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EVAPORATION OF DROPS INTO A GAS FLOW THROUGH A CYLINDRICAL TUBE

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Abstract. Evaporative, mass transfer, from a sequence of monodisperse droplets to a surrounding gas flow inside a cylindrical tube is treated. The evaporation process is modeled as a rod source of vapor (droplets chain) located at the axis of the droplet array. The diffusion equation describing this processes is solved analytically. Diffusion in the axial direction is taken into consideration. The vapor concentration distribution in the gas phase is obtained. The first few eigenvalues E_n for different Peclet number were computed. The obtained solution is used to calculate the mixing cup concentration and the mass transfer coefficient, Sherwood number, for different Peclet number. These results are a powerful tool for description of the diffusion in the pipe and in other applications. The suitable length of the cylinder is obtained from the ratio $\frac{Dz}{2Vr_0^2} = 0.1$. Around this value the effect of Pe can be studied.

Key words: mass transfer, diffusion equation, Sherwood number, mixing cup concentration

1. INTRODUCTION

Recently evaporative mass transfers from an aerosol droplet has been the object of numerous theoretical and experimental studies [1, 2]. Widmann and Davis [2] have provided a nice analysis about the mass transfer between a sequence of drops and a surrounding gas. They considered mass transport associated with a sequence of equally spaced monodisperse droplets moving along the axis of a gas-filled tube. The evaporation process is modeled as a line source of vapor with respect to the gas phase. The vapor concentration distribution in the gas phase is obtained by numerically solving the convective diffusion equation. The above numerical solution is obtained under the assumption: axial diffusion within the gas phase is neglected, the temperature of the droplets is invariant and there is no gas-phase chemical reaction in the cylindrical tube.

The system considered here is consists of droplets traveling downword along the axis of a cylindrical tube. The cylinder is assumed to contain an inert carrier gas and a droplets producing a vapor. The mixture (gas, vapor and droplet stream) are considered to be in steady laminar flow. The concentration of the vapor species within the gas, C_i, due to convection and diffusion is given by [3–5]:

$$v_z \partial_z C_i = D[r^{-1}\partial_r (r\partial_r) + \partial_z^2]C_i + \phi_i, \tag{1}$$

the velocity distribution, v_z, in the flow direction is parabolic and is given by Poiseuille formula such as

$$v_z = 2V(1 - r^2/r_0^2)$$

where D, V, r_0 and ϕ_i are the diffusion coefficient of the vapor into the gas flow, the mean velocity of the flow, the radius of the cylinder ($0 \le r \le r_0$) and the rate of production of the vapor species via chemical reaction, respectively. In equation (1), the terms on the left-hand side represent the effects of convection while the terms on the right-hand side describe the effects of diffusion. Equation (1), with the appropriate boundary conditions, describes the vapor concentration of species i in the gas flow. The boundary conditions, as be taken by the previous authors will be defined as follows [2, 4–7]: At the wall of the cylinder tube, the complete absorption of the vapor and the finiteness of the concentration at the center of the droplet leads to no-flux condition,

$$\begin{aligned} C_i(z, r_0) &= 0 \\ \partial_r C(0, z) &= 0, \end{aligned} \tag{2}$$

The axial dependence of the concentration is due to the droplets flow being stationary, and would not exist if the droplets were moving along the axis of the chain. According to Widmann and Davis approximation, the boundary condition at the droplet (if a is the radius of the droplet) chain is

$$C_i(z, a) = C_i^{sat}, \tag{3}$$

The uniform distribution, initially, of the vapor at the tube inlet leads to:

$$C_i(0, r) = C_{i,0},$$

where $C_{i,0}$ is the concentration at $z = 0$, usually zero for an evaporation study. *i.e.*

$$C_i(0, r) = 0, \tag{4}$$

At large z and when the laminar flow of the concentration distribution is fully established, a balance between the axial convection and radial diffusion is reached. Moreover the concentration distribution does not change with z. Referring to this concentration by C_{ass}, then

