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THE SEMIMICROSCOPIC ALGEBRAIC CLUSTER MODEL WITH BROKEN DYNAMICAL SYMMETRY

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Abstract. We investigate the *Semimicroscopic Algebraic Cluster Model* outside a dynamical symmetry. In order to reduce the number of parameters, the effective charge for the quadrupole moment is determined via a geometrical mapping. For simplicity, we restrict to two spherical clusters and consider the system ^{16}O+$\alpha \rightarrow {}^{20}$Ne. Advances and problems are discussed.

Key words: cluster model, algebraic model, spectroscopic factors.

1. INTRODUCTION

The cluster model of nuclei [1] is, in the harmonic oscillator limit, an equivalent description to the shell model. However, a fully microscopic treatment suffers from practical problems related to the antisymmetrization of the wave function, as can be appreciated by the early works of Refs. [2, 3]. More than ten years ago, an alternative approach was presented [4], known as the

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Semimicroscopic Algebraic Cluster Model (SACM), which is algebraic in structure and the Hamiltonian is a function of group generators. The model space, however, is microscopic and respects the Pauli exclusion principle. This is achieved by selecting only those cluster model states which have a shell model equivalent. The Wildermuth condition [1] is the simplest tool to insure it, however, it is sufficient only for closed shell clusters, while for systems with open shell clusters a more refined procedure is needed. In [5] the complete description of the model and some of its application is given. Further successful applications were published in Refs. [6, 7, 8, 9, 10, 11].

In spite of its great success, the model Hamiltonians were all restricted to dynamical symmetries. This can be understood, because the algebraic solution of a Hamiltonian within a dynamical symmetry is analytical. In [5] the extension to an arbitrary Hamiltonian was discussed but never applied.

Even when a general Hamiltonian is used, the number of parameters rises significantly. When electromagnetic transitions are also required, the number of parameters increases even more, thus rendering the theory as less attractive. One would like, at least, to find a procedure to get rid of parameters in the transition operators and only be left with those in the Hamiltonian. In case this is possible, the power of the model would be enhanced significantly.

Another important physical quantity to be studied in relation with clusterization is the spectroscopic factor. In this respect the situation of the SACM is the following. Though it is free from the Pauli-forbidden states, these are not eliminated as a result of the full antisymmetrization of the wavefunction. Therefore, the calculation of the spectroscopic factor is not straightforward. Nevertheless, it can be done in two ways. i) Due to the fact that the $U(3)$ basis states of this semimicroscopic approach are the same as those of the fully microscopic cluster models, their methods (or results) can be applied, when possible. This approach is, however, limited to relatively simple problems, just like the complete application of the fully microscopic models. On the other hand the SACM is a model which is meant to be useful also for more complicated cases. For this purpose the second method could be used. ii) A phenomenologic approximation to a spectroscopic factor operator can be done, in the same way, how it is realized for other physical quantities [12]. This method is justified by the fact that in the semimicroscopic model (in its simplest form) an operator (which is combined from the antisymmetrizer and the normalization factor of the cluster-wavefunction [13]) is substituted by the unit operator. In order to get the correct spectroscopic factor we can make a better approximation to it in a phenomenological way, i.e. by including parameters, which can be fitted. Both the microscopic and the phenomenologic method for the determination of the spectroscopic factor (or spectroscopic amplitude) is defined for the $U(3)$ basis states. When symmetry-breaking is taking place, and the wave function is obtained as a linear combination of

these basis states, then the spectroscopic factor (amplitude) is determined by the linear-combination coefficients and by the corresponding values for the basis states. In the present work we focus on the phenomenological method.

In Ref. [14] a parametrization of the spectroscopic factor operator was proposed for heavy nuclei. The application referred to the cluster radioactivity, reported for the first time in Ref. [15] and beforehand predicted in [16], which is one of Prof. Poenaru's important contributions. In cluster radioactivity, not only the emission of α particles is observed but also of clusters corresponding to heavier nuclei. This new phenomenon has important implications, like in the formation of super-heavy nuclei. In Ref. [17] the parametrization of the spectroscopic factor was extended further, and applied to nuclei in the second half of the p-shell and the first half of the sd-shell. The agreement with theoretical results of the fully microscopic models [2] is excellent. The hope is that, when applied to light and heavy nuclei, the SACM can contribute in predicting the formation of super heavy nuclei and the properties of cluster radioactivity.

