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*Dedicated to Prof. Dorin N. Poenaru's*
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# ROLE OF NON-AXIAL DEFORMATIONS IN THE FISSION BARRIERS OF HEAVIEST NUCLEI

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*Abstract.* The influence of various non-axial deformations on the fission-barrier height of a heavy nucleus is discussed. Both the quadrupole and hexadecapole shapes of a nucleus are considered. It is shown that the non-axial shapes may reduce the height by up to about 2 MeV. This makes the heights obtained in the case of axial symmetry of a nucleus, which is usually done, unrealistic, at least for some nuclei.

*Key words:* heavy and superheavy nuclei, nuclear deformations, heights of fission barriers.

## 1. INTRODUCTION

One of the basic problems in the studies of heaviest nuclei is the description and predictions of the cross section $\sigma$ for their synthesis (e.g., [1, 2, 3, 4, 5, 6, 7, 8]). The height of the static fission barrier $B_{\rm f}^{\rm st}$ is an important quantity needed for the calculations of $\sigma$. This height is a decisive quantity in the competition between neutron evaporation and fission of the compound nucleus in the process of its cooling. A large sensitivity of $\sigma$ to $B_{\rm f}^{\rm st}$ stresses a need for accurate calculations of $B_{\rm f}^{\rm st}$. For example, a change of $B_{\rm f}^{\rm st}$ by 1 MeV may result in a change of $\sigma$ by about one order of magnitude or even more [4]. The basic role, in reaching this accuracy, is played by the deformation space admitted in the calculations of $B_{\rm f}^{\rm st}$.

Up to the present, the height $B_{\rm f}^{\rm st}$ has been mostly studied in the case of the axial symmetry of a nucleus (e.g., [9, 10, 11, 12, 13, 14]). Studies taking into account non-axial degrees of freedom were less frequent (e.g., [15, 16, 17, 18, 19, 20]).

The objective of this paper is to illustrate the role of non-axial shapes in the barrier heights $B_{\rm f}^{\rm st}$ of heaviest nuclei.

## 2. THEORETICAL MODEL

The potential energy of a nucleus is analyzed within a macroscopic-microscopic approach. The Yukawa-plus-exponential model [21] is taken for the macroscopic part of the energy and the Strutinski shell correction, based on the Woods-Saxon single-particle potential, is used for its microscopic part. Details of the approach are specified in [22].

Especially important in the calculations is the deformation space admitted in them. Generally, a 10-dimensional deformation space is used in our studies. In particular, it includes the general hexadecapole space (if one assumes the reflexion symmetry of a nucleus with respect to all three planes of the intrinsic coordinate system [23]), not considered in earlier studies. The space is specified by the following expression for the nuclear radius $R(\vartheta,\varphi)$ (in the intrinsic frame of reference) in terms of spherical harmonics $Y_{\lambda\mu}$:

$$\begin{aligned} R(\vartheta,\varphi) &= R_0\Big\{1+\beta_2\left[\cos\gamma_2 \mathrm{Y}_{20}+\sin\gamma_2 \mathrm{Y}_{22}^{(+)}\right] \\ &+ \frac{1}{\sqrt{12}}\beta_4\left[(\sqrt{7}\cos\delta_4+\sqrt{5}\sin\delta_4\cos\gamma_4)\mathrm{Y}_{40}\right. \\ &- \left.\sqrt{12}\sin\delta_4\sin\gamma_4\mathrm{Y}_{42}^{(+)}+(\sqrt{5}\cos\delta_4-\sqrt{7}\sin\delta_4\cos\gamma_4)\mathrm{Y}_{44}^{(+)}\right] \\ &+ \beta_6\mathrm{Y}_{60}+\beta_8\mathrm{Y}_{80}+\beta_3\mathrm{Y}_{30}+\beta_5\mathrm{Y}_{50}+\beta_7\mathrm{Y}_{70}\Big\}, \end{aligned} \tag{1}$$

