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GRAVITATIONAL POTENTIAL ENERGY FOR DIFFERENT DISTRIBUTIONS OF MATTER. PRELIMINARY RESULTS*

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Abstract. The distribution of matter in the universe is a topic of high interest nowadays. Considering gravity as the only force held responsible for the behaviour of the large scale structures dynamics we are attempting to make a comparison between the gravitational potential energy in a Newtonian approximation of a fractal distribution of matter with the one in the case of a random distribution. The first results of the simulation of a deterministic fractal distribution of points in a space and a random one are presented. Moreover, taking into account that observations have established that galaxies are distributed along some kind of filaments, we were interested in studying this distribution using the redshift survey of galaxies. The results and difficulties encountered in making a fractal analysis are discussed.

Key words: fractal analysis, gravitational potential energy.

1. INTRODUCTION

In the early days of general relativity, Einstein (1917) argued that a closed homogeneous world model fits well into the general relativity theory and requirements of Mach's principle. Little by little, Hubble (1926), Lemaître (1927) and others developed the idea of homogeneous universe.

Survey of stars in the galaxy consistently shows that most stars (over 60%) are members of binary and multiple star systems. Stars are clustered in galaxies. Galaxies tend to make groups. For example, our Milky Way Galaxy is part of a gathering of about 40 galaxies known as the Local Group. Therefore, it is interesting to discuss the nature of the large-scale matter distribution. Observational astronomy shows that apparently the Universe is not at all homogeneous. The matter distribution in any case is strongly clamped on the scale of stars, galaxies and cluster of galaxies. Could these clusters be some sort of remnant process in the

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distant past? How does clustering evolve in the expanding universe and what does it tell about the nature of the universe?

We tried to study the experimental data on the real distribution of the galaxies in the universe, as obtained from the most recent surveys, and using fractal approach of their distribution. As gravity is the only important force at a large scale, it is interesting to study how a fractal distribution of the luminous matter could influence the gravitational potential energy.

In this paper we will present the preliminary results in the attempt of making a comparison between the gravitational potential energy computed for a fractal and a random distribution of matter in space in a Newtonian approximation.

2. LARGE SCALE DISTRIBUTION OF MATTER IN THE UNIVERSE

One first piece of observational evidence of how the galaxies are distributed in the universe was the CfA redshift survey. The first "slice of the universe" done in 1985 by V. Lapparent, M. Geller, J. Huchra represents the spectroscopic observations of about 1100 galaxies in the strip on the sky with a width of 6 degrees and a length of 130 degrees.

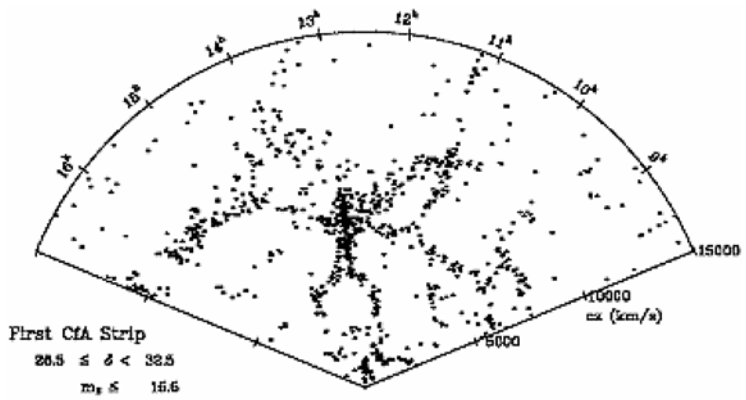

Fig. 1 – First CfA slide (Huchra *et al.* 1985, Smithsonian Astrophysical Observatory).

This map showed that galaxies appear to be distributed on surfaces surrounding large "voids". These voids are roughly spherical and measure 100 million to 400 million light years in diameter. This was treated as important evidence for the existence of dark matter.

Starting from these studies, many phenomenological models regarding the fractal nature of the galaxies distribution were proposed, for example, simple fractal models with an upper homogeneity cut-off (Ruffini *et al.* 1988), or spatial scale-dependent fractal models (Lukash and Novikov 1988).