$$lim_{z\to\infty} C_i(z, r) = C_{ass}; \quad \partial_z C_{ass}(r) = 0, \tag{5}$$

where C_{ass} is the solution of

$$D\frac{1}{r}\partial_r(r\partial_r C_{ass}) + \phi_i = 0, \tag{6}$$

The objective of the present work is to find an analytical solution of equation (1) under the assumptions: (*i*) Axial diffusion and the rate of producing the vapor are taking into consideration. The criteria of including the axial diffusion term are: for gases with higher diffusivities neglecting axial diffusion becomes less satisfactory. (*ii*) The region between the droplets are saturated with the vapor, so the droplets chain will be consider as a rod source with respect to the gas flow. (*iii*) The system (vapor and gas) is isothermal, *i.e.* the temperature is invariant. (*iv*) The gas flow is not affected by the presence of droplets. Following Widmann and Davis, since it is difficult to measure the concentration in the liquid phase, particularly if chemical reaction occurs within the droplet, the gas phase has been chosen. The results can be used in design experimental apparatus and interpretation of the experimental data.

It is more convenient for further analysis to express equation (1) and the relevant boundary conditions in terms of the following dimensionless quantities

$$\kappa(\mu,\eta) = \frac{C_i(z,r)}{C_i^{sat}}, \quad \sigma = \frac{r_0^2\phi_i}{C_i^{sat}D}, \quad \mu = Dz/2Vr_0^2,$$
$$\eta = r/r_0, \quad \xi = \frac{a}{r_0}, \quad Pe = 2Vr_0/D, \quad \kappa_{ass} = \frac{C_{ass}}{C_i^{sat}} \tag{7}$$

where the Peclet number, Pe, in the problem under consideration, characterizes the axial self diffusion with in the gas phase relative to convective transport of species i into the gas flow. The nondimensional diffusion equation and the boundary conditions become

$$(1-\eta^2)\partial_\mu\kappa(\mu,\eta) = \eta^{-1}\partial_\eta[\eta\partial_\eta\kappa(\mu,\eta)] + Pe^{-2}\partial_\mu^2\kappa(\eta,\mu) + \sigma, \tag{8}$$

and

$$\kappa(\mu,1) = \partial_\eta\kappa(0,\mu) = 0, \quad \kappa(\mu,\xi) = 1, \quad \kappa(0,\eta) = 0, \quad lim_{\mu\to\infty}\kappa(\mu,\eta) = \kappa_{ass}(\eta) \tag{9}$$

respectively.

2. ANALYSIS OF THE PROBLEM

The nonhomogeneous differential equation(8), subjected to the boundary conditions (9) can be reduced to a homogeneous equation by using the asymptotic solution κ_{ass}, solution of equation:

$$\frac{1}{\eta}\partial_\eta\{\eta\partial_\eta\kappa_{ass}\}+\sigma=0,$$

This solution is given by

$$\kappa_{ass}=\frac{\sigma}{4}(1-\eta^2)$$

and the appropriate general solution of equation (8) is composed of two parts. the first part is the particular , κ_{ass}, and the second is an unknown function which is set in the form of an infinite series which depend exponentially on the axial coordinate. The suggested solution will be in the form [4, 5, 9]:

$$\kappa(\mu,\eta)=\frac{\sigma}{4}(1-\eta^2)+\sum_{n=1}^{\infty}A_nR_n(\eta)e^{-E_n^2\mu}, \tag{10}$$

where A_n are series coefficients, E_n are eigenvalues and $R_n(\eta)$ are unknown functions. The constants A_n, E_n and the functions R_n will be determined from equations (8) and (9). Substitution of the proposed solution, equation (10) into the original equation, equation (8), leads to an ordinary differential equation for the determination of the unknown functions $R_n(\eta)$. This equation has the form:

$$\eta^{-1}\partial_\eta\{\eta\partial_\eta R_n(\eta)\}+g_n(\eta,E_n)R_n(\eta)=0, \tag{11}$$

where

$$g_n=(1-\eta^2)E_n^2+\frac{E_n^4}{Pe^2}, \tag{12}$$

equation (11) determines the functions R_n and satisfy the boundary conditions (9), which reduce to

$$R_n(1)=\partial_\eta R(0)=0, \tag{13}$$

$$\frac{\sigma}{4}(1-\eta^2)+\sum_{n=1}^{\infty}A_nR_n(\eta)=0, \tag{14}$$

equation (11) can be reduced to the standard form of the confluent hypergeometric equation using the transformation

$$\lambda=E_n\eta^2,\quad F_n(\lambda)=e^{\frac{\lambda}{2}}R_n[\eta(\lambda)].$$

