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ON COULOMBIAN THRESHOLD STATES

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Abstract. This work does approach the description of Coulombian Threshold States in terms of Nuclear Reactions Theory. The physical basis of this approach is the spatial extension, outside the nucleus radius, of the Coulombian Threshold States. The spatial extension factor is discussed according to different physical interpretations. The generalization of the one-channel spatial factor to multichannel reaction systems is obtained. The problem of the spreading of the threshold state in continuum of statistical levels is approached too. The implication of Coulombian Threshold States in problem of Optical Potential Threshold Anomaly is also discussed.

Key words: threshold states, nuclear reactions theory.

1. INTRODUCTION

The highly excited nuclear states ($E_{exc} \sim$ Break-Up Threshold) figured as states with high complexity, requiring statistical approaches for their descriptions. On the other side, one could observe sometimes levels which are discernable on the statistical background.The inhibition of mixing of a special level with the statistical levels is related to a specific nuclear reaction mechanism. For example the Isobaric Nuclear Resonances do not mix with the statistical levels because of Isospin conservation. Another possible mechanism inhibiting the mixing of a special state with the statistical background could be (different) spatial localizations. If the statistical levels are spatially confined to interior of the nucleus and if the special state is extended outside of the channel radius then the overlap of the two kinds of states is diminished; in other words the special state avoids its mixing with statistical levels and it is experimentally discerned as a level superposed on statistical background. The near-zero energy states, both below and above Break-Up Threshold, do fulfill this condition; the wave function's tail does extend out of nucleus radius because of slow decrease of its spatial asymptotic form, $\exp(ikr)$, with k near zero value. (For negative energies $k = i|k|$; for positive energies, $k = k_1 - ik_2$ and k_1 and k_2, positive.) These states are located near zero-energy (in vicinity of escape threshold), i.e. near transition point between Discret and Continuum parts of the Spectrum. One can enumerate, in this respect, various types of spatial extended states both in nuclear and atomic systems: virtual neutron states, neutron halo-states, zero-energy neutron single particle resonances, kinematical threshold states (of astrophysical interest), fission vibrational states (defined with respect to a deformation variable instead of a spatial one), atomic Rydberg states. One can add also the Coulombian Threshold States, related to Coulombian and Centrifugal Threshold. The description of this last type of states is subject of the present work.

The Coulombian Threshold States are connected with energy of two-body Break-Up of a nucleus, $C = a_i + X_i$, $i = 1, 2, \ldots, n$. These levels are composed mostly of clusters a_i and X_i in a definite orbital state. The excitation energy of the cluster level π is $E_\pi = Q_i$, where Q_i is a Break-Up ("threshold") energy for the decomposition $C = a_i + X_i$. The cluster levels of this type were named "Threshold States" (see Baz A.I., 1959; Baz A.I., Zeldovich I.B., and Perelomov A.M., 1971). The main characteristics of the Coulombian Threshold States are:

– they are correlated with fragmentation of the nucleus into two components with zero kinetic energy, $E_\pi - (B_i + B_c + B_l) = 0$ ($B_{n,l,c}$ – binding, centrifugal and Coulomb energies);

– the two fragments have definite relative angular momentum l;

– the Threshold States have a large reduced width for decay in threshold channel i, $\gamma^2_{\pi i} = \gamma^2_W$, and a negligeable reduced width for decay in other open channels a, $\gamma^2_{\pi a} \sim 0$, (γ^2_W – Wigner unit).

In spite of several theoretical approaches (see Kukulin V.I., Krasnopolsky V.M., and Horacek J., 1989), the description of Threshold States is still vague. Qualitative treatments, based on peculiar attractive surface potential just at top of the Coulombian barrier, were proposed. Such barrier potentials could influence the threshold channel decay width but they do not account for other main properties of these states. One needs an approach which describes firstly the global characteristics of Coulombian Threshold States as: energy positions, multichannel decay widths, spreadig width into statistical levels, rotational properties, etc. A Reaction Model for Coulombian Threshold States, responding to these requirements, is presented here. Its physical idea is the spatial extension, outside nuclear radius, of the Threshold State radial wave function. The Coulombian Threshold States are described in this work in framework of the R-Matrix Theory. The formal device for description of spatial extension of wave function, both within nucleus interior and also outside channel radius, is known as the Compression Factor of R-Matrix Theory. This is why the R-Matrix Theory appears to describe appropriatelly the spatial distribution of these states. The R-Matrix description of Threshold States is developed in two variants: the isolated threshold level, and threshold level embeded in the background of statistical levels.