2.1. QUANTIFYING THE HOMOGENEITY OF THE UNIVERSE

If we suppose that stars have a fixed intrinsic luminosity, then those appearing brighter than f, according to the inverse square law, would vary with the distance r as $(f)^{-1/2}$. For a homogeneous distribution, the number counted will vary as the volume r^3. As a result, we expect to find a distribution over luminosity, of the form $(f)^{-3/2}$. The sums over stars of different intrinsic luminosity affect the constant of proportionality but not the power law behaviour. Direct counts of the stars in our Galaxy are strongly influenced by interstellar absorption, so the star system was substantially underestimated. It was shown that the counts in direction well away from our Galaxy are little affected by absorption, so the estimates of thickness of the disc were quite reasonable.

If the distribution had been homogeneous, the number of stars brighter than f would have varied as:

$$N(<m) \propto f^{-3/2} \propto 10^{0.6m} \tag{1}$$

where m is the apparent magnitude. The star counts are different in different directions in the sky and increase with the decreasing f less rapidly than would be expected from the above equation. This is because we are seeing the edge of the universe. In 1936 Hubble's counts in the range of magnitudes $m \sim 19.1$ to 19.6 showed that $N(<m)$ increases less rapidly than the above rule.

Other observations had important effects on our understanding of the large-scale of the universe. For example in the 1930s Hubble noted that the frequency distribution of nebula counts N found in different telescope fields is not Poisson, as would be expected if the galaxies had a random distribution; the general clamping makes for a considerable broader distribution of counts (the distribution of $\log N$ is remarkable close to Gaussian).

Modern deep galaxy counts (1974 ~ 1990) are found to vary with magnitude roughly as $10^{0.45m}$ to depth comparable to c/H_0.

2.2. FRACTAL ANALYSIS OF 2DF REDSHIFT GALAXY SURVEY

The survey presented by Coless *et al.* (2003) makes use of the 2dF multi-fibre spectrograph at the Anglo-Australian Telescope to measure redshifts for over 250,000 galaxies brighter than $b_J = 19.5$ and a further 10,000 galaxies brighter than $R = 21$. The main goals of the survey are to characterize the large-scale structure of the universe and quantify the properties of the galaxy population at low redshifts.

The geometry of the survey was chosen to be an effective compromise between the desire to sparsely sample the largest possible volume in order to determine the power spectrum on very large scales, and the desire to fully sample a representative but compact volume in order to investigate the redshift space distortions and the topology of the galaxy distribution.

The source catalogue for the survey is a revised and extended version of the automated plate measuring (APM) galaxy catalogue (Maddox *et al.* 1990*a-c*).

Using several maps like the one in Fig. 2 we calculated the fractal dimension with a program especially designed. The main issue was to see whether there exists some kind of similarity between those two angles of observation, the one in the South Galactic Pole (SGP) region covering approximately $-37°.5 < \delta < -22°.5$, $21^h40^m < \alpha < 3^h30^m$ and the other in the direction of the North Galactic Pole (NGP) $-7°.5 < \delta < 2°.5$, $9^h50^m < \alpha < 14^h50^m$.

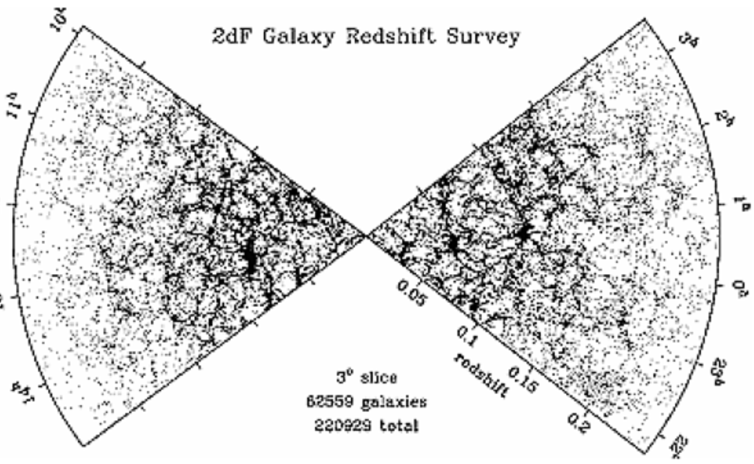

Fig. 2 – The two degree field galaxy redshift survey.

