Romanian Reports in Physics, Vol. 60, No. 2, P. 271–280, 2008

ESTIMATIONS OF THE COLLISION INTEGRAL IN THE CHAOTIC GUN EFFECT*

GHEORGHE DUMITRESCU

Gr. Sc. Ind. Toma N. Socolescu Ploiesti, Romania,
meditatie@yahoo.com

(Received August 14, 2007)

Abstract. The aim of this paper is to estimate the collision time integral of the kinetic equation describing a stochastic mechanism of acceleration. This mechanism, namely the chaotic gun effect, might be an efficient mechanism of injection. The efficiency of the mechanism can be expressed using the collision integral of the charged particles moving in a turbulent electromagnetic field.

Key words: collision integral, chaotic motion, correlation tensor, correlation length, spectral index.

1. INTRODUCTION

The kinetic equation for a charged particle moving in a magnetic field with small scale inhomogeneities and also with strong anisotropy comprises a collision integral in the right-hand part of the equation. For a while there were some attempts to study the behavior of the charged particles in such types of fields as mentioned above. The usual way to resolve the problem was to approach the process to a diffusion mechanism [1]. Some authors claimed the necessity of the strong anisotropy in order to explain the injection of the charged particles across the shock front occurring in supernovae, active galactic nuclei, etc. That is why it is important to investigate some theoretical solutions of the kinetic equation for strong anisotropy.

Melnikov transformed the small-scale collision integral in an approximation of a weak regular magnetic field using unequal parallel and perpendicular correlation lengths to the regular magnetic field [2]. He advocated his approximation on the basis of the great number of types of disturbances of a random magnetic field and also of the fractal structure of the field. Hence, for this assumption he selected the tensor part of the correlation tensor (contained in the

* Paper presented at the Annual Scientific Conference, Faculty of Physics, University of Bucharest, June 1, 2007, Bucharest, Romania.

collision integral) in an isotropic form. Using a diffusion approximation in the kinetic equation, Melnikov found that the form of the spatial diffusion tensor $\kappa_{\alpha\beta}$ also remains the same as in the isotropic case. The only one changing is the formula for the mean free path Λ.

The mean free path slightly differs from its value in isotropic case at $L_{\perp} << L_{||}$ and at $L_{\perp} \approx L_{||}$. Here $L_{\perp}$ and $L_{||}$ are the radii of the charged particle along the perpendicular and parallel direction to the regular magnetic field. But, the mean free path Λ tends to the infinity in a sharply anisotropic small scale random magnetic field at $L_{\perp} >> L_{||}$. In this case random magnetic structures are flattened in the direction of a regular field.

To date the most accepted mechanism able to accelerate charged particles to high energies in astrophysical environments is the diffusive shock acceleration mechanism. But in one of our previous papers we have adopted a special mechanism of acceleration which displays strong anisotropy of the charged particles [6]. Such strong anisotropy may be explained by the anisotropy of the small scale inhomogeneities of the magnetic field.

The fundamental assumption of the theory of diffusive shock acceleration is that accelerated particles diffuse in space, *i.e.* that the particle flux is proportional to the gradient of the particle density (Fick's law). Charged particles deflected by fluctuations in the electromagnetic fields obey this relation only if their velocities are distributed almost isotropically. More precisely, the theory employs an expansion in the ratio of the plasma speed in the shock frame to the particle speed, and the velocity anisotropy is considered to be small in the first order.

At a shock front, the downstream plasma speed is of the same order as the thermal speed of the ions in the plasma, so that the theory of diffusive shock acceleration does not apply to particles whose energy is several times less than the thermal energy. The question of how particles might be accelerated from the thermal pool up to an energy where they can be assumed to diffuse is referred to as the 'injection problem', and cannot be treated within the framework of the diffusive acceleration theory When the anisotropy is very large Fick's law does not work. The anisotropy of the magnetic field may be obtained by different correlation lengths and indices of the power spectrum along two directions. The aim of this paper is to derive and to estimate the collision integral for the special case of strong anisotropy and which cannot be reduced to the isotropic case. In the second section we present the correlation tensor of the second order of the turbulent magnetic field. Then in the third section we make some estimations of the small-scale collision integral.