2. **SINGLE THRESHOLD STATE**

The Threshold States are related to process of escape of a particle (or cluster) from nucleus. In limit of zero energy the nucleus is splited into two fragments, the residual nucleus and an emitted particle. Their relative velocity is near zero; the two fragments remain close together for a while. This configuration, called Threshold State, consists of two clusters in a definite relative movement (orbital angular momentum). The relative movement of the two fragments is described by a radial wave function extended spatially outside channel radius. The overlap of this function to channel wave function becomes very large, resulting in a big reduced width for level's decay in threshold channel. This physical image (break-up into two fragments in a very slow relative movement, radial wave function extended outside channel radius, large reduced width for decay in threshold channel) is subject of the following formal modelling.

The spatial extension of the radial wave function, outside the channel radius a is given by (Lane A.M., 1970),

$$\beta(E) = \frac{\int_0^a |u(r)|^2 dr}{\int_0^a |u(r)|^2 dr + |u(a)/f(a)|^2 \int_a^\infty |f(r)|^2 dr},$$

where $u(r)$ is the radial wave function of the level under consideration and $f(r) = \exp(ikr)$, its channel wave function. For a deep-lying bound state $k = i|k|$, $|k|$ – large, $\beta(E)$ is unity; the spatial extension factor, $\beta(E)$, reaches its minimum value just at zero energy. For the resonant level, above threshold $E > 0$, $k = k_1 - ik_2$ with k_1 and k_2 both positive, a similar definition holds (see e.g. Baz A.I., Zeldovich I.B. and Perelomov A.M. 1971, Ch. VII; also Schnol E.E., 1970 and Schnol E.E., 1971). The $\beta(E)$ factor is a measure of spatial extension of the level's wave function outside channel radius. On the other hand its significant non-unity value is just at or near threshold. We have to put into correspondance the threshold level with its spatial extension outside nucleus radius. Moving a deep bound state towards threshold, its tail extends more and more outside nucleus and this corresponds to the "explosion" of threshold level. This single particle picture does not depend mainly on interior of the nucleus.

The R-Matrix Theory provides a suitable formal device for description the spatial extension (out of nucleus radius) of the radial wave function; this is R-Matrix "compression factor" (Lane A.M. and Thomas R.G., 1958). According to R- Matrix Theory, the "spatial extension factor" is identical to R-Matrix "compression factor".

$$\beta(E) = \frac{1}{1 + \gamma_{\pi n}^2 dS_n/dE},$$

where $L_n = S_n + iP_n$ is logarithmic derivative of the n-threshold channel. The real and imaginary parts of logarithmic derivative are shift-factor, S_n, and penetration-factor, P_n. The level π reduced width for decay in threshold channel n is $\gamma_{\pi n}^2$; in a single particle limit it is proportional to wave function value $|u(a)|^2$ at channel radius a. A resonant level $1/(E_\pi^0 - \Delta_\pi + i\Gamma_\pi)$ is distorted by strong energy dependance of threshold channel terms in the level-shift Δ_π and total width Γ_π, namely $S_n\gamma_{\pi n}^2$ and $P_n\gamma_{\pi n}^2$. By a Taylor expansion of threshold channel shift factor, $S_n(E) = S_n(0) + E(dS_n/dE)_\epsilon$ with energy measured with respect to threshold (zero energy) and $0 < \epsilon < E$, one obtains a renormalization of level's position and width

$$E_\pi = E_\pi^0 - S_n(0)\gamma_{\pi n}^2 \to \beta E_\pi, \quad \Gamma_\pi \to \beta\Gamma_\pi.$$

It is proved analitically for neutrons that $dS_n/dE > 0$ and this results into $\beta < 1$. Far away from threshold the shift factor is nearly constant and $\beta = 1$. For threshold region a strong energy dependance of the shift factor and a large reduced width $\gamma_{\pi n}^2$ result in small values for compression factor β. In the limit $\beta \to 0$ the (single particle) levels are shifted just to threshold.

