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CAN BIANCHI TYPE-II COSMOLOGICAL MODELS WITH A DECAY LAW FOR Λ TERM BE COMPATIBLE WITH RECENT OBSERVATIONS?

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Abstract. Observations of high-redshift type Ia supernovae have placed stringent constraint on the cosmological constant Λ. In this report the behavior of the scale factors and the cosmological tests pertaining to proper distance, luminosity distance redshift, angular diameter distance redshift, and look-back time redshift are analyzed for different possible scenarios where the cosmological term of the form $\Lambda = \beta\frac{\ddot{a}}{a}$, $a(t)$ is the scale factor of the universe and β is a constant. It has been shown that such models are found to be compatible with the recent observations.

Key words: cosmology, Bianchi type II model, variable cosmological constant.

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1. INTRODUCTION

One of the most important and outstanding problems in cosmology is the cosmological constant problem. The recent observations indicate that $\Lambda \sim 10^{-55}$ cm^{-2} while particle physics prediction for Λ is greater than this value by a factor of order 10^{120}. This discrepancy is known as cosmological constant problem. Some of the recent discussions on the cosmological constant "problem" and consequence on cosmology with a time-varying cosmological constant are investigated by Ratra and Peebles [1], Dolgov [2–4] and Sahni and Starobinsky [5]. Recent observations by Perlmutter *et al.* [6] and Riess *et al.* [7] strongly favour a significant and positive value of Λ. Their finding arise from the study of more than 50 type Ia supernovae with redshifts in the range $0.10 \le z \le 0.83$ and these suggest Friedmann models with negative pressure matter such as a cosmological constant (Λ), domain walls or cosmic strings (Vilenkin [8], Garnavich *et al.* [9]). Recently, Carmeli and Kuzmenko [10] have shown that the cosmological relativistic theory (Behar and Carmeli [11]) predicts the value for cosmological constant $\Lambda = 1.934 \times 10^{-35}$ s^{-2}. This value of "Λ" is in excellent agreement with the measurements recently obtained by the High-z Supernova Team and Supernova Cosmological Project (Garnavich *et al.* [9]; Perlmutter *et al.* [6]; Riess *et al.* [7]; Schmidt *et al.* [12]). The main conclusion of these observations is that the expansion of the universe is accelerating.

Several ans*ä*tz have been proposed in which the Λ term decays with time (see Refs. Gasperini [13, 14], Berman [15], Freese *et al.* [16], Özer and Taha [16], Peebles and Ratra [17], Chen and Hu [18], Abdussattar and Viswakarma [19], Gariel and Le Denmat [20], Pradhan *et al.* [21]). Of the special interest is the ans*ä*tz $\Lambda \propto a^{-2}$ (where a is the scale factor of the Robertson-Walker metric) by Chen and Wu [18], which has been considered/modified by several authors (Abdel-Rahaman [22], Carvalho *et al.* [23], Waga [24], Silveira and Waga [25], Vishwakarma [26]). However, not all vacuum decaying cosmological models predict acceleration. Al-Rawaf and Taha and Al-Rawaf [27] and Overdin and Cooperstock [28] proposed a cosmological model with a cosmological constant of the form $\Lambda = \beta \frac{\ddot{a}}{a}$, where a is the scale factor of the universe and β is a constant. Following the same decay law recently Arbab [29] have investigated cosmic acceleration with positive cosmological constant and also analyze the implication of a model built-in cosmological constant. The cosmological consequences of this decay law are very attractive. This law provides reasonable solutions to the cosmological puzzles presently known. One of the motivations for introducing Λ term is to reconcile the age parameter and the density parameter of the universe with recent observational data.

In this paper by considering cosmological implication of decay law for Λ that proportional to $\frac{\ddot{a}}{a}$, the behavior of the scale factors are investigated and the cosmological tests pertaining proper distance, luminosity distance, angular diameter distance, and look-back time in the frame work of Bianchi type-II cosmological model are analyzed. The Einstein de-Sitter results are obtained from our model for the case $b = \frac{2}{3}$.

