

MATHEMATICAL AND GENERAL PHYSICS

TILTED BIANCHI TYPE V BULK VISCOUS COSMOLOGICAL
MODELS IN GENERAL RELATIVITY

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Abstract. Conformally flat tilted Bianchi type V cosmological models in presence of a bulk viscous fluid and heat flow are investigated. The coefficient of bulk viscosity is assumed to be a power function of mass density. The cosmological constant is found to be a decreasing function of time, which is supported by results from recent type Ia supernovae observations. Some physical and geometric aspects of the models are also discussed.

Key words: cosmology, tilted Bianchi type V universe, variable cosmological constant.

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1. INTRODUCTION

The Bianchi cosmologies play an important role in theoretical cosmology and have been much studied since the 1960s. A Bianchi cosmology represents a spatially homogeneous universe, since by definition the spacetime admits a three-parameter group of isometries whose orbits are spacelike hyper-surfaces. These models can be used to analyze aspects of the physical Universe which pertain to or which may be affected by anisotropy in the rate of expansion, for example, the cosmic microwave background radiation, nucleosynthesis in the early universe, and the question of the isotropization of the universe itself [1]. Spatially homogeneous cosmologies also play an important role in attempts to understand the structure and properties of the space of all cosmological solutions of Einstein field equations. A spatially homogeneous cosmology is said to be [4] if the fluid velocity vector is not orthogonal to the group orbits, otherwise the model is said to be *non-tilted* [2]. A tilted model is spatially homogeneous relative to observers whose world lines are orthogonal relative to the group orbits, but is spatially inhomogeneous relative to

observers comoving with the fluid. In a tilted Bianchi cosmology the tilt can become extreme in a finite time as measured along the fluid congruence, with the result that the group orbits become time-like. This means that the models are no longer spatially homogeneous [3].

The general dynamics of tilted models have been studied by King and Ellis [2], and Ellis and King [4]. Ellis and Baldwin [5] have shown that we are likely to be living in a tilted universe and they have indicated how we may detect it. Beesham [6] derived tilted Bianchi type V cosmological models in the scale-covariant theory. A tilted cold dark matter cosmological scenario has been discussed by Cen et al. [7]. Several researchers (Matravers et al. [8], Ftaclas and Cohen [9], Hewitt and Wainwright [10], Lidsey [11], Bali and Sharma [12], Hewitt et al. [13], Horwood et al. [14], Bogoyavlenskii and Novikov [15], Barrow and Sonoda [16], Barrow and Hervik [17], Apostolopoulos [18], Pradhan and Rai [19] and Hervik [20] have studied various aspects of tilted cosmological models.

A considerable interest has been shown to the study of physical properties of spacetimes which are conformal to certain well known gravitational fields. The general theory of relativity is believed by a number of unknown functions – the ten components of g^{ij} . Hence there is a little hope of finding physically interesting results without making reduction in their number. In conformally flat spacetime the number of unknown functions is reduced to one. The conformally flat metrics are of particular interest in view of their degeneracy in the context of Petrov classification. A number of conformally flat physically significant spacetimes are known like Schwarzschild interior solution and Lemaitre cosmological universe.

General Relativity describes the state in which radiation concentrates around a star. Klein [21] worked on it and obtained an approximate solution to Einsteinian field equations in spherical symmetry for a distribution of diffused radiation. Many other researchers (Singh and Sattar [22], Roy and Bali [23]) have worked on this topic and obtained exact static spherically and cylindrically symmetric solutions of Einstein's field equations with exception as well. Roy and Singh [24] have obtained a non-static plane symmetric spacetime filled with disordered radiation. Teixeira, Wolk and Som [25] investigated a model filled with source free disordered distribution of electromagnetic radiation in Einstein's general relativity. The cosmological models with heat flow have been also studied by Coley and Tupper [26], Roy and Banerjee [27]. Recently Bali and Meena [28] have investigated two tilted cosmological models filled with disordered radiation of perfect fluid and heat flow.