where $\gamma_2$ is the Bohr quadrupole non-axiality parameter, $\delta_4$ and $\gamma_4$ are the hexadecapole non-axiality parameters [23], and the dependence of $R_0$ on the deformation parameters is determined by the volume-conservation condition. The functions $\mathrm{Y}_{\lambda\mu}^{(+)}$ are defined as:

$$\mathrm{Y}_{\lambda\mu}^{(+)}=\frac{1}{\sqrt{2}}\left[\mathrm{Y}_{\lambda\mu}+(-1)^{\mu}\mathrm{Y}_{\lambda-\mu}\right], \quad \text{for } \mu\neq 0. \tag{2}$$

The regions of variation of the deformation parameters are

$$\beta_\lambda\geq 0 \quad (\lambda=2,3,\ldots,8), \tag{3}$$

$$0^\circ\leq\gamma_2\leq 60^\circ, \tag{4}$$

$$0^\circ\leq\delta_4\leq 180^\circ, \quad 0^\circ\leq\gamma_4\leq 60^\circ. \tag{5}$$

In our studies, the deformation parameters $\beta_3$, $\beta_5$, $\beta_7$ are only used to show that the potential energy of the studied nuclei is not influenced by the reflection-asymmetric shapes at both the equilibrium and the saddle-point configurations.

Details of the calculations are given in [24].

## 3. ILLUSTRATION OF THE RESULTS

Figure 1, taken from [25], shows an example of the ground-state static fission barrier for the superheavy nucleus $^{278}112$ (this is the compound nucleus in the reaction which has lead to the discovery of the element 112 [26]). One can see that a rather high barrier is obtained for this very heavy nucleus, which is entirely created by effects of shell structure in energy of this nucleus. Without this structure (see macroscopic part of the energy, $E_{\rm macr}$), no barrier is obtained. The largest shell correction to the macroscopic part of the energy is obtained at the (deformed) equilibrium point (about 6 MeV), smaller (about 1.8 MeV) at the first, and the smallest (about 0.5 MeV) at the second saddle point. Significant shell corrections at the saddle points are worth to be noticed, as these corrections are quite often neglected in various estimates of the static fission barriers of superheavy nuclei.

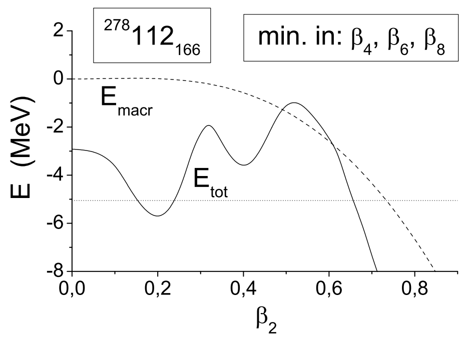


Fig. 1 – Static spontaneous-fission barrier calculated for the nucleus $^{278}112$ in the two cases: when only the macroscopic ($E_{\rm macr}$) and when the total ($E_{\rm tot}$) energy of it is considered [25].

The height of the barrier is defined as the difference between the potential energy at the highest saddle point and the ground-state energy. The latter is the potential energy at the equilibrium point, increased by the zero-point energy in the fission degree of freedom, for which 0.7 MeV is taken [27]. Thus, as a matter of fact, we are only interested in the two values of the potential energy: at the equilibrium point $\beta_\lambda^0$ and at the highest saddle point $\beta_\lambda^s$. To find, however, these points, knowledge of the energy in a large deformation region is needed.