After applying the transformation equation (11) becomes:

$$\lambda\frac{d^2F_n}{d\lambda^2}+(b_n-\lambda)\frac{dF_n}{d\lambda}+a_nF_n=0; \tag{15}$$

where

$$a_n = 0.5 - 0.25 E_n \left(1 - \frac{E_n^2}{Pe^2}\right), \quad b_n = 1 \quad \text{for} \quad n = 1, 2, 3, ...,$$

The solution of equation (15) is known to be the Kummer function, defined by the series

$$F_n = {}_1F_1(a_n, b_n, \lambda) = \sum_{k=0}^{\infty} \frac{(a_n)_k \lambda^k}{(b_n)_k k!}, \quad \text{where} \quad (a_n)_k = \prod_{i=0}^{k-1} (a_k + i),$$

since $b_n = 1$ for all n, it follows that $(b_n)_k = k!$. and the final solution of equation (11) in terms of the original variable η takes the final form

$$R_n(\eta) = e^{-E_n \eta^2/2} \sum_{k=0}^{\infty} \frac{(a_n)_k E_n^k \eta^{2k}}{(k!)^2}; \quad \text{for} \quad n = 1, 2, \dots, \tag{16}$$

The boundary condition $R_n(1) = 0,$ equation (13), requires the eigenvalues E_n to be the zeros of the equation

$$R_n(1) = e^{\frac{E_n}{2}} \sum_{k=0}^{\infty} \frac{(a_n)_k E_n^k}{(k!)^2} = 0, \tag{17}$$

The computed numerical results for E_n were given in Table 1. The series coefficients A_n will be determined by the boundary condition defined in equation (14) and it is given by (see the Appendix):

$$A_n = \frac{-\sigma}{4} \frac{\sum_{k=0}^{\infty} \frac{(a_n)_k 2^{k+2}}{k!} \left[\frac{-G_n}{2} \beta_0 + \frac{(G_n + 1)(k+1)}{E_n} \beta_1 - \frac{2(k+2)(k+1)}{E_n^2} \beta_2 \right]}{\sum_{k=0, l=0}^{\infty} \frac{(a_n)_k (a_n)_l (k+l)!}{(k!)^2 (l!)^2} \left[G_n \beta_3 - \frac{(k+l+1)}{E_n} \beta_4 \right]} \tag{18}$$

where

$$\beta_0 = 1 - e^{-E_n/2} \sum_{j=0}^{k} \frac{E_n^j}{2^j j!}, \quad \beta_1 = \beta_0 - e^{-E_n/2} \frac{(E_n/2)^{k+1}}{(k+1)!},$$

$$\beta_2 = \beta_1 - e^{-E_n/2} \frac{(E_n/2)^{k+2}}{(k+2)!}, \quad \beta_3 = 1 - e^{-E_n} \sum_{j=0}^{k+l} \frac{E_n^j}{j!},$$

$$\beta_4 = \beta_3 - e^{-E_n} \frac{E_n^{k+l+1}}{(k+l+1)!}, \quad G_n = \frac{2E_n^2}{Pe^2} + 1$$

Finally the solution, equation (10), takes the final form by using the boundary condition, $\kappa(\mu, \xi) = 1$, which leads to

$$\frac{\sigma}{4}(1-\xi^2) + \sum_{n=1}^{\infty} A_n R_n(\xi) e^{-E_n^2 \mu} = 1, \tag{19}$$

and the final expression for the concentration distribution becomes

$$\kappa(\mu, \eta) = \frac{1-\eta^2}{1-\xi^2} - \sum_{n=1}^{\infty} A_n e^{-E_n^2 \mu} \left\{ \frac{1-\eta^2}{1-\xi^2} R_n(\xi) - R_n(\eta) \right\} \tag{20}$$

For purposes of designing an apparatus to determine the length required to achieve a specified gas concentration it is useful to determine the mixing cup concentration and the mass transfer coefficient, the Sherwood number.