Most cosmological models assume that the matter in the universe can be described by 'dust' (a pressure-less distribution) or at best a perfect fluid. Nevertheless, there is good reason to believe that - at least at the early stages of the universe - viscous effects do play a role (Israel and Vardalas [29], Klimek [30], Weinberge [31]). For example, the existence of the bulk viscosity is equivalent to slow process of restoring equilibrium states (Landau and Lifshitz [32]). The observed physical phenomena such as the large entropy per baryon and remarkable

degree of isotropy of the cosmic microwave background radiation suggest analysis of dissipative effects in cosmology. Bulk viscosity is associated with the GUT phase transition and string creation. Thus, we should consider the presence of a material distribution other than a perfect fluid to have realistic cosmological models (see Gron [33]) for a review on cosmological models with bulk viscosity). The model studied by Murphy [34] possessed an interesting feature in that the big bang type of singularity of infinite spacetime curvature does not occur to be a finite past. However, the relationship assumed by Murphy between the viscosity coefficient and the matter density is not acceptable at large density. The effect of bulk viscosity on the cosmological evolution has been investigated by a number of authors in the framework of general theory of relativity (Pavon [35], Padmanabhan and Chitre [36], Johri and Sudarshan [37], Maartens [38], Zimdahl [39], Santos *et al.* [40], Pradhan, Sarayakar and Beesham [41], Kalyani and Singh [42], Singh, Beesham and Mbokazi [43], Pradhan *et al.* [44]). This motivates to study cosmological bulk viscous fluid model.

Models with a dynamic cosmological term $\Lambda(t)$ are becoming popular as they solve the cosmological constant problem in a natural way. There is significant observational evidence for the detection of Einstein's cosmological constant, Λ or a component of material content of the universe that varies slowly with time and space and so acts like Λ . Recent cosmological observations by High -z Supernova Team and Supernova Cosmological Project (Garnavich *et al.* [45], Perlmutter *et al.* [46], Riess *et al.* [47], Schmidt *et al.* [48]) suggest the existence of a positive cosmological constant Λ with magnitude $\Lambda(G\hbar/c^3) \approx 10^{-123}$. These observations on magnitude and red-shift of type Ia supernova suggest that our universe may be accelerating with a large function of the cosmological density in the form of the cosmological Λ -term. Earlier researchers on this topic, are contained in Zeldovich [49], Weinberg [50], Dolgov [51], Bertolami [52], Ratra and Peebles [53], Carroll, Press and Turner [54]. Some of the recent discussions on the cosmological constant "problem" and consequence on cosmology with a time-varying cosmological constant have been discussed by Dolgov [55], Tsagas and Maartens [56], Sahni and Starobinsky [57], Peebles [58], Padmanabhan [59], Vishwakarma [60], and Pradhan *et al.* [61]. This motivates us to study the cosmological models in which Λ varies with time.

Recently Bali and Meena [62] have investigated two conformally flat tilted Bianchi type V cosmological models filled with a perfect fluid and heat conduction. Conformally flat tilted Bianchi type V cosmological models in presence of a bulk viscous fluid and heat flow are investigated by Pradhan and Rai [63]. In this paper, we propose to find tilted Bianchi type V cosmological models in presence of a bulk viscous fluid and heat flow with variable cosmological term Λ and we will generalize the solutions of Refs. [62, 63]. This paper is organized as

follows. The metric and field equations are presented in Section 2. In Section 3, we deal with the solutions of the field equations in presence of bulk viscous fluid. In Section 4, we give the concluding remarks.

2. THE METRIC AND FIELD EQUATIONS

We consider the Bianchi type V metric in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{2x} (dy^2 + dz^2), \quad (1)$$

where A, B are function of t only.