To get some orientation in the values of the barrier heights $B_{\rm f}^{\rm st}$ of heaviest nuclei and their behavior as functions of the proton, $Z$, and the neutron, $N$, numbers, let us look at Fig. 2. The heights are calculated within two models: a macroscopic-microscopic (MM) one [10] and the extended Thomas-Fermi plus Strutinski integral (ETFSI) model [9]. One can see that quite large values of $B_{\rm f}^{\rm st}$ (about 6–8 MeV) are obtained for even so large atomic numbers as $Z = 120$. It is also seen that the difference between the two results is quite large (up to about 3 MeV). Also the dependencies of $B_{\rm f}^{\rm st}$ on both $Z$ and $N$, obtained within the two models, are very different. This suggests a need of improvements in the approaches. Both results are obtained with the assumption of axial symmetry of the nuclei.

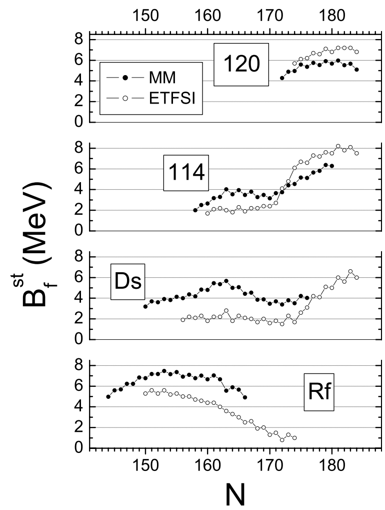


Fig. 2 – Static fission barriers heights $B_{\rm f}^{\rm st}$ calculated by a macroscopic-microscopic (MM) and the ETFSI methods [25].

Effect of the quadrupole non-axial deformation $\gamma_2$ on the barrier height $B_{\rm f}^{\rm st}$ is illustrated in Fig. 3 [28]. The figure shows a contour map of the potential energy of the nucleus $^{250}$Cf plotted as a function of $\beta_2 \cos\gamma$ and $\beta_2 \sin\gamma$. The line $\gamma$=0° corresponds to the axially symmetric (with respect to $Oz$ axis) prolate shapes, and the line $\gamma$=60° is corresponding to the axially symmetric (with respect to the $Oy$ axis) oblate shapes of the nucleus. The line $\gamma$=30° corresponds to shapes with maximal non-axiality. One can see that in the case of axial symmetry the saddle point (denoted by the symbol "+") has the energy 3.8 MeV, while non-axiality shifts the saddle to the point denoted by the symbol "×" and decreases its energy to 2.0 MeV, i.e. by 1.8 MeV. (The energy is normalized so, that its macroscopic part is zero at spherical shape of a nucleus). This means that the reduction of the saddle-point energy (and, thus, of the barrier height $B_{\mathrm{f}}^{\mathrm{st}}$) of this nucleus by non-axiality is quite large. One can show [29] that only the inclusion of the non-axial shapes leads to the barrier height $B_{\mathrm{f}}^{\mathrm{st}}$=5.8 MeV of the nucleus $^{250}$Cf, which is close to the measured value $B_{\mathrm{f}}^{\mathrm{st}}$=5.6± 0.3 MeV [30]. To obtain this, however, one needs to use in the calculations a more dimensional deformation space (which also includes the deformations $\beta_4, \beta_6$ and $\beta_8$ [29]) than the 2-dimensional one taken in Fig. 3.

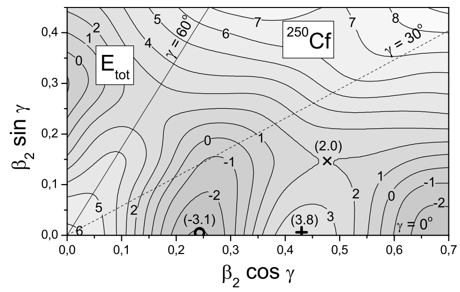


Fig. 3 – Contour map of the total potential energy calculated for the nucleus $^{250}$Cf. Numbers at the contour lines specify the value of the energy. Position of the saddle point is marked by the symbol "+", when the axial symmetry of the nucleus is assumed, and by the symbol "×", when non-axiality is taken into account. Position of the equilibrium point is denoted by the sign "∘". Numbers in parentheses give values of the energy at these points [28].