Some final discussions in the paper point out a few other problems concerning our derivation.

2. THE FORM OF THE SMALL-SCALE COLLISION INTEGRAL

The motion of a relativistic charged particle in a magnetic field with small scale inhomogeneities has the kinetic equation of the form

$$\frac{\partial f}{\partial t}+\mathbf{v}\frac{\partial f}{\partial \mathbf{r}}+\mathbf{F}(\mathbf{r},t)\frac{\partial f}{\partial \mathbf{p}}=0 \tag{1}$$

where $\mathbf{F}(\mathbf{r}, t)$ is the total force acting on the charged particle and $f(\mathbf{r},\mathbf{p},t)$ the distribution function. The equation can be averaged by the use of the method of Vedenov *et al.* [1]

$$\begin{aligned}\frac{\partial \tilde{f}}{\partial t}+\mathbf{v}\frac{\partial \tilde{f}}{\partial \mathbf{r}}-(\mathbf{\Omega}\grave{\mathbf{o}}_1)\tilde{f} &= \int_0^\infty d\tau\left\{\left(\frac{ec}{\varepsilon}\right)^2\grave{\mathbf{o}}_{2\alpha}\mathrm{T}_{\alpha\beta}(\Delta\mathbf{r}(\tau),\tau)\grave{\mathbf{o}}_{2\beta}+\right.\\ &\left.+e^2\left(\frac{\partial}{\partial \mathbf{p}_\alpha}\right)\mathrm{K}_{\alpha\beta}(\Delta\mathbf{r}(\tau),\tau)\frac{\partial}{\partial \mathbf{p}_\beta}-\frac{e^2c}{\varepsilon}\left(\grave{\mathbf{o}}_{2\beta}S_{\alpha\beta}\frac{\partial}{\partial \mathbf{p}_\alpha}+\frac{\partial}{\partial \mathbf{p}_\alpha}S_{\alpha\beta}\grave{\mathbf{o}}_{2\beta}\right)\right\}\times\\ &\times\tilde{f}(\mathbf{r}-\Delta\mathbf{r}(\tau),\mathbf{p}-\Delta\mathbf{p}(\tau),t-\tau)\end{aligned} \tag{2}$$

where

$$\mathrm{T}_{\alpha\beta}(\mathbf{r},t)=\langle\,\mathbf{B}^{st}_{\alpha}(\mathbf{r}_1,\mathrm{t}_1)\mathbf{B}^{st}_{\beta}(\mathbf{r}_2,\mathrm{t}_2)\rangle,\quad \mathbf{r}=\mathbf{r}_1-\mathbf{r}_2 \tag{3}$$

$$\mathrm{K}_{\alpha\beta}(\mathbf{r},t)=\langle\,\mathbf{E}^{st}_{\alpha}(\mathbf{r}_1,\mathrm{t}_1)\mathbf{E}^{st}_{\beta}(\mathbf{r}_2,\mathrm{t}_2)\rangle,\quad \mathrm{t}=\mathrm{t}_1-\mathrm{t}_2 \tag{4}$$

$$\mathrm{S}_{\alpha\beta}(\mathbf{r},t)=\langle\,\mathbf{E}^{st}_{\alpha}(\mathbf{r}_1,\mathrm{t}_1)\mathbf{B}^{st}_{\beta}(\mathbf{r}_2,\mathrm{t}_2)\rangle \tag{5}$$

are the correlation tensors of the second order, and

$$\mathbf{\Omega}=\frac{e\mathbf{B}_0c}{\varepsilon} \tag{6}$$

$$\frac{e\mathbf{B}_0}{c}\left[\mathbf{v},\frac{\partial}{\partial \mathbf{p}}\right]=\frac{e\mathbf{B}_0\,c}{\varepsilon}\grave{\mathbf{o}}_1 \tag{7}$$

$$\frac{e\mathbf{B}_\mathrm{b}}{c}\left[\mathbf{v},\frac{\partial}{\partial \mathbf{p}}\right]=\frac{e\mathbf{B}_\mathrm{b}\,c}{\varepsilon}\grave{\mathbf{o}}_2 \tag{8}$$

Here $\mathbf{B_0}$ is the strength of the uniform magnetic field along the z axis, $\mathbf{B}_\mathrm{b}$ is the strength of the magnetic inhomogeneities, e is the charge and ε is the energy of the particle.