The Einstein's field equations (in gravitational units $c = 1, G = 1$) read as

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi T_i^j, \quad (2)$$

where R_i^j is the Ricci tensor; $R = g^{ij} R_{ij}$ is the Ricci scalar; Λ is the variable cosmological constant and T_i^j is the stress energy-tensor in the presence of bulk stress given by

$$T_i^j = (\rho + \bar{p}) v_i v^j + \bar{p} g_i^j + q_i v^j + v_i q^j, \quad (3)$$

and

$$\bar{p} = p - \zeta v^i_{;i} \quad (4)$$

Here ρ, p, \bar{p} and ζ are the energy density, isotropic pressure, effective pressure and bulk viscous coefficient respectively and v_i is the flow vector satisfying the relations

$$g_{ij} v^i v^j = -1, \quad (5)$$

$$q_i q^i > 0, \quad (6)$$

$$q_i v^i = 0, \quad (7)$$

where q_i is the heat conduction vector orthogonal to v_i . The fluid flow vector has the components $\left(\frac{\sinh \lambda}{A}, 0, 0, \cosh \lambda \right)$ satisfying Eq. (5) and λ is the tilt angle. The Einstein's field equations (2) for the line element (1) has been set up as

$$-8\pi[(\rho + \bar{p})\sinh^2 \lambda + \bar{p} - 2Aq_1 \sinh \lambda] = \frac{2B_{44}}{B} + \left(\frac{B_4}{B}\right)^2 - \frac{1}{A^2} - \Lambda, \quad (8)$$

$$-8\pi\bar{p} = \frac{A_{44}}{A} \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{1}{A^2} - \Lambda, \quad (9)$$

$$-8\pi[-(\rho + \bar{p})\cosh^2 \lambda + \bar{p} - 2Aq_1 \sinh \lambda] = \frac{2A_4 B_4}{AB} + \left(\frac{B_4}{B}\right)^2 - \frac{3}{A^2} - \Lambda, \quad (10)$$

$$-8\pi[(\rho + \bar{p})A \sinh \lambda \cosh \lambda + A^2 q_1 (\cosh \lambda + \sinh \lambda \tanh \lambda)] = \frac{2A_4}{A} - \frac{2B_4}{B} - \Lambda, \quad (11)$$

where the suffix 4 at the symbols A, B denotes ordinary differentiation with respect to t .

3. SOLUTION OF THE FIELD EQUATIONS

Equations (8) – (11) are four independent equations in eight unknowns $A, B, \rho, p, \zeta, q, \Lambda$ and λ . For the complete determinacy of the system, we need four extra conditions.

First we assume that the spacetime is conformally flat which leads to

$$C_{2323} = \frac{1}{3} \left[\frac{A_{44}}{A} - \frac{B_{44}}{B} - \frac{A_4 B_4}{AB} + \frac{B_4^2}{B^2} \right] = 0 \quad (12)$$

and secondly, we assume

$$A = B^n, \quad (13)$$

where n is any real number. Eqs. (12) and (13) lead to

$$\frac{B_{44}}{B} + (n-1) \frac{B_4^2}{B^2} = 0. \quad (14)$$

From Equations (8), (10) and (13), we have

$$-4\pi[(\rho + \bar{p})\cosh 2\lambda + 4B^n q_1 \sinh \lambda] = \frac{B_{44}}{B} - n \frac{B_4^2}{B^2} + \frac{1}{B^{2n}}, \quad (15)$$

and

$$-4\pi(\rho + \bar{p}) = \frac{B_{44}}{B} - (n+1) \frac{B_4^2}{B^2} - \frac{2}{B^{2n}} - \Lambda. \quad (16)$$

Equations (9), (12) and (16) lead to

$$-n(2n-3)\frac{B_4^2}{B^2} - (2n-1)\frac{B_{44}}{B} - \frac{1}{B^{2n}} = 4\pi(\rho + \bar{p}). \quad (17)$$

Equations (11) and (13) lead to

$$-16\pi q_1 B^n \sinh \lambda = \frac{[2(n-1)B_4 - \Lambda B] \tanh 2\lambda}{B^{n+1}} + 4\pi(\rho + \bar{p}) \sinh 2\lambda \tanh 2\lambda. \quad (18)$$