"Cutting" the image on the vertical we obtain the value of 1.81 for the fractal dimension of the galaxies distribution in the left side and 1.83 for the other side. When we did the same "cutting" on the horizontal we obtained for both cases a value of the fractal dimension of 1.77.

Analysing the variance of the fractal dimension with the redshift, we observed an almost linear increase of this dimension with the distance (Fig. 3).

The box counting fractal dimension of the analysed survey of galaxies shows a few interesting features.

The first observed characteristic is the approximately linear dependence of the fractal dimension on the redshift of the galaxies. The second one is the increase of the fractal dimension towards a value of 2, at higher redshift in the range z = 0.05–0.25.

Another aspect is that there are no significant differences in the fractal behaviour for the two analysed sides of the map.

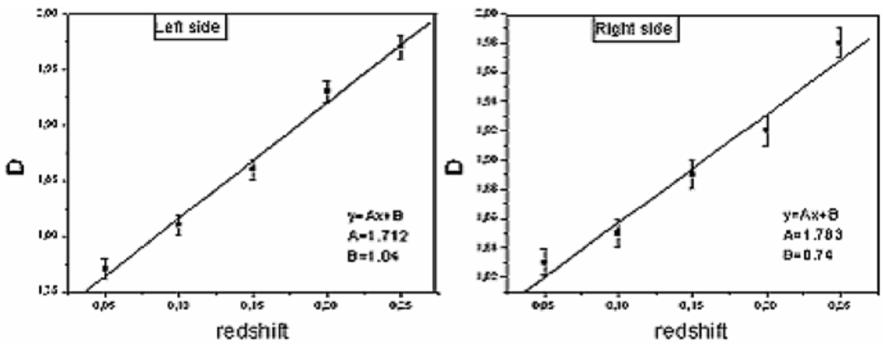

Fig. 3 – Dependence of D with the redshift for both left and right parts of the map.

The difficulty consists in analysing a 3D structure using a 2D image. Considering the "great wall" approximation, at this stage we did not use the 3D data of the galaxies distribution.

These preliminary results do not take into account the corrections necessary to reduce the Z parameter to the actual distances between the galaxies from the survey.

3. GRAVITATIONAL POTENTIAL ENERGY FOR DIFFERENT DISTRIBUTIONS

In the case of clusters the virial theorem is used for mass determination:

$$2\langle E_{kin}\rangle + \langle E_{pot}\rangle = 0 \tag{2}$$

In these estimates, however, it is necessary to remember that the virial theorem can only be used under certain conditions: a closed system in a state of mechanical equilibrium, and it applies to the time average of the system. Whether the observed clusters do indeed fulfil these conditions or, in other words, whether they are relaxed, is not easy to determine, and therefore the application of the virial theorem is questionable. Assuming nevertheless that it is applicable, the kinetic energy for N galaxies in a cluster is given by

$$\langle E_{kin}\rangle = \frac{1}{2} N \langle m v^2\rangle \tag{3}$$

and with $1/2N(N-1)$ independent galaxy pairs the potential energy is

$$\langle E_{pot}\rangle = \frac{1}{2} G N(N-1) \frac{\langle m^2\rangle}{\langle r\rangle} \tag{4}$$

With $(N-1) \sim N$ and $N(m) = M$ an estimate of the dynamic mass is given by

$$M \approx \frac{2\langle r\rangle\langle v^2\rangle}{G} \tag{5}$$

Considering the Newtonian theory, with "universal time" and rectangular space coordinates, the trajectories of neutral particles is given by (Misner *et al.* 1970)

$$\frac{\mathrm{d}^2 x^j}{\mathrm{d}t^2} + \frac{\mathrm{d}\phi}{\mathrm{d}x^j} = 0 \tag{6}$$

where ϕ is the Newtonian potential.