For heavy nuclei the SACM has to be extended [18] and first applications, concerning the preferences of clusterization, were published in [19, 20, 21, 22]. In these systems the *effective* $SU(3)$ irreducible representation (irrep) [23, 24, 25] has to be determined, which can be considered as the average contribution of many $SU(3)$ shell model irreps.

In this contribution we will propose a procedure how to treat Hamiltonians which do not exhibit a dynamical symmetry. A geometrical mapping is used to estimate, and thus to fix, the effective charges. In order to keep it simple and as a first step, only a system with two spherical clusters (^{16}O+α $\rightarrow^{20}$Ne) is considered. This simplifies the structure of the Hamiltonian, the model space and helps to elucidate the main ideas of the general approach. Besides the spectrum, also some E2-transitions will be calculated and compared to experiment. The question of spectroscopic factors will be discussed and the problem how to extend it to the general description. The main idea of this contribution is to report on the first steps done in treating the SACM outside a dynamical symmetry and on the difficulties encountered.

The paper is structured as follows: In Section 2 the model is presented: the Hamiltonian, the transition and the spectroscopic factor operators is defined. In Section 3 the model will be applied to ^{16}O+α. In Section 4 conclusions are drawn.

2. THE MODEL

In the SACM the structure of a cluster with isospin zero, to which we will restrict for illustrative reasons, is described as in the shell model by an orbital

$SU(3)$ irreducible representation (irrep). A spherical nucleus is represented by the scalar (0,0). Therefore, the shell model $SU(3)$ of the two spherical clusters does not play an active role. The relative motion is described via a $SU_R(3)$ group and the irrep is $(n_\pi, 0)$, where n_π is the number of relative oscillation quanta. The group chain, classifying the particular problem, is given by [26]

$$\begin{array}{ccccc} U_R(3) & \supset SU_R(3) \supset & SO(3) \supset & SO(2) \\ n_\pi & (n_\pi, 0) & L & M\ . \end{array} \tag{1}$$

L is the angular momentum and M its projection. The Pauli exclusion principle is easily implemented through the Wildermuth condition [1]. For the system ^{16}O+α, this implies to raise the four nucleons from the α particle to the sd-shell. Therefore, the minimal number of relative oscillation quanta has to be $n_0 = 8$ and only values $n_\pi \geq 8$ are allowed.

A possible Hamiltonian, for systems of two spherical clusters, is given by

$$\begin{aligned} \boldsymbol{H} \ = \ & \hbar\omega \boldsymbol{n}_\pi + a_1 \boldsymbol{C}_2(SU_R(3)) + a_2 \boldsymbol{L}^2 + a_3 \boldsymbol{C}_3(SU_R(3)) \\ & + b_1 \left[(\boldsymbol{\pi}^\dagger \cdot \boldsymbol{\pi}^\dagger) - R^2 \boldsymbol{\sigma}^{\dagger 2} \right] \left[(\boldsymbol{\pi} \cdot \boldsymbol{\pi}) - R^2 \boldsymbol{\sigma}^2 \right], \end{aligned} \tag{2}$$

where the third-order Casimir operator $\boldsymbol{C}_3(SU_R(3))$ of the $SU_R(3)$ group gives higher order contributions, important for the relative positions of excited states. The first line corresponds to interactions in the dynamical symmetry of the group chain (1). A $\boldsymbol{K}^2$ operator is not considered because the states of ^{16}O+α in question [27] will all correspond to $K = 0$ bands. The second line would correspond, for $R^2 = 1$, to the dynamical $O(4)$ symmetry interaction. The inclusion of a further parameter R^2 is an essential difference, which allows to fix the position of the relative minimum in the SACM [29]. The $\boldsymbol{C}_2(SU_R(3))$ is the second-order Casimir operator of the $SU_R(3)$ group, with eigenvalue $n_\pi(n_\pi + 3)$. Its relation to the quadrupole and angular momentum operators is [30]

$$\boldsymbol{C}_2(SU(3)) = \frac{1}{4}\boldsymbol{Q}^2 + \frac{3}{4}\boldsymbol{L}^2. \tag{3}$$

The eigenvalue of the third-order Casimir operator is $n_\pi(2n_\pi + 3)(n_\pi + 3)$.