The above kinetic equation (2) may describe the charged particle behavior in the chaotic gun effect. This is a mechanism of stochastic acceleration in a random magnetic field with small inhomogeneities. The random magnetic field $\mathbf{B}_\mathrm{b}$ lies along z axis, while the random electric field $\mathbf{E}_b$ is along the y axis. Both are produced by the accelerated charged particles in their own synchrotron emission. As the numerical simulations of the chaotic gun effect have displayed, at every complet gyration of the charged particle, the particle receives a kick of energy when it reaches the wave vector direction. Hence the particle is deflected strictly along the x direction perpendicular to the uniform magnetic field $\mathbf{B}_0$. The acceleration takes place along the wave vector direction and there is no angular deflection. That is why we will omit in the equation (2) all the terms containing the **ò** operator. The **ò** operator is the angular variation of the velocity direction. The equation (2) becomes simpler [6]

$$\partial\tilde{f}/\partial t + \mathbf{v}\left(\partial\tilde{f}/\partial\mathbf{r}\right) =$$
$$= \int_0^\infty d\tau\left\{e^2\left(\frac{\partial}{\partial\mathbf{p}_\alpha}\right)\mathrm{K}_{\alpha\beta}(\Delta\mathbf{r}(\tau),\tau)\frac{\partial}{\partial\mathbf{p}_\beta}\right\}\tilde{f}(\mathbf{r}-\Delta\mathbf{r}(\tau),\mathbf{p}-\Delta\mathbf{p}(\tau),t-\tau) \tag{9}$$

After some manipulations the right-hand part of the equation (9) turns out to be

$$\int_0^\infty d\tau e^2\left(\frac{\partial}{\partial p_\alpha}\right)\mathrm{K}_{\alpha\beta}(\Delta\mathbf{r}(\tau),\tau)\frac{\partial\tilde{f}}{\partial p_\beta} =$$
$$= \frac{2^{-2s_y+1}\pi e^2c^2\langle \mathrm{E}_{\mathrm{st}}^2\rangle k_{\min y}\Gamma^2\left(\frac{s_y}{2}+1\right)}{\varepsilon\Gamma^2\left(\frac{s_y}{2}-\frac{1}{2}\right)\Gamma^2\left(s_y+\frac{1}{2}\right)}[k_{\min,y}\mathrm{v}_\beta\tau]^{s_y}K_{s_y}[k_{\min,y}\mathrm{v}_\beta\tau]\times$$
$$\times\int d\tau\{\tau\frac{\partial}{\partial a}[K_{s_y}(a)\times a^{s_y}]\frac{\partial f}{\partial p_{y\beta}}+$$
$$+\frac{2^{-2s_{\perp y}+1}\pi e^2c^2\langle \mathrm{E}_{\mathrm{st}}^2\rangle k_{\min,\perp y}\Gamma^2\left(\frac{s_{\perp y}}{2}+1\right)}{\varepsilon\Gamma^2\left(\frac{s_{\perp y}}{2}-\frac{1}{2}\right)\Gamma^2\left(s_{\perp y}+\frac{1}{2}\right)}[k_{\min,\perp y}\mathrm{v}_\beta\tau]^{s_{\perp y}}K_{s_{\perp y}}[k_{\min,\perp y}\mathrm{v}_\beta\tau]\times$$
$$\times\int d\tau\{\tau\frac{\partial}{\partial b}[K_{s_{\perp y}}(b)\times b^{s_{\perp y}}]\frac{\partial f}{\partial p_{\perp y\beta}}+$$
$$+\frac{2^{-\left(s_y+s_{\perp y}-1\right)}\pi e^2c^2<B_{st}^2>k_{\min,y}\Gamma\left(\frac{s_y}{2}+1\right)\Gamma\left(\frac{s_{\perp y}}{2}+1\right)}{\varepsilon\Gamma\left(\frac{s_y-1}{2}\right)\Gamma\left(\frac{s_{\perp y}-1}{2}\right)\Gamma\left(s_y+\frac{1}{2}\right)\Gamma\left(s_{\perp y}+\frac{1}{2}\right)}$$