Up to the present, the hexadecapole non-axiality has been usually treated in an approximate way (e.g., [15, 16, 17, 18]). (It will be shown later that the approximation is not satisfactory). Only very recently, a general non-axial hexadecapole shapes (as specified in Eq. (1)) have been used in calculations [31]. They are described by two non-axiality parameters: $\delta_4$ and $\gamma_4$. The role of the parameter $\delta_4$ in the potential energy of the nucleus $^{262}$Sg ($Z = 106$) is illustrated in Fig. 4 [32]. One can see that the parameter does not influence the energy of the considered nucleus at its equilibrium point (denoted by circle in the figure), but it strongly decreases (by about 1.5 MeV) the energy at its saddle point (denoted by cross). As in the earlier descriptions of the hexadecapole non-axiality (e.g., [15, 16, 17, 18]), the value of this important parameter ($\delta_4$) was assumed to be constant, the descriptions could not be good.

The role of the parameter $\gamma_4$ is illustrated in Fig. 5 [32]. It is seen that the role of this parameter is very small. In particular, it does not modify the energy at the equilibrium point or at the saddle point of the nucleus $^{262}$Sg, leaving, thus, the barrier height $B_{\mathrm{f}}^{\mathrm{st}}$ unchanged.

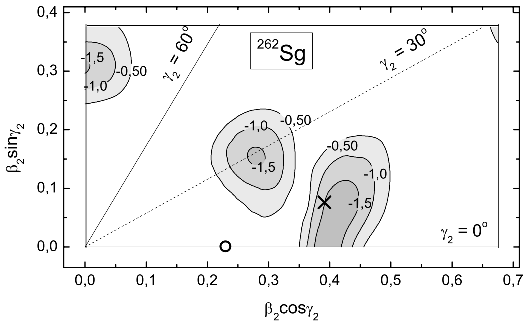


Fig. 4 – Contour map of the difference: $E(\beta_2, \gamma_2; \beta_4^{\min}, \delta_4^{\min}, \gamma_4^{\min}) - E(\beta_2, \gamma_2; \beta_4^{\min}, \delta_4 = 0, \gamma_4^{\min})$, i.e. for the effect on energy of the hexadecapole non-axial deformation of $^{262}$Sg described by the parameter $\delta_4$ [32].

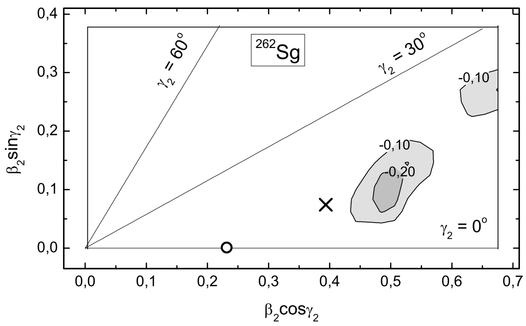


Fig. 5 – Same as in Fig. 4, but for the difference: $E(\beta_2, \gamma_2; \beta_4^{\min}, \delta_4^{\min}, \gamma_4^{\min}) - E(\beta_2, \gamma_2; \beta_4^{\min}, \delta_4^{\min}, \gamma_4 = 0)$, i.e. for the effect on energy of the hexadecapole non-axial deformation of $^{262}$Sg described by the parameter $\gamma_4$ [32].

In conclusion, one can say that the examples of nuclei considered in this paper show the importance of non-axial deformations of a heavy nucleus. These deformations may strongly modify its potential energy, in particular the height of its fission barrier, making the result obtained in the case of axial symmetry completely unrealistic, at least for some of the nuclei.

Writing this article to the special issue devoted to the 70th Anniversary of Professor Dorin Poenaru, I would like to express my great appreciation of his very important contribution to many fields of nuclear physics, in particular to the studies of exotic decay and of $\alpha$ decay.

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