The mixing of the shells is done via the last interaction term in (2), which will be partly responsible for the increase in the B(E2) transition values. This mixing can also be increased by lowering the higher $n\hbar\omega$ states through enlarging the negative a_1 parameter. The role of the parameter R^2 is to have a handle on the possible mixing interaction, where larger values increase the mixing. In geometrical terms [31] the R^2 parameter shifts the minimum of the relative position.

The E2-transition operator is defined as

$$\boldsymbol{T}^{(E2)}_\mu = \left(\frac{5}{16\pi} \right)^{\frac{1}{2}} e^{(2)}_R \boldsymbol{Q}^{(2)}_{R,\mu}, \tag{4}$$

again assuming that the two clusters are spherical, i.e. they do not contribute to the B(E2)-transition values. The $\boldsymbol{Q}^{(2)}_{R,\mu}$ is the mass quadrupole operator, thus, the $e^{(2)}_R$ must include a factor Z/A. The effective charge $e^{(2)}_R$ carries also the units, because $Q^{(2)}_{R,\mu}$ is defined without them.

Usually, the effective charge $e^{(2)}_R$ is treated as a parameter, which implies an addition to those of the Hamiltonian. This might not sound so bad for the case we study. However, one has to bear in mind that we want to extend the consideration to, in general, two deformed clusters. In that case, there are three effective charges for the E2-transition operator, one each for cluster No. 1 and No. 2 and one for the relative motion. There will also be further transition operators, for the description of E1-, M1- and E3-transitions. Thus, in total we would have 12 new parameters. It should be clear that this situation has to be avoided. Somehow we have to find a way to estimate these effective charges and in this way reduce the number of free parameters, which results in an increase of the predictive power of the model.

To estimate the effective charge $e^{(2)}_R$ we start from the geometrical mapping of the quadrupole transition operator within the SACM [29]. The result is

$$\langle e^{(2)}_R \boldsymbol{Q}^{(2)}_{R,\mu} \rangle \approx \sqrt{6} e^{(2)}_R \, [r \times r]^{[2]}_\mu \left(\frac{m\omega}{2\hbar} \right), \tag{5}$$

where [30]

$$\boldsymbol{Q}^{(2)}_{R,\mu} = \sqrt{6} \left[\pi^\dagger \times \tilde{\pi} \right]^{[2]}_\mu . \tag{6}$$

For $\mu = 0$, the expression in Eq. (5) has to be equal to $\frac{Z}{A}(2z^2 - x^2 - y^2)$, which is the zero component of the charge quadrupole operator. This results in the effective charge

$$e^{(2)}_R = 2\frac{Z}{A} \left(\frac{\hbar}{m\omega} \right), \tag{7}$$

where m is the mass of a nucleon.

To complete the theory, we give the structure of the algebraic spectroscopic factor, as it was proposed in Ref. [17]. The agreement with microscopically calculated spectroscopic factors [2] is nearly perfect, for systems with one cluster being an α particle. For two spherical clusters, the spectroscopic factor within a dynamical symmetry is given by

$$S = \exp\left(A + Bn_\pi + CC_R(n_\pi, 0) + D\Delta n_\pi\right). \tag{8}$$

In the original definition of Ref. [17] additional operators and $SU(3)$ isoscalar factors appear, which are only important when one or two clusters are deformed. They disappear when both clusters are spherical. The parameters were fitted to the results of a full microscopic treatment [2] and the result is $A = 3.6163$, $B = -0.361$, $C = -0.0086654$ and $D = 1.9090$. The first term

(A) is just a normalization, the second term (B) is related to the overlap of the two clusters [17]. The third term (C) gives modifications to the overlap due to the $SU(3)$ structure of the states. The fourth term increases the overlap with raising $\hbar\omega$ excitations. Of course, the parameter may change when no dynamical symmetry is considered by adjusting them to experimental data.

