ANALYTICAL APPROXIMATIONS OF THE NIEL IN SEMICONDUCTOR DETECTORS FOR HEP*

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Abstract. An important tool for the development of devices (detectors, cell solar, electronic circuit components) for high energy accelerator facilities or for space utilization, where new missions and experiments will be operated, is to find new materials with harder radiation properties. The radiation fields in these environments are extremely complex and the tests of the behaviour of different materials and devices for concrete situations are difficult to realise and very expensive. Thus, scaling of degradation effects would represent a useful tool and it is the main aim of the present contribution. Some analytical expressions for NIEL that suggest possible scaling formula are given.

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Key words: displacement damage, Lindhard theory.

1. GENERAL ASPECTS OF THE EFFECTS OF RADIATION IN MATERIALS

Bulk damage represents the limiting factor for detector lifetime, electronic components or other devices in the experiments with hostile radiation environment. These processes have been studied for several decades, but until now the phenomena are not completely clarified.

Radiation effects in semiconductor devices include:

a) Transient ionisation effects due to the generation of electron-hole pairs. These effects are used to detect the passage of high energy particles in particle detectors, or in the case of electronic components, if a critical charge in the sensitive volume of certain electronics components is exceeded, Single Event Effects (SEE) may result in a catastrophic failure of the electronic components.

b) The long-term ionisation effects appear in insulating areas of a device: charges produced by ionising radiation may be trapped and thus immobilised. This can affect (possibly permanently) the electrical properties of the device.

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c) Displacement effects. The well-ordered crystalline lattice structure could be disturbed by radiation displacing some of the lattice elements. Important manifestations of displacement damage are the formation of mid-gap states, which are either generation-recombination centres, or trapping centres for the charge carriers. Because the primary defects have very high mobilities, they interact mutually or with pre-existent impurities evolving toward partial recovery, but also forming complex defects. Consequently, electrical parameters of the device are affected. The current of reversed-biased p-n diodes is increased or, in forward-biased junctions or non-depleted regions, recombination is enhanced, a charge loss resulting. The effective carrier concentration is modified and also effective trapping probabilities for electrons and holes. Both categories of effects are cumulative, but we would like to underline that the nuclei displaced from lattice positions generate also different dynamic evolutions of defects depending on different factors: threshold energy of displacement, rate of generation of primary defects, concentration of pre-existent impurities, temperature.

The fist step in the estimation of bulk degradation is to calculate the primary degradation, quantitatively characterized by the Non Ionizing Energy Loss (NIEL) \[1\] or by the Concentration of Primary Defects (CPD) \[2\]. Unfortunately, these quantities are not directly measurable. In the second step, these defects, having high mobility, interact mutually or with other defects and impurities in the lattice, evolving in a complied manner to a “stable” equilibrium state of degradation.

\textit{NIEL} is the rate at which the energy of the incoming particle is lost per unit length of the material, in nonionizing processes. It is the analogue of the linear energy transfer (\textit{LET}) or stopping power for ionizing events.

In the last period, different groups claimed that changes of electrical properties of semiconductor devices could be proportional to NIEL \[3\] and different ways of scaling experimental data are searched \[4\] for various irradiation conditions, radiation fields and materials. Unfortunately, contrary to the common belief, even a significant violation of NIEL scaling can still be consistent with experimental data \[5\]. Until now, there is no real theoretical – microscopic understanding, of why radiation damage should scale with NIEL.

The radiation fields in the interaction cavity at the new high energy facilities in HEP as LHC, SLHC and VLHC for example, or in space, where new missions and experiments will be operated, are extremely complex; tests for concrete studies of the behaviour of different materials and devices are difficult and very expensive. Thus, scaling of degradation effects is of high interest.

The present discussion tries to bring some possible analytical clarifications. NIEL depends on the incoming particle, its energy and the properties of the target mainly by the type(s) of interaction of the particle with target nuclei and on the partition of particle’s kinetic energy into displacements and ionisation.
2. NIEL FORMALISM

In all forthcoming discussion, the particle of interest is the recoil, produced in the primary interaction of the incoming particle with the target nucleus: depending on type and energy of the incident particle, interactions could be elastic and/or inelastic, eventually with the fragmentation of the target and/or projectile.