$$[k_{\min,\perp y} \mathrm{v}_\beta \tau]^{s_{\perp y}} K_{s_{\perp y}} [k_{\min,\perp y} \mathrm{v}_\beta \tau] \times \int d\tau \left\{ \tau \frac{\partial}{\partial a} [K_{s_y,\alpha}(a) \times a^{s_y}] \frac{\partial f}{\partial p_{\perp y,\beta}} \right\} +$$

$$+ \frac{2^{-\left(s_y + s_{\perp y} - 1\right)} \pi e^2 c^2 < B_{st}^2 > k_{\min,\perp y} \Gamma\left(\frac{s_y}{2} + 1\right) \Gamma\left(\frac{s_{\perp y}}{2} + 1\right)}{\varepsilon \Gamma\left(\frac{s_y - 1}{2}\right) \Gamma\left(\frac{s_{\perp y} - 1}{2}\right) \Gamma\left(s_y + \frac{1}{2}\right) \Gamma\left(s_{\perp y} + \frac{1}{2}\right)} \times \tag{10}$$

$$\times [k_{\min,y} \mathrm{v}_\beta \tau]^{s_y} K_{s_y} [k_{\min,y} \mathrm{v}_\beta \tau] \times$$

$$\times \int d\tau \left\{ \tau \frac{\partial}{\partial b} [K_{s_{\perp y},\alpha}(b) \times b^{s_{\perp y}}] \frac{\partial f}{\partial p_{y,\beta}} \right\}$$

where we used a correlation tensor of the magnetic field

$$\mathrm{K}_{\alpha\beta}(\mathrm{k}, \omega) = \frac{2 \langle \mathrm{E}_{\mathrm{st}}^2 \rangle k_{\min,y}^{s_y - 1} \, k_{\min,\perp y}^{s_{\perp y} - 1} \, \Gamma\left(\frac{s_y}{2} + 1\right) \Gamma\left(\frac{s_{\perp y}}{2} + 1\right)}{\left(k_{\min,y}^2 + k_{y,\alpha}^2\right)^{s_y/2+1} \left(k_{\min,\perp y}^2 + k_{\perp y,\beta}^2\right)^{s_{\perp y}/2+1} \Gamma\left(\frac{s_y}{2} - \frac{1}{2}\right) \Gamma\left(\frac{s_{\perp y}}{2} - \frac{1}{2}\right)} \tag{11}$$

where s_y and $s_{\perp y}$ are the spectral indices toward y axis and any other direction perpendicular to y axis, respectively, $s_y \neq s_{\perp y}$, with the following properties:

(i) when k_y (or $k_{\perp y}) \to 0$ while $k_{\perp y}$ (or k_y) remains constant, then $\mathrm{T}_{\alpha\beta}(\mathrm{k}, \omega) \sim$ const.

(ii) when k_y (or $k_{\perp y}) \to \infty$ while $k_{\perp y}$ (or k_y) remains constant, then $\mathrm{T}_{\alpha\beta}(\mathrm{k}, \omega) \propto k_z^{-\frac{1}{2} - s_z}$ (or $\mathrm{T}_{\alpha\beta}(\mathrm{k}, \omega) \propto k_{\perp z}^{-\frac{1}{2} - s_{\perp z}}$), *i.e.*, the scaling in both y and $\perp y$ directions are not the same.

(iii) $s_y > 1$; $s_{\perp y} > 1$

$\Gamma(a)$ is gamma function and $K_\nu(z)$ is the modified Bessel function of the second order.

For instance, numerical experiments showed strong anisotropy along y axis for $k_{\min y}^{-1} = 100$, $k_{\min \perp y}^{-1} = 400$, $s_y = 0.4$, $s_{\perp y} = 0.75$ [3].