Another physical interpretation of the compression factor is its reciprocal β^{-1} is the "enhancement factor" of the probability to find the level near threshold (Lane A.M., 1970).

There is also other alternative interpretation of the β factor (Ata M.S. and Hategan C., 1988). A slight generalization of β factor is

$$\beta = \frac{1}{1 + \gamma_{\pi n}^2 [S_n(E) - S_n(E_r)]/(E - E_r)}.$$

This quantity for a threshold level (energy $E_r = E_\pi - S_n(E_r)\gamma_{\pi n}^2 = 0$) is just ratio between the Reduced R-Matrix $R_r(b)$ and the R-Matrix itself where natural boundary conditions $b = S_n(E_r)$ were used in definitions: $R_r(b) = (R^{-1} - b)^{-1}$ and $S_n(b) = S_n(E) - b$.

$$\beta = R_r(b)R^{-1} = (1 - Rb)^{-1}.$$

The R-Matrix contains informations only on inner part of configuration space for the open channel system. The Reduced R-Matrix contains additional information on threshold channel and its outer channel region. The

β factor expresses the effect of the threshold channel on the reaction system. It enters as a "renormalization factor" of R-Matrix parameters (energy, width). If β is unity the system does not feel the threshold channel. This happens far away from threshold (where $S_n(E)$ does not change much with energy, $S_n(E) \sim S_n(E_r)$) or for negligeable reduced width $\gamma_{\pi n}^2 \sim 0$. But it is essentially different from unity for those resonances which interplay with n-threshold ($S_n(E)$-strong energy dependent) and which have a large reduced width for decay in threshold channel (single particle or cluster character). These remarks result into another formal definition of threshold state π of energy E_π and width Γ_π:

$$|E_\pi - E_{thr}| < \Gamma_\pi, \quad \Gamma_{\pi n} \sim \Gamma_\pi.$$

Only these states (coincident with threshold and decaying preferentially in threshold channel) have a β factor differing essentially from unity. A significant threshold effect in cross-section of an open channel occurs onlly as consequence of a neutron threshold state.

Concluding the cardinal role played by β factor, it deserves a matrix generalization for the multichannel systems:

$$\beta = R_S R^{-1} = (1 - RS)^{-1}.$$

The diagonal shift-factor matrix, S, is splited into an open-channel part, S_0, and the threshold channel component ΔS, $S \to S_0 + \Delta S$ with only one non-zero component S_n in ΔS. Now the matrix $(R^{-1} - S)$ becomes $[(R^{-1} - S_0) - \Delta S]$ and

$$R_S = R_0 + r \frac{S_n}{1 - R_n S_n} r^T,$$

with r defined by $r = \|R_{1n}, R_{2n}, \ldots, R_{Nn}, R_{nn}\|$ both for the N open channels and the threshold one $n = N + 1$. This formula is similar to the Reduced R- Matrix one, but in addition it includes the threshold channel, too. For the single channel case the generalized matrix β reduces to the single channel factor $\beta = R_S R_0^{-1}$.

The spatial extension factor β could be expressed in terms of Kapur-Peierls Matrix too,

$$R_{KP} = (R^{-1} - L)^{-1},$$

with $L = S + iP$ the logarithmic derivative matrix,

$$R_{KP} = (R_S^{-1} - iP)^{-1}.$$

By analogy we define

$$\beta_{KP} = R_{KP} (R_{KP}^0)^{-1},$$

which for a single channel reduces to

$$\beta_{KP} = (1 - R_n L_n)^{-1}.$$

The compression factor becomes complex; its modulus can be used as a measure of spatial extension of the level wave function, outside channel radius.