Table 1

Eigenvalues E_n for different Peclet number Pe

Pe	E_1	E_2	E_3	E_4	E_5	E_6
3	2.1199	3.7099	4.8082	5.7009	6.4728	
5	2.3853	4.5109	5.9765	7.1579	8.1744	9.0798
10	2.5969	5.5469	7.7138	9.4592	10.9532	12.2780
50	2.6994	6.5959	10.3389	13.8542	17.1271	20.1710
100	2.7031	6.6575	10.5827	14.4370	18.2033	21.8748

Table 2

Series coefficients A_n for different Peclet number Pe

Pe	A_1	A_2	A_3	A_4	A_5	A_6
3	0.2850	–0.0539	0.0179	–0.00775	0.00278	
5	0.2861	–0.05932	0.02157	–0.00954	0.00474	–0.00348
10	0.2865	–0.06193	0.02721	–0.01341	0.00675	–0.00493
50	0.2863	–0.05147	0.02357	–0.01637	0.01533	–0.01953
100	0.2864	–0.05009	0.02099	–0.01332	0.01254	–0.03226

3. MIXING CUP CONCENTRATION

The mixing cup concentration is equivalent to the mean concentration at any cross-section of the cylinder. It is defined as the ratio of the flux of vapor at a given cross-section to the flux of the host medium (gas) through the same cross-section. Therefore, the mixing cup concentration, $< C_i >$ is given by

$$< C_i > = \frac{\int_a^{r_0} v_z C_i 2\pi r dr}{\int_a^{r_0} v_z 2\pi r dr}$$

In dimensionless quantities, mixing cup concentration becomes

$$< \kappa > = \frac{4}{(1-\xi^2)^2} \int_\xi^1 \eta(1-\eta^2)\kappa(\mu, \eta) d\eta \tag{21}$$

substituted from equation (19), leads to

$$\begin{aligned} < \kappa > &= \frac{4}{(1-\xi^2)^2} \int_\xi^1 \eta(1-\eta^2) \left\{ \frac{1-\eta^2}{1-\xi^2} - \sum_{n=1}^{\infty} A_n e^{-E_n^2 \mu} \left[\frac{1-\eta^2}{1-\xi^2} R_n(\xi) - R_n(\eta) \right] \right\} d\eta = \\ &= \frac{2}{3} - \sum_{n=1}^{\infty} A_n e^{-E_n^2 \mu} \left\{ \frac{2}{3} R_n(\xi) + \frac{2}{(1-\xi^2)^2} \sum_{k=0}^{\infty} \frac{(a_n)_k 2^{k+1}}{k! E_n} \left[\beta_5 + \frac{2(k+1)}{E_n} \beta_6 \right] \right\} \end{aligned} \tag{22}$$

where

$$\beta_5 = \sum_{j=0}^{k} \frac{E_n^j}{2^j j!} \left[e^{-E_n \xi^2/2} \xi^{2j} - e^{-E_n/2} \right], \quad \beta_6 = \sum_{j=0}^{k+1} \frac{E_n^j}{2^j j!} \left[e^{-E_n \xi^2/2} \xi^{2j} - e^{-E_n/2} \right],$$

and

$$R_n(\xi) = e^{-E_n \xi^2/2} \sum_{k=0}^{\infty} \frac{(a_n)_k E_n^k \xi^{2k}}{(k!)^2}; \quad \text{for} \quad n = 1, 2, \ldots, \tag{23}$$