If we consider a number of points in space, where we approximate each point with a galaxy, the gravitational potential energy depends on the sum $\Sigma 1/d$, where d are the distances between all randomly chosen two points.

3.1. GRAVITATIONAL POTENTIAL ENERGY FOR RANDOM DISTRIBUTIONS

In order to see the variation of the gravitational potential energy for different distributions of material points in a two dimensional space, we developed a program that generates certain numbers of points with a random distribution in space and computes the sum of all possible distances between all the generated points and also the sum of the distances with respect to a point in particular. One of our goals was to see how this sum is varying when maintaining a certain number of points and increasing the area in which these points are distributed.

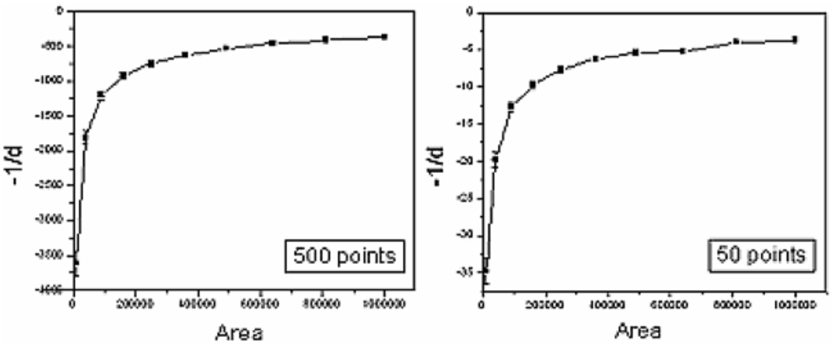

Fig. 4 – Dependence of the $\Sigma(-1/d)$ factor with the area for 500 points and 50 points.

We can observe that this sum factor is, as expected, decreasing with the increase of the surface area where we generate points.

If we increase the surface area and we maintain a constant number of points, we can observe the distribution of the combinations of distances between points.

The following distance histograms are computed for an area, which is increased by a factor f that has the values 9, 25 and 100, respectively.

Moreover, we plotted the variance of this sum, with the number of generated points, when the area is maintained constant. The result is shown in Fig. 6. The

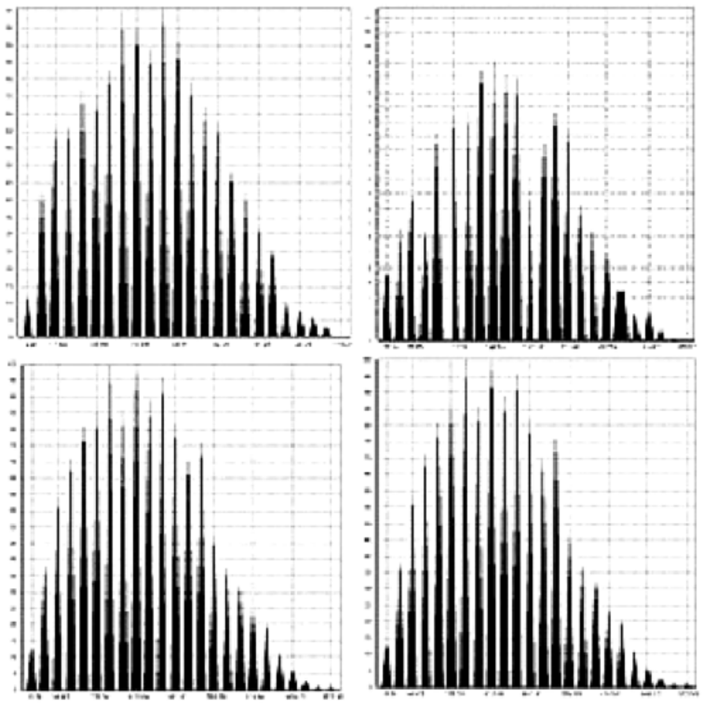

Fig. 5 – Distance histogram, representing the number of points distribution with the distance for four different areas, when the number of points remains constant.