One can add some remarks on Molecular Aspects of Coulombian Heavy-Ions Threshold States. An analytical formula for calculus of Coulombian Threshold States' energies has been obtained; it uses analytical formulae for logarithmic derivative in heavy fragmentation limit (Coulomb parameter, $\eta \gg 0$).

$$dL/dE = (1/2E)[L - L^2 + (2ma^2)/h^2(V - E) - \eta dL/d\eta],$$

with V – threshold channel interaction potential, m – reduced mass and a – channel radius. Evaluating minimum of β-factor one obtains an approximative formula for the energy of Threshold States in limit of heavy ions fragmentation (see Hategan C., 1983).

$$E_{TS} = B_c + (h^2/2ma^2)[-4\eta - L(L+1)/2\eta + L^2(L+1)^2/32\eta^3],$$

with B_c- coulombian/centrifugal barrier. Observe that $L = 0$ threshold level is below effective threshold B_c. (According to some models for nuclear molecules their energy is just B_c.) Also an exact numerical approach to calculus of Coulombian Heavy-Ions Threshold States energies has been done (Duma M., Hategan C., and Ionescu R.A., 1993). It is based on calculus of minimum for "Compression Factor β". Numerical evaluations of minimum of β parameters for different nuclear systems as $Ne^{20}, Mg^{24}, Si^{28}$, etc. result also into conclusion that the threshold's state energy is located just below the coulombian-centrifugal threshold (Hategan C., 1983). Both numerical and analytical approaches agree pretty well with experimental centroids of nuclear molecules (Cindro, 1981).

In next paragraph we discuss about effect of Channel Interaction on Threshold States. Let assume an attractive potential superposed on coulombian/centrifugal barrier. Such additional channel interaction will result into change of channel's logarithmic derivative and, consequently, of Threshold State's energy. A similar situation does appear in Fission Theory (see Bjornholm S., and Lynn J.E., 1980). The attractive potential, superposed on fission barrier, does accomodate the fission vibrational states. One could approach the problem of fission vibrational states in two formal ways: either to associate to a class of R-Matrix states or to modify the channel logarithmic derivative. The last method is named "extended penetration factor or logarithmic derivative". By analogy we could suppose the existence of "vibrational states" in a binuclear attractive potential superposed on channel coulombian/centrifugal barrier. The Coulombian Threshold State is then splited in these vibrational states, resulting in substructures (similar to "Intermediate Structure" phenomenon). The split of R-Matrix in two components results into renormalization of channels logarithmic derivatives (see Lane A.M., and Thomas R.G., 1958)

$$L = (L^{-1} - R)^{-1} = L + L/[(LR)^{-1} - 1].$$

Assuming vibrational resonances in attractive potential

$$R_{nn} = \Sigma_\lambda \gamma_{\lambda n}^2/[E_\lambda - E - iW_\lambda],$$

one obtains for channel logarithmic derivative

$$L = L + \Sigma_\lambda L^2\gamma_{\lambda n}^2/[E_\lambda - E - iL\gamma_{\lambda n}^2 - iW_\lambda]$$

reflecting its resonant structure. One could calculate the transmission coefficients according to usual formula

$$T_n = 1 - |[1 - RL^*]/[1 - RL]|^2 = 4P_n ImR/|1 - RL|^2 =$$
$$= 4P_n W\gamma_{\lambda n}^2/[(E_\lambda - S\gamma_{\lambda n}^2 - E)^2 + (P\gamma_{\lambda n}^2 + W)^2].$$

Observe that in weak-absorption limit (W – small) one obtains sharp resonant structures both in logarithmic derivative and transmission coefficient. In the large absorption limit these resonant structures are washed out. This aspect should be approached in more detail.

Concluding this chapter, the Threshold State is a special quasistationary state, coincident in energy with threshold, also having a large reduced width ($\simeq$ Wigner unit) for decay in the threshold channel. The reduced width is a measure of single particle character of the level in interior region. The probability of finding a pair of threshold particles out of channel radius is proportional to the threshold channel reduced width; a very large reduced width results into level "explosion" out of channel radius. The Threshold State is described, in first approximation, as a Single Particle State coincident with threshold; its overlap $\gamma_{\pi n}$ with threshold channel is very large, i.e. it has a large escape width $\Gamma_\pi^\uparrow$. By residual interactions the Single Particle Resonant State is spread out over compound nucleus actual levels (next paragraph).