Such an exponential ansatz was also used, with success [14], for the description of cluster radioactivity [15, 16], to which Prof. Poenaru did important contributions [32]. The parametrization given in Ref. [14], however, did not include the $SU(3)$ isoscalar factors, though both clusters were deformed. The inclusion of these factors will surely improve the description of cluster radioactivity.

3. APPLICATION TO ^{16}O+$\alpha \rightarrow {}^{20}$Ne

We would like to adjust the spectrum of ^{20}Ne to our model Hamiltonian, assuming that this nucleus can be described by two spherical clusters. We have to be careful which states to choose, because only those can be included in a fit which show a clear α-cluster structure in experiment. For the states, listed in Table 1, we used the compilation of Refs. [27, 28]. In Table 2 some E2-transition values are given as far as they were available by experiment [27]. The E2-transition values are in Weisskopf units.

In order to determine the parameters of the Hamiltonian, we use a fitting routine, written by us. As the minimization procedure the MINUIT routine of the CERN library was applied [34]. The maximal number of π-bosons for the system considered is $N = 15$. The quality of the fit with respect to the energy is determined via the function

$$\Phi^2 = \sum_i \frac{(E_{\rm exp} - E_{\rm theo})^2}{E_{\rm exp}^2} {\rm w}(i), \tag{9}$$

where i refers to the running index of experimental data and w(i) is a value to increase or lower the weight for particular states. Except for the second fit presented below, where for the $3_1^- \rightarrow 1_1^-$ B(E2)-transition the weight 4 was chosen, all weights are 1. For the transition values a similar function is defined. The question of which fit is best is not only determined by the Φ^2 value (including the B(E2)-contributions) but also by judging with the eye the final result. The reason of it lies in possible changes of the weight values from fit to fit and on the personal judgment about the details of the numerical results. The corresponding Φ^2 values are, respectively, 0.50 and 0.25. The second seems to be better, but as explained above it depends on the personal judgment. For each fit, the Φ^2 obtained corresponds to a global minimum.

Table 1

List of α-cluster states. J is the spin, k the order of the state, where only α-cluster bands taken into account, and the energy is given in units of MeV. "Exp" corresponds to experiment and "Theo-1" and "Theo-2" to two different theoretical results. States of different bands are separated by a horizontal line. The experimental data are taken from [27]. In some cases two possible assignments to experimental states are possible. In these cases, we give the alternative experimental value within a bracket. The second $K^\pi = 0^-$ α-cluster band is actually assigned as a 1^- band in experiment, contrary to the theoretical interpretation.

K^π	J_k^π	Exp. [MeV]	Theo-1 [MeV]	Theo-2 [MeV]
0^+	0_1^+	0.	0.	0.
0^+	2_1^+	1.634	1.177	1.192
0^+	4_1^+	4.248	3.924	3.975
0^+	6_1^+	8.778	8.242	8.353
0^+	8_1^+	11.951	14.130	14.331
0^-	1_1^-	5.788	5.097	5.672
0^-	3_1^-	7.156	7.060	7.635
0^-	5_1^-	10.262	10.592	11.134
0^-	7_1^-	13.692	15.694	16.122
0^-	9_1^-	17.430	22.367	22.619
0^+	0_2^+	8.450	8.288	8.998
0^+	2_2^+	8.800	9.466	10.162
0^+	4_2^+	10.790	12.212	12.882
0^+	6_2^+	12.585	16.528	17.162
0^+	8_2^+	17.295	22.414	23.012
0^-	1_2^-	8.854	8.447	5.931
0^-	3_2^-	10.406	10.409	7.858
0^-	5_2^-	12.713	13.940	11.364
0^-	7_2^-	16.581	19.041	16.503
0^-	9_2^-	20.686	22.367	23.275
		(21.06)		
0^+	0_3^+	10.970	10.626	10.805
0^+	2_3^+	12.324	11.803	11.925
0^+	4_3^+	12.943	14.550	14.538
0^+	6_3^+	15.72	18.868	18.642
		(15.97)		
0^+	8_3^+	18.617	24.755	24.233
		(18.957)		

Changing the weights also changes the landscape of the function Φ^2 and, thus, the position of the global minimum changes.