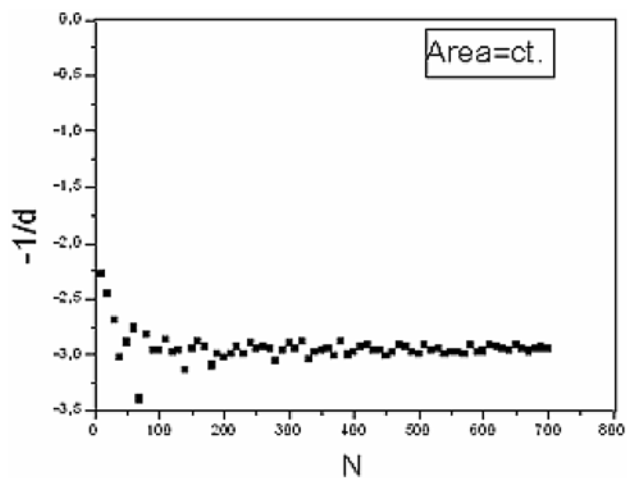

Fig. 6 – The sum factor dependence on N, for a constant area, where N is the number of points.

distance histograms are presented in Fig. 7, where the number of points is increased by a factor that has the values 5, 10 and 50, respectively.

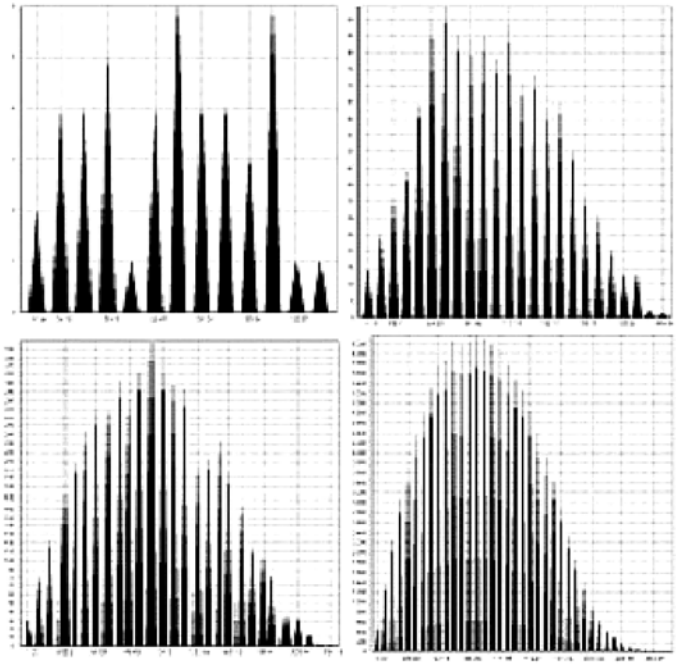

Fig. 7 – Distance histograms, representing the number of points distribution with distance for four different numbers of points.

3.2. GRAVITATIONAL POTENTIAL ENERGY FOR FRACTAL DISTRIBUTIONS

Our next objective was to see how the gravitational potential energy is influenced by a fractal distribution of matter. We are taking into consideration two fractal models, one for a Koch-type evolution and another one for a Sierpinski-type evolution.

We compared these distributions with the random ones, generated for the same number of points and the same area, as the one mentioned before.

In Figs. 8 and 9 are shown the distance histograms for a fractal distribution of Koch-type and a random one with the same number of points and same area, while in Figs. 10 and 11 we present the distance histograms, using the same procedure, but for a Sierpinski-type distribution.

Even though we used a fractal set generated by a deterministic algorithm, not a stochastic one, the total “gravitational potential energy”, in the first approximation, does not differ much from the equivalent structure of a random distribution in the same conditions.

For a Koch distribution, the factor $\Sigma -1/d$ is -1.7364 in area units, while for its corresponding random distribution it is -1.9396. In the case of a Sierpinski distribution, we have a factor of -0.8368 and -0.6138 for the random one.