3. THRESHOLD STATE IN MULTILEVEL SYSTEMS

The Coulombian Threshold States are highly excited states ($\sim$ – Coulomb-Centrifugal barrier); at such excitation energies there is a high density of states, even for light-medium nuclei. The Threshold State appears as a state embeded in a background of (statistical) levels. However it is not disolved in the statistical levels, showing up as a discernable level superposed on statistical background. This situation is similar to the case of Isobaric Analogue Resonances. These resonances are levels with a given (good) isospin number, superposed on background levels with different isospin. The isospin conservation is the mechanism which prevents the spreading of the Isobaric Analogue

Resonance in the statistical levels. In the case of Threshold States, the underlying mechanism which does inhibit their decay in statistical levels, is their large extension in the threshold channel, outside its radius. Formally this is expressed by a large overlap of the level to the channel, i.e. a large reduced width for decay in threshold channel. As the total reduced width cannot excede the Wigner unit, this condition results into small values of reduced widths for decay in complementary reaction channels. The out of channel radius part of level wave function does not overlap to nucleus inner states; the last ones are complicated multiparticle states forming the statistical background. The spatial extension in the threshold channel of the level under consideration is responsable also for decoupling from statistical levels, resulting in a smaller spreading width. The states with small spreading widths, as compared to the escape one, are called intermediate or doorway states. The doorway aspect of the Threshold State has to be described as a special state embeded in the sea of statistical levels. The reaction model for descriptin of Threshold States embeded in statistical levels is similar to external mixing model for Isobaric Analogue Resonances. In the external mixing model the approximative isospin conservation is involved. In this description of Threshold States, embeded in statistical levels, the spatial extension of the threshold level is used (Hategan C., 1983).

In order to describe the interplay between a special state (an Isobaric Analogue Resonance, a Threshold State, or an usual Doorway State) and the statistical levels one has to split the K- or R-Matrix or Kapur-Peierls Matrix into two corresponding parts (see Lane A.M., 1969). The two parts will be labeled, in the following, by letters π (for Threshold State) and β (for background statistical levels μ)

$$R_{KP} = R^{\pi} + R^{\beta} = \gamma^{\pi} * \gamma^{\pi}/(E_{\pi} - E) + \Sigma_{\mu}\gamma^{\mu} * \gamma^{\mu}/(E_{\mu} - E).$$

This results into decomposition of the Collision Matrix in two parts, corresponding to R^{π} and R^{β}, for studying the problem of the immersion of the single level π in the sea of statistical levels μ. One can study either the influence of statistical levels on the threshold level, or the influence of the threshold level on the statistical ones (Ata M.S., and Hategan C., 1988).

In this approach one assumes that the statistical background could be described as a set of non-overlaping levels. (An exact treatment does not involve this assumption (Hategan C., Comisel H. and Ionescu R.A., 2004).) The only physical assumption concerns the reduced widths of the threshold level π for decay in the open channels a and in n threshold channel, $\gamma_{\pi a} \ll \gamma_{\pi n}$. It appears that the decay of statistical levels μ in open channels is not affected to much by the presence of the threshold level. The statistical levels are decoupled from threshold channel by a factor $f^2 = \beta^2$ which can be named "de-enhancement factor". This factor is a measure of the decoupling of ordinary statistical levels from the threshold channel.

The threshold state π has more interesting properties. If the energy-averaged statistical backgroud is diagonal (basic assumption in Hauser-Feshbach Statistical Model) then the threshold state's coupling to open channels is not modified. If this statistical assumption does not hold then the Threshold State can decay in open (non-threshold) channels only by a two-step process. This process was called Quasi-Resonant Scattering (Dorobantu V. and Hategan C., 1991). The resonance's denominator of the threshold level, immersed in background of statistical levels, results into evaluation of the spreading width. It is proved that the Threshold State's spreading width is propotional to the statistical background Strength Function of threshold channel, $\Gamma_{\pi}^{\downarrow} \sim S_n^{\beta}$. As the background Strength Function is proportional to level density of statistical levels one obtains the "Damping Width Postulate" which is common to theories of simple structures lying at high excitation energies (Lane's Theory of "Line Broadening").