If the particle has energy $E$, after the interaction processes (for particle stopped in the target), its energy is imparted to the atoms ($E_1$) and to the electrons ($E_2$).

Using some approximations in Lindhard’s theory [6], the equation satisfied by $E_1$, is

$$E_1 = \int_{T=0}^{E_1} E_1(T) \frac{d\sigma}{dT} dT$$

$i$ representing the stopping cross-sections for nuclei and electrons respectively, and $T$ the energy transferred.

Formally, the energy range of a particle could be divided into three regions, with different ratios of nuclear to electronic stopping powers [7].

In the low energy region, the energy of the particle is mainly lost in the production of displacements.

$$E_1 \approx E_c \left\{ -12 + 6 \left[ 1 + 2 \left( \frac{E_c}{E} \right)^{1/2} \right] \log \left[ 1 + \left( \frac{E}{E_c} \right)^{1/2} \right] \right\}$$

for energies where $E/E_c$ is less than unity.

Eq. (2) could be approximated by its series expansion. In the limit $E/E_c \to 0$, the following expression is obtained:

$$E_1 \approx E \cdot \left[ 1 - (E/E_c)^{1/2} \right]$$

<i.e. in the first approximation it depends only on the energy of the particle, and does not depend neither on the particle, nor on the medium (target).

For energies where $E/E_c$ is somewhat less than unity, from the series expansion of eq. (2) around $E/E_c = 1$:

$$E_1 \approx E_c \left( 0.067 + 0.478 \cdot \frac{E}{E_c} - 0.067 \cdot \left( \frac{E}{E_c} \right)^2 \right)$$

or, keeping only the first two terms,

$$E_1 \approx 0.478 \cdot E + 0.067 \cdot E_c$$
So, the expression of $E_1$ is a sum of two terms: one of them depends only on the energy of the particle, and the other only on the characteristics of the particle and target (medium).

In the second region:

$$E_1(E) \propto C_1 E_b \left[ 1 - \frac{1}{4\sqrt{2}} \xi^{-1/4} \log \frac{\xi^{1/2} + \sqrt{2}\xi^{1/4} + 1}{\xi^{1/2} - \sqrt{2}\xi^{1/4} + 1} - \frac{1}{2\sqrt{2}} \xi^{1/4} \arctan \frac{\sqrt{2}\xi^{1/4}}{1 - \xi^{1/2}} \right] + C_2 \xi^{-1/4} E_b^{-1/4}$$

\[ (6) \]

In these equations the following notations have been used:

$$\xi(E) = \frac{S_e}{S_n} \approx \frac{E}{E_b} \quad (7)$$

$C_1$ is of the order of unity and $C_2$ is small, usually negative.

The energy $E_b$ represents the value where the stopping cross sections (ionization and nuclear) are equal.

For the analysis of terms in eq. (6), it is useful to note with "$P$" the quantity in square brackets:

$$P = 1 - \frac{1}{4\sqrt{2}} \xi^{-1/4} \log \frac{\xi^{1/2} + \sqrt{2}\xi^{1/4} + 1}{\xi^{1/2} - \sqrt{2}\xi^{1/4} + 1} - \frac{1}{2\sqrt{2}} \xi^{1/4} \arctan \frac{\sqrt{2}\xi^{1/4}}{1 - \xi^{1/2}}$$

The energy $E_c$ is the limit of the first region, and it has the explicit forms:

$$E_C[MeV] \equiv \begin{cases} 5 \times 10^{-4} \cdot Z_{med}^{1/3} \cdot \frac{Z_{pr}^{1/3}}{(A_{pr} + A_{med})^2} & Z_{pr} \leq Z_{med} \leq A_{pr} \leq A_{med} \\ 1.25 \times 10^{-4} \cdot Z_{med} \cdot \frac{(A_{pr} + A_{med})^2}{A_{pr}} & Z_{pr} \leq Z_{med} \leq A_{pr} \geq A_{med} \end{cases} \quad (8)$$