2. THE METRIC AND FIELD EQUATIONS

In an orthogonal frame, the metric for LRS Bianchi type II is given by [31]

$$ds^2 = \eta_{ij}\sigma^i\sigma^j, \quad \eta_{ij} = diag(-1, 1, 1, 1), \tag{1}$$

where the Cartan bases σ^i are given by

$$\sigma^0 = dt, \quad \sigma^1 = S(t)\omega^1, \quad \sigma^2 = R(t)\omega^2, \quad \sigma^3 = R(t)\omega^3, \tag{2}$$

where $R(t)$ and $S(t)$ are the metric functions. Taking (x, y, z) as local coordinates the invariant basis ω^i is given by

$$\omega^1 = dy + xdz, \quad \omega^2 = dz, \quad \omega^3 = dx. \tag{3}$$

The usual energy-momentum tensor is modified by addition of a term

$$T_{ij}^{vac} = -\Lambda(t) g_{ij}, \tag{4}$$

where $\Lambda(t)$ is the cosmological term and g_{ij} is the metric tensor. Thus the energy-momentum tensor is

$$T_{ij} = (p+\rho)u_i u_j + p g_{ij}, \tag{5}$$

where p and ρ are, respectively, the energy and pressure of the cosmic fluid, and u_i is the fluid four-velocity such that $u^i u_i = 1$.

For energy-momentum tensor and the LRS Bianchi type II spacetime (1), Einstein's field equations

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi T_{ij} - \Lambda(t) g_{ij} \tag{6}$$

yields the following set of three independent equations

$$\frac{2\dot{R}\dot{S}}{RS} + \frac{\dot{R}^2}{R^2} - \frac{S^2}{4R^4} = 8\pi\rho - \Lambda, \tag{7}$$

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} - \frac{3S^2}{4R^4} = -8\pi p - \Lambda, \tag{8}$$

$$\frac{\ddot{S}}{S} + \frac{\ddot{R}}{R} + \frac{\dot{R}\dot{S}}{RS} + \frac{S^2}{4R^4} = -8\pi p - \Lambda\ . \tag{9}$$

The average scale factor $a(t)$ is defined as

$$a(t) = (R^2 S)^{\frac{1}{3}}. \tag{10}$$

The Hubble parameter and deceleration parameter are respectively defined by

$$H = \frac{\dot{a}}{a}, \quad q = -\frac{\ddot{a}a}{\dot{a}^2}. \tag{11}$$

An over dot indicates a derivative with respect to time t. The energy conservation equation $T^i_{j;i} = 0$ leads to

$$\dot{\rho} + (\rho + p)\left(\frac{2\dot{R}}{R} + \frac{\dot{S}}{S}\right) = -\frac{\dot{\Lambda}}{8\pi}. \tag{12}$$

For complete determinacy of the system, we consider a perfect-gas equation of state

$$p = \gamma\rho, \quad 0 \le \gamma \le 1. \tag{13}$$

It is worth noting here that our approach suffers from a lack of Lagrangian approach. There is no known way to present a consistent Lagrangian model satisfying the necessary conditions discussed in the paper.

3. SOLUTION OF THE FIELD EQUATIONS

The research on an exact solutions is based on some physically reasonable restriction used to simplify the Einstein's equations. In the neighbourhood of our galaxy today, the ratio of shear σ to Hubble constant H is given by $\frac{\sigma}{H} \le 0.3$. Following Roy *et al.* [32], Bali and Jain [33] and Pradhan *et al.* [34], we assume that the expansion (θ) in the model is proportional to the eigen value σ_1^1 of the shear tensor σ_i^j. This condition leads to

$$S = R^{\alpha}, \tag{14}$$

where α is the proportionality constant.

In case of stiff fluid, Eqs. (7) and (8), with the use of (14), reduces to

$$(\alpha + 1)\frac{\ddot{R}}{R} + (\alpha + 1)^2\frac{\dot{R}^2}{R^2} = -2\Lambda(t). \tag{15}$$

In term of the average scale factor $a(t)$, Eq. (15) can be expressed as

$$\frac{3}{(\alpha + 2)}\frac{\ddot{a}}{a} + \frac{6}{(\alpha + 2)}\frac{\dot{a}^2}{a^2} = -\frac{2\Lambda(t)}{(\alpha + 1)}. \tag{16}$$

We propose a phenomenological decay law for Λ of the form [27, 28]

$$\Lambda = \beta\left(\frac{\ddot{a}}{a}\right), \tag{17}$$

where β is constant. Overdin and Cooperstock [28] have pointed out that the model with $\Lambda \propto H^2$ is equivalent to above form.