From Eqs. (15) and (18), we obtain

$$\frac{B_{44}}{B} - \frac{nB_4^2}{B^2} + \frac{1}{B^{2n}} = -\frac{4\pi(\rho + \bar{p})}{\cosh 2\lambda} + \frac{[2(n-1)B_4 - \Lambda B] \tanh 2\lambda}{B^{n+1}}. \quad (19)$$

Equations (17) and (19) lead to

$$\begin{aligned} \frac{B_{44}}{B} - \frac{nB_4^2}{B^2} + \frac{1}{B^{2n}} &= \frac{1}{\cosh 2\lambda} \left[n(2n-3)\frac{B_4^2}{B^2} + (2n-1)\frac{B_{44}}{B} + \frac{1}{B^{2n}} \right] + \\ &+ \frac{[2(n-1)B_4 - \Lambda B] \tanh 2\lambda}{B^{n+1}}. \end{aligned} \quad (20)$$

Equation (14) can be rewritten as

$$\frac{B_{44}}{B_4} + \frac{(n-1)B_4}{B} = 0, \quad (21)$$

which on integration leads to

$$B = n^n (\alpha t + \beta)^{\frac{1}{n}}, \quad (22)$$

where α, β are constants of integration. Hence we obtain

$$A^2 = n^2 (\alpha t + \beta)^2, \quad (23)$$

$$B^2 = n^n (\alpha t + \beta)^{\frac{2}{n}}. \quad (24)$$

Hence the geometry of the spacetime (1) reduces to the form

$$ds^2 = -dt^2 + n^2 (\alpha t + \beta)^2 dx^2 + [n(\alpha t + \beta)]^{\frac{2}{n}} e^{2x} (dy^2 + dz^2). \quad (25)$$

After the suitable transformation of coordinates, the metric (25) takes the form

$$ds^2 = -\frac{dT^2}{2} + n^2 T^2 dX^2 + n^{\frac{2}{n}} T^{\frac{2}{n}} e^{2X} (dY^2 + dZ^2). \quad (26)$$

The effective pressure and density of the model (26) are given by

$$8\pi\bar{p} = 8\pi(p - \xi\theta) = -\frac{(\alpha^2 - 1)}{n^2T^2} - \Lambda, \quad (27)$$

$$8\pi\rho = \frac{3(\alpha^2 - 1)}{n^2T^2} + \Lambda, \quad (28)$$

where θ is the scalar of expansion calculated for the flow vector v^i and given is as

$$\theta = \frac{2K + (n+2)k\alpha}{nT}. \quad (29)$$

The tilt angle λ is given by

$$\cosh^2 \lambda = k^2, \quad (30)$$

$$\sinh^2 \lambda = K^2, \quad (31)$$

where k and K are constants given by

$$k^2 = \frac{(n\alpha - 1)^2}{(\alpha^2 - 1)(2n^2\alpha^2 - n\alpha^2 - 1) + 2\alpha^2(n-1)\sqrt{(n^2 - n)(1 - \alpha^2)}}, \quad (32)$$

$$K^2 = \frac{(n-1)(2n\alpha^2 - n\alpha^4 - \alpha^2) - 2\alpha^2(n-1)\sqrt{(n^2 - n)(1 - \alpha^2)}}{(\alpha^2 - 1)(2n^2\alpha^2 - n\alpha^2 - 1) + 2\alpha^2(n-1)\sqrt{(n^2 - n)(1 - \alpha^2)}}. \quad (33)$$

For the specification of ξ , we assume that the fluid obeys an equation of state of the form

$$p = \gamma\rho, \quad (34)$$

where $\gamma(0 \leq \gamma \leq 1)$ is a constant. Thus, given $\theta(t)$ we can solve the system for the physical quantities. Therefore to apply the third condition, let us assume the following *ad hoc* law [38, 39]

$$\xi(t) = \xi_0\rho^m, \quad (35)$$

where ξ_0 and m are real constants. If $m = 1$, Eq. (34) may correspond to a radiative fluid [50], whereas $m = \frac{2}{3}$ may correspond to a string-dominated universe.