One result corresponds to a small shell mixing and the other one to a bigger mixing. The resulting parameters for the first theoretical result (Theo-1) are $a_1 = -0.35519\,\mathrm{MeV}$, $a_2 = 0.19570\,\mathrm{MeV}$, $a_3 = -0.0019495$, $b_1 = -0.0019285\,\mathrm{MeV}$ and $R^2 = 7.41087$. For the second theoretical result

Table 2

Experimental and theoretical calculated E2-transition values of the first positive and first negative parity band, in Weisskopf units (W.U.) [33]

$J_k^\pi(i) \to J_k^\pi(f)$	B(E2)-Exp. [W.U.]	B(E2)-Theo-1 [W.U.] [W.U.]	B(E2)-Theo-2 [W.U.]
$2_1^+ \to 0_1^+$	20.3±1.	21.53	22.46
$4_1^+ \to 2_1^+$	22.0±2.	27.27	28.44
$6_1^+ \to 4_1^+$	-	23.10	24.13
$8_1^+ \to 6_1^+$	-	13.70	14.34
$3_1^- \to 1_1^-$	50.0±8.	32.73	49.69
$5_1^- \to 3_1^-$	-	33.38	61.21
$7_1^- \to 5_1^-$	-	26.88	76.18
$9_1^- \to 7_1^-$	-	15.55	78.80

they are $a_1 = -0.33821$, $a_2 = 0.175429$, $a_3 = -0.002876$, $b_1 = -0.057425$ and $R^2 = 3.3323$. In both cases, the $\hbar\omega$=13.19 MeV value is determined via $45/A^{\frac{1}{3}} - 45/A^{\frac{2}{3}}$ [35] and as the maximal number of π-bosons we choose $N = 15$.

The results of the fits are summarized in Fig. 1. As for the subindex k, it refers to the order within the set of α-cluster states, i.e., other lower lying states are not included in the compilation. In the first fit (middle panel), the energies are well reproduced as are the B(E2)-transition values of the ground state band. Note, that within this band the theoretical B(E2)-values start to decrease with larger spin. The experimental B(E2)-value in the first negative-parity band, however, is not well reproduced. The b_1 turns out to be small and, thus, mixing between shells are small. This is reflected in the average value of n_π, which is practically 8 for the ground state and 9 for the first exited 1^- state. For completeness, we give the average n_π values for the 0_2^+, 1_2^- and 0_3^+ states, which are about 10, 15 and 14 respectively. The last too are at the limit of the model space.

In the second fit we obtained a larger mixing of the shells, which is reflected in a larger value of the parameter b_1. The average value of n_π turned out to be about 8.2 for the ground state. Thus, the B(E2)-value for $2_1^+ \to 0_1^+$ is slightly larger. In contrast to this, the average value for n_π has increased from 9 to 11.1 for the first exited 1^- state. This increased mixing of shells results in the larger B(E2)-value for the $3_1^- \to 1_1^-$ transition. Note, that in the theoretical calculation the B(E2)-values continue to increase within the negative parity band. With respect to the energies, the second exited 1^- state turns out to be too low, but is in good agreement for higher spins. In a more refined fit, with more parameters, this could probably be resolved. The main message is that a larger mixing, due to the $O(4)$ part, is important in order to reproduce the larger B(E2)-values for higher bands. For completeness, we