Using (17) in (16) we get

$$\frac{\ddot{a}}{a} = A_0\frac{\dot{a}^2}{a^2}, \tag{18}$$

where

$$A_0 = \frac{6(\alpha+1)}{3(\alpha+1)+2\beta(\alpha+2)}. \tag{19}$$

After integrating (18) we obtain

$$a(t) = \left(\frac{c_1 t}{b}\right)^b, \tag{20}$$

where c_1 is an integrating constant and

$$b = \frac{1}{1-A_0}. \tag{21}$$

By using (20) in (17) we obtain

$$\Lambda(t) = \frac{B_0}{t^2}, \tag{22}$$

where

$$B_0 = \frac{6\beta(\alpha+1)\left[3(\alpha+1)+2\beta(\alpha+2)\right]}{\left[9(\alpha+1)+2\beta(\alpha+2)\right]^2} \tag{23}$$

and

$$16\pi\rho(t) = \frac{B_1}{t^2} - \frac{1}{2}\left(\frac{c_1 t}{b}\right)^{\frac{6b(\alpha-2)}{(\alpha+2)}}, \tag{24}$$

where

$$B_1 = \frac{3b}{(\alpha+1)^2}\left[b(2+3\alpha-3\alpha^2)+(\alpha+1)(\alpha+2)\right]. \tag{25}$$

It is observed that the cosmological constant Λ (for $\alpha > 2$) is positive and the deceleration parameter is calculated as

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -A_0 = -\frac{6(\alpha+1)}{[3(\alpha+1)+2\beta(\alpha+2)]}. \tag{26}$$

The age of the universe is calculated as

$$t_0 = bH_0^{-1} = \frac{H_0^{-1}}{(1-A_0)}. \tag{27}$$

4. NEOCLASSICAL TESTS (PROPER DISTANCE $d(z)$)

The proper distance between the source and observer is given by

$$d(z) = a_0 \int_a^{a_0} \frac{da}{a\dot{a}} = a_0 \int_t^{t_0} \frac{dt}{a}. \tag{28}$$

Using Eq. (20) in (28) we obtain

$$d(z) = H_0^{-1} \frac{b}{(1-b)} \left[1 - (1+z)^{\frac{(b-1)}{b}} \right], \tag{29}$$

where $b = \frac{1}{1-A_0}$ and $(1+z) = \frac{a_0}{a}$ = redshift.

For small z, Eq. (29) reduces to

$$H_0 d(z) = z - \frac{1}{2}\frac{z^2}{b} + \ldots \tag{30}$$

Using Eq. (26) in (30) reduces to

$$H_0 d(z) = z - \frac{1}{2}(1+q)z^2 + \ldots \tag{31}$$

From Eq. (29), it is observed that the distance d is maximum at $z = \infty$. Hence

$$d(z=\infty) = \frac{H_0^{-1}}{A_0} = H_0^{-1} \left[\frac{3(\alpha+1) + 2\beta(\alpha+2)}{6(\alpha+1)} \right]. \tag{32}$$

It is remarkable that Einstein de-sitter is a special case of our result (29). It is observed from Eq. (29) that d is maximum in *de-Sitter* when $\frac{b-1}{b} \to 1$ *i.e.* $A_0 \to -1$ and minimum in ES when $\frac{b-1}{b} \to -\frac{1}{2}$ *i.e.* $A_0 \to -\frac{1}{2}$.