4. SHERWOOD NUMBER

In order to designing an apparatus to determine the length required to achieve a specified exit gas concentration it is useful determine the mass transfer coefficient. The mass transfer coefficient, the Sherwood number Sh_μ, is defined by:

$$Sh_\mu = \frac{v_d}{f_d r_0} \frac{\left. \frac{\partial \kappa(\mu, \eta)}{\partial \eta} \right|_{\eta=\xi}}{[1 - < \kappa >]} \tag{24}$$

where v_d is the droplet velocity and f_d is the droplet frequency. The gas-phase concentration gradient at $\eta = \xi$, is obtained from equation (20) as

$$\left. \frac{\partial \kappa(\mu, \eta)}{\partial \eta} \right|_{\eta=\xi} = \frac{-2\xi}{1-\xi^2} + \sum_{n=1}^{\infty} A_n e^{-E_n^2 \mu} R_n(\xi) \left[\frac{2\xi}{1-\xi^2} + \frac{2k}{\xi} - E_n \xi \right] \tag{25}$$

The overall or average Sherwood number is obtained by integrating equation (25) over the length of the column of the vapor.

$$< Sh_{\mu} > = \frac{\int_0^{\mu} Sh_{\mu}(\mu')d\mu'}{\int_0^{\mu} d\mu'} = \frac{1}{\mu}\int_0^{\mu} Sh_{\mu}(\mu')d\mu' \tag{26}$$

5. RESULTS AND DISCUSSION

In this paper a mathematical solution has been presented for the diffusion equation (1). This solution gives the concentration of vapor $\kappa(\mu, \eta)$ in the gas phase and its used to calculate the mixing cup concentration and Sherwood number. The mixing cup concentrations are given in Fig. 1 for different

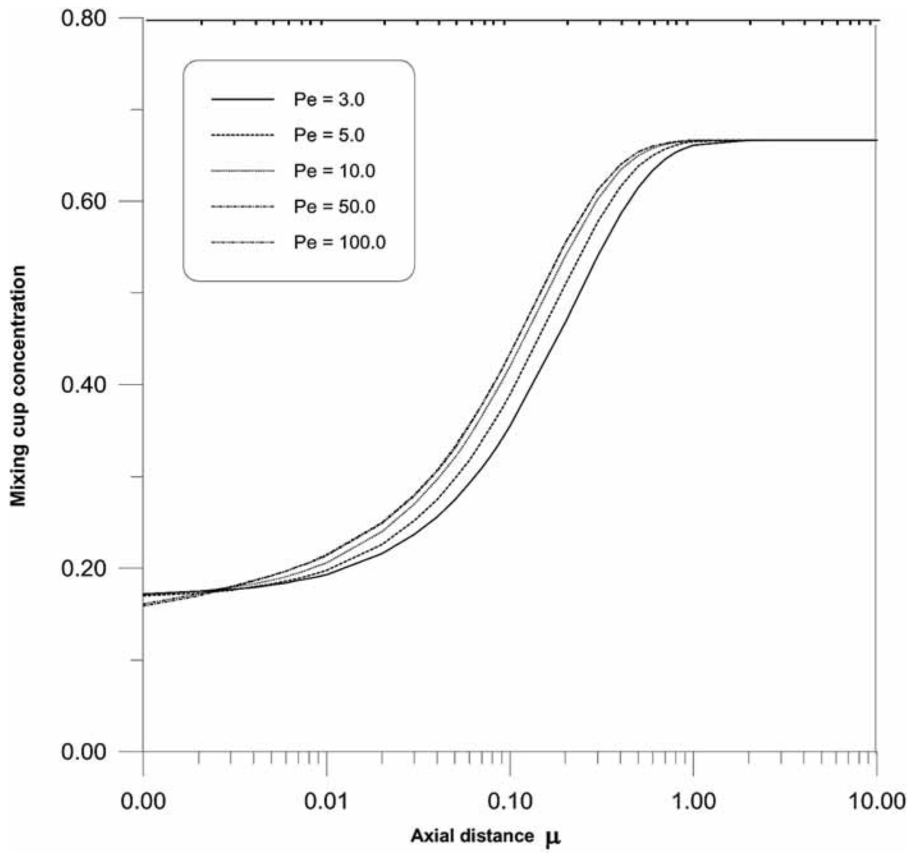

Fig. 1 – The mixing cup concentrations for different dimensionless axial distance μ and different Peclet number *Pe*.