However, more realistic models [40] are based on lying the regime $0 \leq m \leq \frac{1}{2}$.

3.1. MODEL I: SOLUTION FOR $\xi = \xi_0$

When $m = 0$, Eq. (35) reduces to $\xi = \xi_0 = \text{constant}$ and hence Eq. (27), with the use of (34) and (28), leads to

$$4\pi(1+\gamma)\rho = \frac{4\pi\xi_0[2K+(n+2)k\alpha]}{nT} + \frac{(\alpha^2-1)}{n^2T^2}. \quad (36)$$

Eliminating $\rho(t)$ between (28) and (36), we get

$$(1+\gamma)\Lambda = \frac{8\pi\xi_0[2K+(n+2)k\alpha]}{nT} + \frac{(\alpha^2-1)(1+3\gamma)}{n^2T^2}. \quad (37)$$

3.2. MODEL II: SOLUTION FOR $\xi = \xi_0\rho$

When $m = 1$, Eq. (35) reduces to $\xi = \xi_0\rho$ and hence Eq. (27), with the use of (34) and (28), leads to

$$4\pi\rho = \frac{(\alpha^2-1)}{nT[n(1+\gamma)T] - \xi_0\{2K+(n+2)k\alpha\}}. \quad (38)$$

Eliminating $\rho(t)$ between (28) and (38), we get

$$\Lambda = \frac{(\alpha^2-1)}{nT} \left[\frac{2}{[n(1+\gamma)T] - \xi_0\{2K+(n+2)k\alpha\}} - \frac{3}{nT} \right]. \quad (39)$$

3.3. SOME PHYSICAL AND GEOMETRIC PROPERTIES OF THE MODELS

From Eqs. (37) and (39), we observe that the cosmological constant in both the models is a decreasing function of time and it approaches a small and positive value for large T (i.e. the present epoch) which is supported by the results from recent type Ia supernovae observations (Garnavich et al. [45], Perlmutter et al. [46], Riess et al. [47], Schmidt et al. [48]).

The weak and strong energy conditions, we have, in Model I

$$\rho + p = \frac{(\alpha^2-1)}{4\pi n^2T^2} + \frac{\xi_0\{2K+(n+2)k\alpha\}}{nT}, \quad (40)$$

$$\rho - p = \frac{(\alpha^2 - 1)(1 - \gamma)}{4\pi(1 + \gamma)n^2T^2} + \frac{(1 - \gamma)\xi_0\{2K + (n + 2)k\alpha\}}{(1 + \gamma)nT}, \quad (41)$$

$$\rho + 3p = \frac{(\alpha^2 - 1)(1 + 3\gamma)}{4\pi(1 + \gamma)n^2T^2} + \frac{(1 + 3\gamma)\xi_0\{2K + (n + 2)k\alpha\}}{(1 + \gamma)nT}, \quad (42)$$

$$\rho - 3p = \frac{(\alpha^2 - 1)(1 - 3\gamma)}{4\pi(1 + \gamma)n^2T^2} + \frac{(1 - 3\gamma)\xi_0\{2K + (n + 2)k\alpha\}}{(1 + \gamma)nT}. \quad (43)$$

In Model II, we have

$$\rho + p = \frac{(1 + \lambda)(\alpha^2 - 1)}{4\pi nT[n(1 + \lambda)T - \xi_0\{2K + (n + 2)k\alpha\}]}, \quad (44)$$

$$\rho - p = \frac{(1 - \lambda)(\alpha^2 - 1)}{4\pi nT[n(1 + \lambda)T - \xi_0\{2K + (n + 2)k\alpha\}]}, \quad (45)$$

$$\rho + 3p = \frac{(1 + 3\lambda)(\alpha^2 - 1)}{4\pi nT[n(1 + \lambda)T - \xi_0\{2K + (n + 2)k\alpha\}]}, \quad (46)$$

$$\rho - 3p = \frac{(1 - 3\lambda)(\alpha^2 - 1)}{4\pi nT[n(1 + \lambda)T - \xi_0\{2K + (n + 2)k\alpha\}]}, \quad (47)$$

The reality conditions $\rho \geq 0$, $p \geq 0$ and $\rho - 3p \geq 0$ impose further restrictions on both of these models.