5. LUMINOSITY DISTANCE

Luminosity distance is the another important concept of theoretical cosmology of a light source. It is defined in such a way as generalizes the inverse-square law of the brightness in the static Euclidean space to an expanding curved space [24].

$$d_L = \left(\frac{L}{4\pi l} \right)^{\frac{1}{2}}, \tag{33}$$

where L is the total energy emitted by the source per unit time, l is the apparent luminosity of the object. Therefore one can write

$$d_L = d(1+z). \tag{34}$$

Using Eq. (29) in (34) reduces

$$H_0 d_L = (1+z)\left(\frac{b}{1-b} \right) \left[1 - (1+z)^{\frac{b-1}{b}} \right]. \tag{35}$$

For small z, Eq. (35) gives

$$H_0 d_L = z + \frac{1}{2}(1-q)z^2 + ... \tag{36}$$

6. ANGULAR DIAMETER

The angular diameter distance is a measure of how large objects appears to be. As with the luminosity distance, it is defined as the distance that an object of known physical extent appears to be at, under the assumption of the Euclidean geometry.

The angular diameter d_A of a light source of proper distance d is given by

$$d_A = d_L (1+z)^{-2}. \tag{37}$$

Applying Eq. (35) we obtain

$$d_A = H_0 d_L = (1+z)\left(\frac{b}{1-b}\right)\frac{\left[1-(1+z)^{\frac{b-1}{b}}\right]}{(1+z)}. \tag{38}$$

Usually d_A has a minimum (or maximum) for some $Z = Z_m$.

The angular diameter and luminosity distances have similar forms, but have a different dependence on redshift. As with the luminosity distance, for nearly objects the angular diameter distance closely matches the physical distance, so that objects appear smaller as they are put further away. However the angular diameter distance has a much more striking behaviour for distant objects. The luminosity distance effect dims the radiation and the angular diameter distance effect means the light is spread over a large angular area. This is so-called surface brightness dimming is therefore a particularly strong function of redshift.

7. LOOK-BACK TIME

The radiation travel time (or look-back time) $t - t_0$ for photon emitted by a source at instant t and received at t_0 is

$$t - t_0 = \int_a^{a_0} \frac{da}{\dot{a}}, \tag{39}$$

where a_0 is the present average scale factor of the universe.

Equation (20) can be rewritten as

$$a = B' t^b, \quad B' = \text{constant}. \tag{40}$$

This follows that

$$\frac{a_0}{a} = 1 + z = \left(\frac{t_0}{t}\right)^b, \tag{41}$$

which implies

$$t = t_0 (1+z)^{-\frac{1}{b}}. \tag{42}$$

From Eqs. (29) and (42), we obtain

$$t_0 - t = bH_0^{-1}\left[1 - (1+z)^{-\frac{1}{b}}\right], \tag{43}$$

which gives

$$H_0(t_0 - t) = b\left[1 - (1+z)^{-\frac{1}{b}}\right]. \tag{44}$$

For small z, we can obtain

$$H_0(t_0 - t) = z - \left(1 + \frac{q}{2}\right)z^2 + \ldots. \tag{45}$$

From Eqs. (42) and (44), we observe that at $z \to \infty$, $H_0 t_0 = b$ (constant). If $b = \frac{2}{3}$ gives the well-known Einstein-de Sitter result

$$H_0(t_0 - t) = \frac{2}{3}\left[1 - (1+z)^{-\frac{3}{2}}\right]. \tag{46}$$

8. CONCLUSIONS

In this paper we have analyzed the Bianchi type II stiff fluid cosmological models with varying Λ term of the form $\Lambda = \beta\left(\frac{\ddot{a}}{a}\right)$. The results for the cosmological tests are compatible with the present observations. To solve the age parameter and density parameter one require the cosmological constant to be positive or equivalently the deceleration parameter to be negative. This imply an accelerating universe. It is observed that the Λ and ρ vanish at $t \to \infty$ and infinite at $t \to 0$. The proper distance, the luminosity distance-redshift, the angular diameter distance-redshift, and look-back time-redshift for the model are presented. These tests are found to depend on β. The Einstein de-Sitter result is obtained for the case $b = \frac{2}{3}$. It is a general belief among cosmologists that more precise observational data should be achieved in order to make more definite statements about the validity of cosmological models (Charlton and Turner [35]). We hope that in the near future, with the new generation of the telescope, the present situation could be reversed.

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