dimensionless axial distance μ and different Peclet number *Pe*. Near the inlet, mixing cup concentration does not change with μ. For $0.01 \le \mu \le 1.0$ is increases monotonically very fast as μ increases. This behavior may arise from the fact that the concentration at the entrance of the cylinder is not in general uniform and its governed by the unsteady mass balance equation. For $\mu > 1$ the concentration is independent on both *Pe* and μ. So our calculations indicate that the suitable length of the cylinder is given from the ratio $\frac{Dz}{2Vr_0^2} = 0.1.$ Fig. 2 shows the dependence of the mass transfer coefficient, Sherwood number on *Pe*. For all values of μ Sherwood number increase with increasing *Pe* for $Pe \le 10$. For $Pe > 10$ Sherwood number is independent on *Pe*.

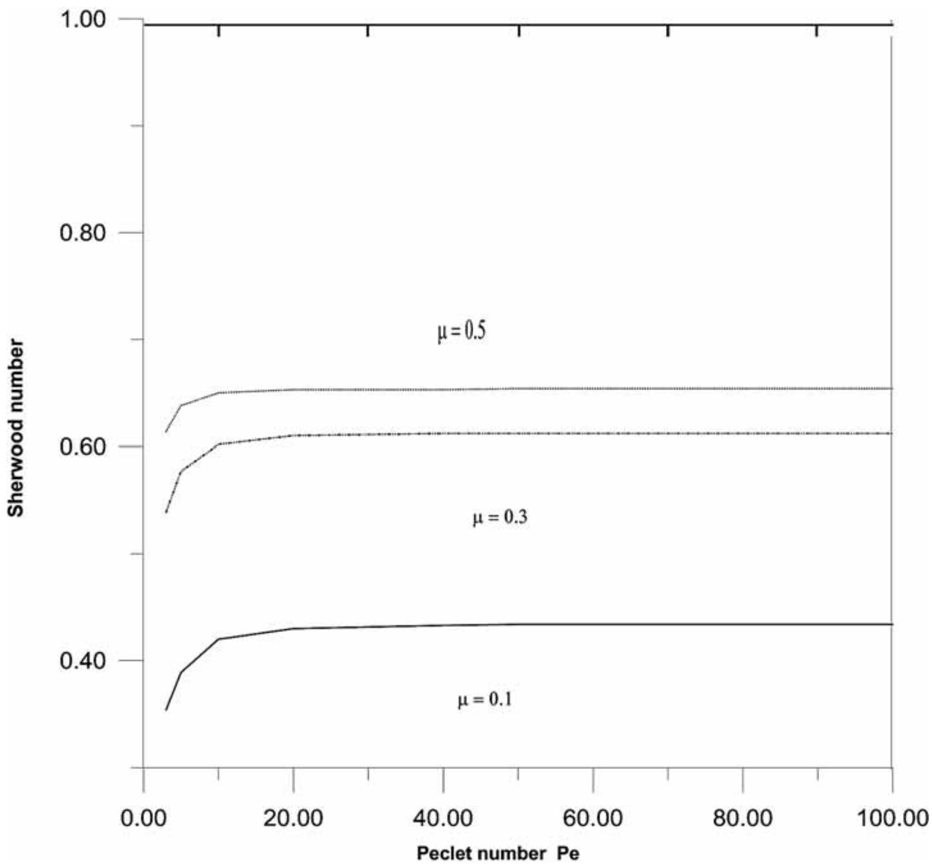

Fig. 2 – Dependence of the mass transfer coefficient, Sherwood number on *Pe*.