The flow vector v^i and heat conduction vector q^i for the models (26) are obtained as

$$v^1 = \frac{K}{nT}, \quad (48)$$

$$v^4 = k, \quad (49)$$

$$q^1 = \frac{k\{(\alpha^2 - 1)kK + (n - 1)\alpha\}}{4\pi n^3T^3(k^2 + K^2)}, \quad (50)$$

$$q^4 = \frac{K \{ (\alpha^2 - 1)kK + (n-1)\alpha \}}{4\pi n^3 T^3 (k^2 + K^2)}. \quad (51)$$

The rate of expansion H_i in the direction of X, Y, Z -axes are given by

$$H_1 = \frac{\alpha}{T}, \quad (52)$$

$$H_2 = H_3 = \frac{\alpha}{nT}. \quad (53)$$

The non-vanishing components of shear tensor (σ_{ij}) and rotation tensor (ω_{ij}) are obtained as

$$\sigma_{11} = \frac{2}{3}nk^2T[(n-1)k\alpha - K], \quad (54)$$

$$\sigma_{22} = \sigma_{33} = \frac{(nT)^{\frac{2}{n}-1}e^{2X}}{3}[(1-n)k\alpha - 2K], \quad (55)$$

$$\sigma_{44} = \frac{2K^2}{3nT}[(n-1)k\alpha - K], \quad (56)$$

$$\sigma_{14} = \frac{K}{3}[2(n-1)k^2 + 2kK - 3n], \quad (57)$$

$$\omega_{14} = n\alpha K. \quad (58)$$

The models, in general, represent shearing and rotating universes. The models start expanding with a big bang at $T = 0$ and the expansion in the models decreases as time increases and the expansion in the models stops at $T = 8$ and $\alpha = -\frac{2K}{(n+2)k}$. Both density and pressure in the models become zero at $T = 8$. For

$a = 1, n = 1$, we observe that heat conduction vector $q^1 = q^4 = 0$. When $T \rightarrow \infty$, $v^1 = 0, v^4 = \text{constant}, q^1 = q^4 = 0$. Since $\lim_{T \rightarrow \infty} \frac{\sigma}{\theta} \neq 0$, the models do not approach isotropy for large values of T . There is a real physical singularity in the model at $T = 0$.

In case $\Lambda = 0$ and $\xi_0 = 0$, metric (26) with expressions p, ρ, θ and s for this model are same as that of solution (2.27) of Bali and Meena [62]. In case $\Lambda = 0$, metric (26) with expressions $\bar{p}, \bar{\rho}, \bar{\theta}$ and $\bar{\sigma}$ for this model are same as that of solution (56) of Pradhan and Rai [63].

4. CONCLUSION

In this paper we have described a new class of conformally flat tilted Bianchi type V magnetized cosmological models with a bulk viscous fluid as the source of matter. Generally, the models are expanding, shearing and rotating. In all these models, we observe that they do not approach isotropy for large values of time T in the presence of magnetic field. It is seen that the solutions obtained by Bali and Meena [62] and Pradhan and Rai [63] are particular cases of our solutions.

The coefficient of bulk viscosity is assumed to be a power function of mass density. The effect of bulk viscosity is to introduce a change in the perfect fluid model. We also observe here that the conclusion of Murphy [34] about the absence of a big bang type of singularity in the finite past in models with bulk viscous fluid is, in general, not true.

The cosmological constant in all models in Sections 3.1 and 3.2, are decreasing function of time and they all approach a small positive value at late time. These results are supported by the results from recent supernovae Ia observations recently obtained by High - Z Supernova Team and Supernova Cosmological Project [45–48].

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