The asymptotic ("plateau") limit of $E_c(E)$ corresponding to the third region has the following dependence on the charge and mass numbers of the particle and target (medium):

$$E_p[MeV] \approx 5.2 \times 10^{-4} \left( \frac{Z_{part}^{2/3} + Z_{med}^{2/3}}{Z_{part}^{2/3}} \right)^2 \cdot \frac{A_{part}^3}{(A_{part} + A_{med})^2} \quad (9)$$

In Fig. 1, the dependence of $E_1/E_C$ as a function of $E/E_C$ is represented with continuous line, as given by eq. (2), together with its series expansions at the end of the interval (dashed).
This region is continued with the second region, represented by equation (6). In order to analyse the dependence of $E_1(E)$, we represented in Fig. 2 the factor $P$, which is the quantity in square brackets in eq. (6). The region of interest is the region after the minimum. One could observe that this factor could be approximated by the value 1.

In the region of highest energies (region 3), the displacement energy increases slowly toward its asymptotic value – plateau of Lindhard curve.
The resulting dependence of the displacement energy for some particular cases is represented in the Fig. 3, on the whole region of variation of $E$. The arrows correspond to the corresponding values of $E_c$. The asymptotic limits are also represented.

![Graph showing the displacement energy as a function of the energy of incident particle for the carbon ion in carbon, the silicon ion in silicon and the germanium ion in germanium.](image)

Let the displacement energy be the Lindhard factor, $L$. Thus, the NIEL is expressed by the following formula.

\[
NIEL(E) = \frac{N_A}{A_{med}} \sum_i \left[ L(E) \frac{d\sigma}{d\Omega} d\Omega \right]
\]  

(10)

Here the following notations have been used:
- $E$ is the kinetic energy of the particle
- $\frac{d\sigma(E,\Omega)}{d\Omega}$ is the differential cross section for nuclear displacement,
- $L$ is the Lindhard factor
- $N_A$ is Avogadro’s number,
- $A_{med}$ is the atomic mass of the target.

Thus, the approximate dependence of NIEL is:
where $F(E, \vartheta)\; F'(E, \vartheta)\; F^*(E, \vartheta)$ and $F^{**}(E, \vartheta)$ are kinematic angular factors characteristics for interaction processes.

In this equation, the particle could be either identical or different of target nuclei. As specified before, in the case of elastic interactions, the particle is identical with target nuclei, and in the case of fragmentation of the target or of the projectile it has lower charge and mass number than the target / projectile. All open channels must be considered. Cross sections from experimental data or in the frame of a model are to be considered.

The comparison of NIEL with experiment cannot be done directly, due to the fact that another step in the chronological order of the production of permanent degradation is missing from the picture illustrated here. NIEL refers to the energy channelled into atomic displacements, i.e., to a number of vacancies and interstitials. In the semiconductors of interest (e.g. Si, Ge, GaAs, other) these defects have a high mobility and could interact mutually or with other defects and impurities in the lattice, producing “stable” degradation, which is measured. It depends on the number of vacancies and interstitials, but also on the concentrations of defects and impurities, on temperature, in a rather complicated manner [8]. Only the generation rate of primary defects scales with NIEL:

$$G_R = N \frac{A}{N_A} \frac{1}{2E_d} NIEL \cdot \varphi$$

where: $N$ – atomic density of the target, $A$ – atomic mass of the target, $N_A$ – Avogadro’s number, $E_d$ – threshold energy for displacements, $\varphi$ – particle flux.

So, a possible scaling of the degradation produced by irradiation depends both on NIEL and on the evolution of primary defects.
SUMMARY

Some analytical approximations useful in the calculation of nonionizing energy loss (NIEL) are given, starting from the general hypotheses of the Lindhard theory. The results obtained in the hypotheses considered, could be applied for all situations: hadrons or heavy ions as irradiation particles, keeping into account the peculiarities of the interactions. These results could represent a useful tool in obtaining scaling relations for NIEL.

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