APPENDIX

Following Hsu [8], the specific form of the function g_n, equation (12), allow us to derive an orthogonality relation. Partial derivative of g_n relative to E_n leads to

$$\frac{1}{4E_m}\frac{\partial g_m}{\partial E_m}=\frac{1}{2}(1-\eta^2)+\frac{E_m^2}{Pe^2}\geq 0, \tag{27}$$

Let $R_n(\eta, E_n)$ and $R_m(\eta, E_m)$ (which will be written as R_n and R_m in the following) be solutions of equation (11) for E_n and E_m, respectively. For this two solutions, equation (11), leads to

$$\int_0^1 R_m\left\{\partial\eta[\eta\partial\eta R_n]+g_nR_n\eta\right\}-R_n\left\{\partial\eta[\eta\partial\eta R_m]+g_mR_m\eta\right\}d\eta=0.$$

Integration by parts leads to

$$\begin{aligned}&\left\{\eta R_m\frac{\partial R_n}{\partial\eta}-\eta R_n\frac{\partial R_m}{\partial\eta}\right\}_0^1-\int_0^1\eta\left\{\frac{\partial R_n}{\partial\eta}\frac{\partial R_m}{\partial\eta}-\frac{\partial R_m}{\partial\eta}\frac{\partial R_n}{\partial\eta}\right\}d\eta+\\&+\int_0^1(g_n-g_m)\eta R_nR_m d\eta=0\end{aligned} \tag{28}$$

due to the boundary conditions (13) the first term is eliminated $R_n(1)= =\partial_\eta R_n(0)=0$, while the second term vanishes identically. Substituting for the function g from equation (12), the last equation reduces to useful relation,

$$\left\{E_n^2-E_m^2\right\}\int_0^1\left[(1-\eta^2)+\frac{E_n^2+E_m^2}{Pe^2}\right]\eta R_nR_m d\eta=0,$$

Provided $E_n\neq E_m$, by using equation (27), this equation can be written as:

$$\frac{1}{4}\left\{E_n^2-E_m^2\right\}\int_0^1\left[\frac{1}{E_n}\frac{\partial g_n}{\partial E_n}+\frac{1}{E_m}\frac{\partial g_m}{\partial E_m}\right]\eta R_nR_m d\eta=0,$$

for $n\neq m$, the above equation gives

$$\int_0^1\left[\frac{1}{E_n}\frac{\partial g_n}{\partial E_n}+\frac{1}{E_m}\frac{\partial g_m}{\partial E_m}\right]\eta R_nR_m d\eta=0, \tag{29}$$

and for $n = m$, gives

$$\begin{aligned}\frac{1}{2E_n}\int_0^1\left\{\frac{\partial g_n}{\partial E_n}\eta R_n^2\right\}d\eta&\equiv\int_0^1\left[(1-\eta^2)+\frac{2E_n^2}{Pe^2}\right]\eta R_n^2 d\eta,\\&\equiv\int_0^1\left[G_n-\eta^2\right]\eta R_n^2 d\eta\neq 0\end{aligned} \tag{30}$$

where

$$G_n = 1 + \frac{2E_n^2}{Pe^2}.$$

Equations (29) and (30) can be combined in a compact form by the relation:

$$\frac{1}{4}\int_0^1 \left[\frac{1}{E_n}\frac{\partial g_n}{\partial E_n} + \frac{1}{E_m}\frac{\partial g_m}{\partial E_m}\right]\eta R_n R_m d\eta = I_n \delta_{nm}$$
$$\int_0^1 \left[(1-\eta^2) + \frac{E_n^2 + E_m^2}{Pe^2}\right]\eta R_n R_m d\eta = I_n \delta_{nm} \quad (31)$$

which can be used as an orthogonality relation.