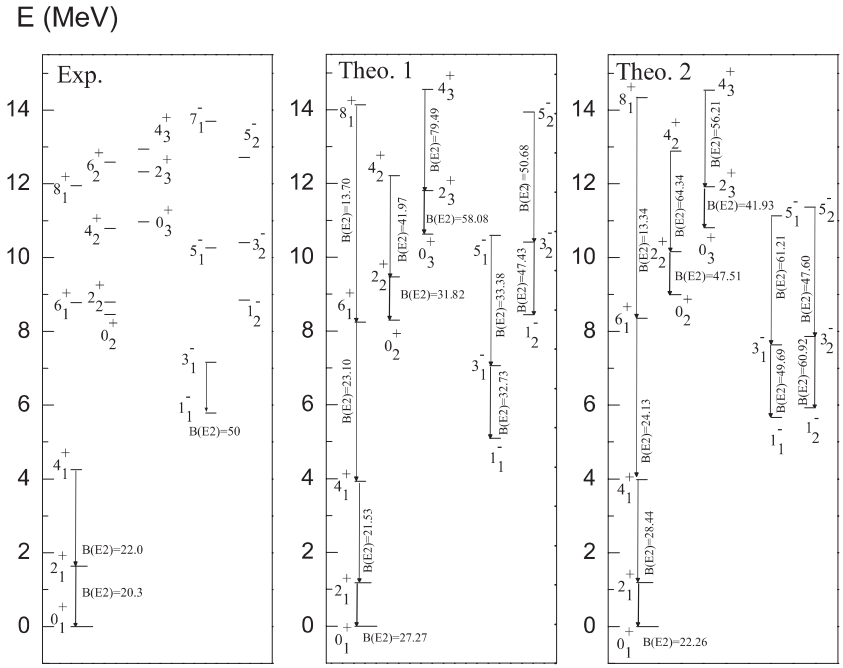

Fig. 1 – Spectrum of the α-cluster states below 14 MeV of ^{20}Ne within the SACM. The transition values, in Weisskopf units (W.U.), are indicated by arrows. The left panel shows the experimental results, while the middle and right panel gives the two theoretical calculations, denotes as "Theo-1" and "Theo-2". The subindex refers to the order of α-cluster states, not including other states which are lower in energy. For details, please consult the text.

give the average n_π values for the 0_2^+, 1_2^- and 0_3^+ states, which are about 12.5, 15 and 13.2 respectively, indicating a large mixing.

In principle, we can determine also the spectroscopic factors of the states as obtained in the fits reported. In the fit called "Theo-1" the mixing is very small. As a consequence, the average values of n_π, $C_R(n_\pi, 0)$ and Δn_π are nearly the same as in the dynamical symmetry limit. Therefore, the values of the spectroscopic factors are very close to those reported in [17]. This is different for the fit called "Theo-2". In the ground-state band, the values are still near the ones without a shell mixing and, thus, the spectroscopic factors will also be very near to those obtained in the dynamical symmetry limit, using the same parameters. This is different for the exited 1^- band. The difference in the n_π value, compared without mixing, ranges in this band from 3 to 4. Because of the large parameter value in front of $\Delta \boldsymbol{n}_\pi$ in Eq. (8) the spectroscopic factor increases. No experimental data for this band are known to the authors and, thus, a new fit is not possible.

4. CONCLUSIONS

The *Semimicroscopic Algebraic Cluster Model* (SACM) outside a dynamical symmetry chain was investigated and applied to a system of two spherical clusters. The geometric mapping of the model was used to determine the effective charge of the E2-transition operator. The way of determining the spectroscopic factor within a broken symmetry was also given. The application showed that the SACM can be extended with success outside a dynamical symmetry. The estimation of the effective charges of the transition operators increases the predictive power of the SACM, by reducing sensibly the number of parameters.

The result of our approach encourages us to continue to investigate systems with at least one cluster deformed, like ^{20}Ne+$\alpha \rightarrow {}^{24}$Mg. In case the model is successful, the extension to heavy nuclei and the determination of properties of cluster radioactivity will be possible also for Hamiltonians outside a dynamical symmetry.

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