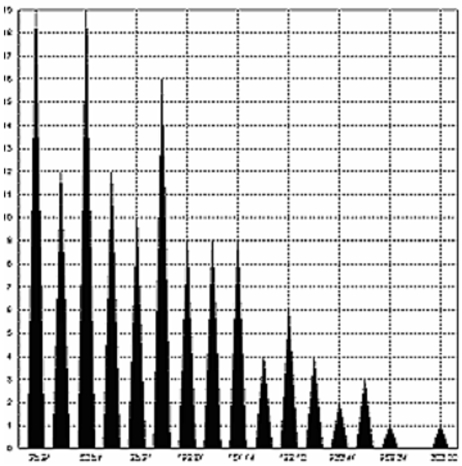

Fig. 8 – Distance histogram for a Koch-type distribution.

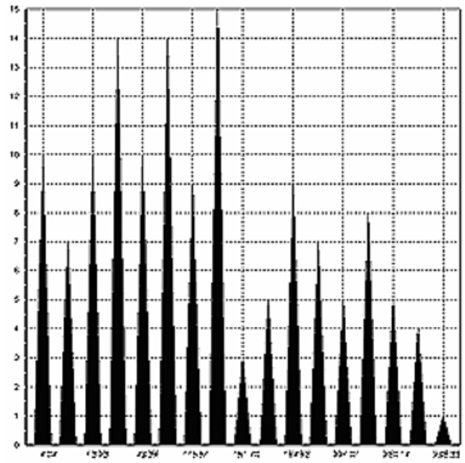

Fig. 9 – Distance histogram for a random similar distribution.

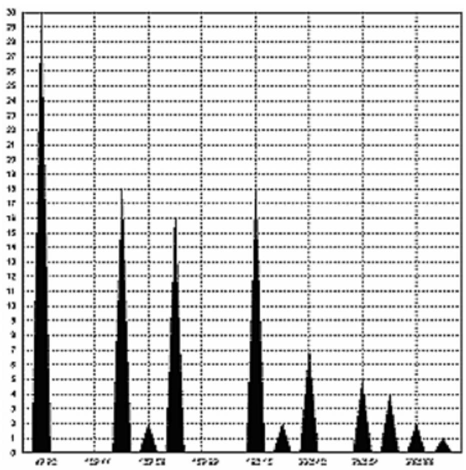

Fig. 10 – Distance histogram for Sierpinski distribution.

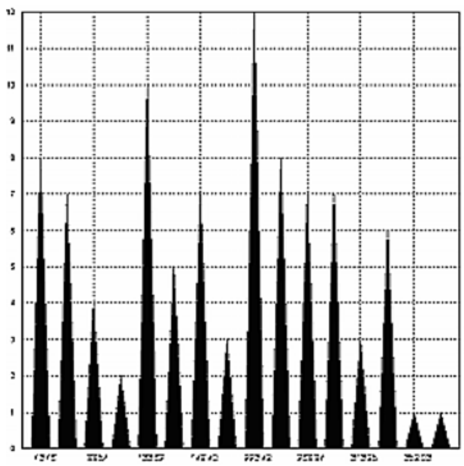

Fig. 11 – Distance histogram for a random similar distribution.

These preliminary results do not take into account the edge effects that may be important when the number of points and the surface area where these points are generated is quite limited.

4. CONCLUSIONS

The gravitational potential energy when increasing the number of generated points for a constant area shows the tendency of reaching a constant value for a large number of points. This suggests that the contribution of distant particles on the total potential energy per pair interactions is not significant.

In the first approximation the total "gravitational potential energy" does not show a significant difference when considering a fractal distribution of points then when considering a random one.

These are preliminary results that show the necessity of using a much larger number of "particles" in order to evaluate the effect of spatial distribution on the total potential energy of the system. Also, we must improve our statistics knowing the approximate case considered here, *i.e.*, a sheet of particles that mimic a 2D structure.

The results of the fractal analysis presented in Section 2.2 are difficult to interpret in this approximation.

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