Multiplying equation(14) by

$$\left\{\frac{1}{2}(1-\eta^2) + \frac{E_m^2}{Pe^2}\right\}\eta R_m \equiv \frac{1}{2}\left[G_m - \eta^2\right]\eta R_m$$

and integrating over the interval $0 \le \eta \le 1$, the following relation is obtained

$$\int_0^1 \sum_{n=1}^{\infty} A_n R_n \frac{1}{2}\left[G_m - \eta^2\right]\eta R_m d\eta = \frac{-\sigma}{4}\int_0^1 (1-\eta^2)\frac{1}{2}\left[G_m - \eta^2\right]\eta R_m d\eta$$

In this equation, the summation can be divided into two parts, one for $m = n$ and the other for $m \neq n$, *i.e.*

$$\frac{A_m}{4E_m}\int_0^1 \left\{\frac{\partial g_m}{\partial E_m}\eta R_m^2\right\} d\eta + \sum_{n=1, m\neq n}^{\infty} A_n \int_0^1 \left\{\frac{1}{2}(G_n - \eta^2)\eta R_n R_m\right\} d\eta,$$
$$= \frac{1}{4E_m}\left\{\int_0^1 \left\{\frac{\partial g_m}{\partial E_m}\eta R_m\right\}\left\{-\frac{\sigma}{4}(1-\eta^2)\right\}\right\} d\eta, \quad (32)$$

where the relation $\frac{\partial g_m}{\partial E_m} = 2E_m[G_m - \eta^2]$ is used. By using the orthogonality relation given by equation (30), the second term in equation (32) can be written as

$$\sum_{n=1, m\neq n}^{\infty} A_n \int_0^1 \left\{\frac{1}{2}(G_n - \eta^2)\eta R_n R_m\right\} d\eta,$$
$$\sum_{n=1, m\neq n}^{\infty} A_n \int_0^1 \left\{\left[\frac{1}{2}(1-\eta^2) + \frac{E_m^2}{Pe^2}\right]\eta R_n R_m\right\} d\eta =$$
$$= \sum_{n=1, m\neq n}^{\infty} A_n \int_0^1 \left\{\left[-\frac{1}{2}\frac{E_m^2 - E_n^2}{Pe^2} + \frac{E_m^2}{Pe^2}\right]\eta R_n R_m\right\} d\eta =$$

$$= \sum_{n=1, m\neq n}^{\infty} A_n \frac{E_m^2 - E_n^2}{2Pe^2} \int_0^1 \eta R_n R_m d\eta \quad \text{for} \quad m = 1, 2, 3, ...$$

and, equation (32) can be written as

$$\frac{1}{2} I_m A_m + \sum_{n=1, m\neq n}^{\infty} B_{mn} A_n = C_m, \tag{33}$$

where

$$\begin{aligned} I_m &= \frac{1}{2E_m} \int_0^1 \frac{\partial g_m}{\partial E_m} \eta R_m^2 d\eta \\ B_{mn} &= \frac{E_m^2 - E_n^2}{2Pe^2} \int_0^1 \left[\eta R_m R_n\right] d\eta \quad \text{for} \quad n \neq m \\ C_m &= -\frac{\sigma}{4} \frac{1}{4E_m} \int_0^1 \left\{ \frac{\partial g_m}{\partial E_m} \eta R_m \right\} (1 - \eta^2) d\eta, \end{aligned} \tag{34}$$

As Pe goes to infinity, equation (34) shows that $\lim_{Pe\to\infty} B_{mn} = 0$. Tan and Hsu suggested that neglecting of B_{mn} in equation (33) leads to errors of the order 3% in the value of A_m, for Peclet numbers ≤ 3. [9, 10] However, A_m will be calculated approximately from the relation

$$\begin{aligned} A_m &= \frac{2C_m}{I_m} = \\ &= -\frac{\sigma}{4} \frac{\int_0^1 \left\{ \frac{\partial g_m}{\partial E_m} \eta R_m(\eta) \right\} (1 - \eta^2) d\eta}{\int_0^1 \left\{ \frac{\partial g_m}{\partial E_m} \eta R_m^2(\eta) \right\} d\eta} = \\ &= -\frac{\sigma}{4} \frac{\int_0^1 \left[G_m - \eta^2\right] R_m(\eta)(1 - \eta^2) \eta d\eta}{\int_0^1 \left[G_m - \eta^2\right] R_m^2(\eta) \eta d\eta} \end{aligned} \tag{35}$$

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