The Threshold State is highly excited state, embeded in a continuum of statistical levels. The Threshold State has a small overlap to inner compound nucleus states because of its spatial extension, out of channel radius.The Threshold State is decoupled from statistical levels by the "de-enhancement" factor β,resulting in a small spreading width $\Gamma_{\pi}^{\downarrow}$. The Micro-Giant Threshold State is an additional example of "Line-Broadening" in Nuclear Physics. Both the "doorway" nature of the Threshold State as well as the mechanism preventing its spreading in statistical continuum originate in its very large spatial extension (out of channel radius).

4. **ON COULOMBIAN THRESHOLD STATES AND OPTICAL POTENTIAL ANOMALY**

A decrease of the real part of Optical Potential ΔV, centred at Coulombian threshold, was reported in literature (see e.g. Satchler G.R., 1991); this effect was called Threshold Anomaly of the Optical Potential. At same Coulombian Threshold energy, the imaginary part W of the Optical Potential sharply decreases when energy falls below the Coulomb barrier. The Optical Potential plays a cardinal role in calculus of fusion reactions and, consequently, in the time-reversed reactions of fragmentation. In this chapter the Optical Potential Threshold Anomaly is related to Coulombian Threshold States. The role of Coulombian Threshold States in producing the Optical Potential Threshold

Anomaly is approached in terms of Feshbach's Projector Theory for Effective Interactions (see e.g. Kuo T.T.S., 1981); this approach is based on β renormalization factor of Threshold State.

According to Feshbach Theory, the reaction systems is split into two parts by the projection operators P and Q, on retained and eliminated channels, respectively

$$1 = \langle \Psi_\pi | \Psi_\pi \rangle = \langle P\Psi_\pi | P\Psi_\pi \rangle + \langle Q\Psi_\pi | Q\Psi_\pi \rangle.$$

In terms of Effective Hamiltonian one obtains a relation connecting the two norms, $\|P\Psi_\pi\|$ and $\|Q\Psi_\pi\|$, of the state π with energy E_π,

$$-(dE_\pi/d\omega)_{(\omega=E_\pi)} \langle P\Psi_\pi | P\Psi_\pi \rangle = \langle Q\Psi_\pi | Q\Psi_\pi \rangle$$

and the norm becomes (Kuo T.T.S., 1981),

$$\langle P\Psi_\pi | P\Psi_\pi \rangle = 1/(1 - (dE_\pi/d\omega_{E_\pi}) = \beta.$$

In next step one defines the hamiltonian for independent (uncoupled) P-reaction system

$$(PHP)|P\Psi_\pi\rangle = E_\pi^0 |P\Psi_\pi\rangle.$$

The shift of the level π due to coupling to Q-system, $\Delta E_\pi = E_\pi - E_\pi^0$, is evaluated assuming a linear approximation, accepted in R-Matrix Theory (Lane A.M. and Thomas R.G., 1958)

$$\Delta E_\pi = E_\pi - E_\pi^0 = \Delta E (dE_\pi/d\omega)_{(\omega=E_\pi)}.$$

One obtaines that the additional term in effective Optical Potential, $\Delta U = \Delta V + iW$, is

$$\langle P\Psi_\pi | \Delta U | P\Psi_\pi \rangle = \Delta E(\beta - 1), \quad \beta = 1/(1 + \gamma_{\pi n}^2 dL_n/dE).$$

