \frac{|N|^4}{r^2}\log\left(\frac{r}{r_0}\right)\right] \tag{32}$$

A particular solution of this system equation can be read as:

$$\Xi = \alpha C_1, \quad b(r) = C_1$$

and

$$a(r) = \frac{\sqrt{u(r)}}{u(r)}$$

where $u(r)$ has the form

$$u(r) = C_2 r^2 - 4\kappa e^2 {m_0}^2 |N|^4 - 8\kappa e^2 {m_0}^2 |N|^4 \log\left(\frac{r}{r_0}\right) \tag{33}$$

Using these results the electric charge [4, 7] can be computed:

$$Q = \int j_4 dV \tag{34}$$

where

$$j_4 = ie\frac{e^{-(\Xi t+\Sigma)}}{b(r)^2}\left[\left(\bar{\Phi}\Phi_{,t} - \bar{\Phi}_{,t}\Phi\right) + 2ieb(r)e^{(\Xi t+\Sigma)}\bar{\Phi}\Phi A_1\right] \tag{35}$$

Considering the electric charge current component, the charge magnitude is

$$Q = \kappa e^{-(\Xi t+\Sigma)} e|N|^2 I_1 \tag{36}$$

where a set of integration limit was used in order to bypass the singularities

$$I_1 = \int_z^R \frac{p_1}{\sqrt{-p_2 + \ln(r)}} dr \tag{37}$$

and

$$p_1 = \sqrt{8\kappa} e|N|^2 m_0 \tag{38}$$

$$p_2 = \frac{8\kappa e^2 |N|^4 m_0^2 \ln(r_0) - 1 + 4\kappa e^2 |N|^4 m_0^2}{8\kappa e^2 |N|^4 m_0^2} \tag{39}$$

Computing (37) we get

$$I_1 = p_1\left(-iG(i\sqrt{-p_2 + R})\right)e^{p_2}\sqrt{\pi} + iG(i\sqrt{p_2})e^{p_2}\sqrt{\pi}$$

where

$$G(x)=\frac{2}{\sqrt{\pi}}\int_{0}^{\lambda}e^{-t^2}dt=\frac{2}{\sqrt{\pi}}\left[F(\lambda)-F(0)\right]$$

where $F(x)=erf(x)$. The magnitude I_1 admits a finite value for a R non-finite limit in opposition with the anterior cases [4, 7 and 10].

These values for the electric charge current components are computed in order to get a comparative expression of the previous results [4, 7 and 10]. The obtained values and the asymptotic trend in the large distance domains could be considered acceptable. Despite the MAPLE analytical software algorithm used in getting these results [11], the magnitude for the boson system's electric charge in large space-time kinds, has to be evaluated numerically.

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