If this matrix element does not depend on specific state π (see e.g. Davydov A.S., 1958), one obtains

$$\Delta V + iW = \Delta E(\beta - 1)$$

or even (because both ΔV and W are negative)

$$|\Delta V| + i|W| = \Delta E(1 - \beta).$$

(The last assumption should be correct in the Optical Model limit which involves only Single Particle States). At this level of derivation, the "compression factor" is evaluated according to R-Matrix Theory, resulting into

$$|\Delta V| + i|W| = \Delta E((1 - \beta_R) + i\gamma_{\pi n}^2 dP_n/dE),$$

$$\beta_R = 1/(1 + \gamma_{\pi n}^2 dS_n/dE).$$

It results that the polarization term of the effective Optical Potential is dependent on the real part of the β factor

$$|\Delta V| = \Delta E(1 - \beta_R),$$

while the imaginary part W is proportional to the energy derivative of the threshold channel penetration factor

$$|W| = \Delta E \gamma_{\pi n}^2 dP_n/dE.$$

One has to remark that the energy dependence of β_R factor and of dP_n/dE do reproduce the energy dependences of the polarization $|\Delta V|$ and of absorptive $|W|$ terms in Optical Potential respectively (dip/resonance and S- shape forms). We have done calculations of β_r and of energy derivative of penetration factors by assuming that the reduced width of the Coulombian Threshold State is just Wigner unit; this assumption on reduced width is congruent with Optical Model description of Single Particle States/Resonances. The above approach to Optical Potential Threshold Anomaly results into conclusion, the threshold anomaly of the Optical Potential $\Delta V + iW$ is strongly dependent on reduced width of the threshold state $\gamma_{\pi n}^2$, i.e. if the Optical Potential U does support a genuine Single Particle Threshold State ($\gamma_{\pi n}^2$ attains maximum value), then the Optical Potential Threshold Anomaly is strong. If there is no Threshold State, i.e. the reduced width vanishes, then there is not a Threshold Anomaly in Optical Potential. This is a new physical result firstly obtained here.

5. **CONCLUSIONS**

The problem of Coulombian Threshold States was formulated by Baz; he proposed a qualitative theory based on spatial extension of wave function's tail outside channel radius. Moreover he extended this view assuming an attractive potential at channel radius in order to increase the probability to find threshold partners just at channel radius.

The present work does approach the Coulombian Threshold States in quantitative terms of Nuclear Reactions Theory. The Threshold States are located just at frontier between Discrete Spectrum (which is subject of Nuclear Spectroscopy) and Continous Spectrum (which is subject of Nuclear Reactions). This is why both spectroscopical and reaction aspects have to be involved in description of Threshold States. The extension of the Threshold States, outside channel radius, is quantitatvely described in terms of Compression Factor of R-Matrix Theory. The present approach does present other three different physical interpretations of the Spatial Extension of wave function: Compression Factor, Enhancement Factor of probability to find the level near threshold and as Ratio between Reduced R-Matrix and R-Matrix itself. The spectroscopical aspects of the Threshold States are quantitatively given by the reduced width for decay in threshold channel. The problem of attractive potential (proposed by Baz) in the threshold channel is qualitativelly approached in R-Matrix terms, similar to an approach used in Fission Theory. This approach results into conclusion that the Threshold State is splited into substuctures of "vibrational" nature; this peculiar aspect needs additional investigations. The one-channel Threshold States are investigated analytically in limit of Heavy Ions fragmentation. It is proved that the Threshold States are organized in rotational bands; in this respect the Heavy Ions Threshold States have properties of Nuclear Molecules.

Another aspect of the Nuclear Molecule, namely the Damping Width Postulate, is approached by extending the description of Threshold State for multilevel-multichannel systems.The one-channel Spatial Extension Factor is generalized to multichannel and multilevel reaction systems. The main problem is that of the spreading of the Threshold State in continuum of statistical levels. The spatial extension of Threshold State outside channel radius results into its decoupling from background of statistical levels. On proves that the "de-enhancement factor", i.e the decoupling from statistical levels, is the sub-unitary Threshold Compression Factor. The Spreading Width of the Threshold State decreases while the Escape Width is increased by β or $1/\beta$ quantities, respectively. One proves also that in statistical limit of fluctuating non-diagonal Collision- Matrix elements, the Threshold State is statistically decoupled from open complementary channels.

The implication of Coulombian Threshold States in problem of Optical Potential Threshold Anomaly is also studied. It is emphasized the role of Coulombian Threshold States in producing significant changes of the real polarization term of the Optical Potential and also its relation to imaginary potential.

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