We used in the equation(10) the following variables

$$a = k_{\min,y}(\mathrm{v}_y \tau) \quad \text{and} \quad b = k_{\min,\perp y}(\mathrm{v}_{\perp y} \tau) \tag{12}$$

In the right hand of the equation (10) we can perform the following derivations

$$\frac{\partial f}{\partial p_y} = \frac{c^2}{\varepsilon}\frac{\partial f}{\partial \mathrm{v}_y} = \frac{c^2}{\varepsilon} ik_y \tau f \tag{13}$$

$$\frac{\partial f}{\partial p_{\perp y}} = \frac{c^2}{\varepsilon}\frac{\partial f}{\partial \mathrm{v}_{\perp y}} = \frac{c^2}{\varepsilon} ik_{\perp y} \tau f \tag{14}$$

Therefore the equation (10) may be rewritten as

$$\begin{aligned}
<\widehat{M}f^{(1)}> = {} & \frac{2^{-s_y+1}\pi e^2 c^4 \langle \mathrm{E}_{\mathrm{st}}^2\rangle ik_y \Gamma\left(\frac{s_y}{2}+1\right)}{\varepsilon^2 \Gamma\left(\frac{s_y}{2}-\frac{1}{2}\right)\Gamma\left(s_y+\frac{1}{2}\right)} [k_{\min,y}\mathrm{v}_\beta \tau]^{s_y} K_{s_y}[k_{\min,y}\mathrm{v}_\beta\tau] \times \\
& \times \left\{ \frac{2^{-s_y} k_{\min,y}\Gamma\left(\frac{s_y}{2}+1\right)}{\Gamma\left(\frac{s_y-1}{2}\right)\Gamma\left(s_y+\frac{1}{2}\right)} \int \tau^2 d\tau [2 s_y a^{s_y-1} K_{s_y}(a) - a^{s_y} K_{s_y+1}(a)] f + \right. \\
& \left. + \frac{2^{-s_{\perp y}} k_{\min,\perp y}\Gamma\left(\frac{s_{\perp y}}{2}+1\right)}{\Gamma\left(\frac{s_{\perp y}-1}{2}\right)\Gamma\left(s_{\perp y}+\frac{1}{2}\right)} \int \tau^2 d\tau [2 s_{\perp y} b^{s_{\perp y}-1} K_{s_{\perp y}}(b) - b^{s_{\perp y}} K_{s_{\perp y}+1}(b)] f \right\} + \\
& + \frac{2^{-s_{\perp y}+1}\pi e^2 c^4 \langle \mathrm{E}_{\mathrm{st}}^2\rangle ik_{\perp y} \Gamma\left(\frac{s_{\perp y}}{2}+1\right)}{\varepsilon^2 \Gamma\left(\frac{s_{\perp y}}{2}-\frac{1}{2}\right)\Gamma\left(s_{\perp y}+\frac{1}{2}\right)} [k_{\min,\perp y}\mathrm{v}_\beta \tau]^{s_{\perp y}} K_{s_{\perp y}}[k_{\min,\perp y}\mathrm{v}_\beta\tau] \times \\
& \times \left\{ \frac{2^{-s_{\perp y}} k_{\min,\perp y}\Gamma\left(\frac{s_{\perp y}}{2}+1\right)}{\Gamma\left(\frac{s_{\perp y}-1}{2}\right)\Gamma\left(s_{\perp y}+\frac{1}{2}\right)} \int \tau^2 d\tau [2 s_{\perp y} b^{s_{\perp y}-1} K_{s_{\perp y}}(b) - b^{s_{\perp y}} K_{s_{\perp y}+1}(b)] f + \right. \\
& \left. + \frac{2^{-s_y} k_{\min,y}\Gamma\left(\frac{s_y}{2}+1\right)}{\Gamma\left(\frac{s_y-1}{2}\right)\Gamma\left(s_y+\frac{1}{2}\right)} \int \tau^2 d\tau [2 s_y a^{s_y-1} K_{s_y}(a) - a^{s_y} K_{s_y+1}(a)] f \right\}
\end{aligned} \tag{15}$$

The right-hand part of the equation (9) was labeled $<\widehat{M}f^{(1)}>$ in the equation (15).

The equation (15) displays the complex dependence of collision integral on the strength of the magnetic field, on the correlation lengths and on the spectral indices of the power of the random magnetic fields. On the other hand, one must mention that we recover the exponential dependence of the collision integral on time and space through the modified Bessel function of the second order.

3. SOME ESTIMATIONS OF THE COLLISION INTEGRAL

The choice of different magnitudes of the correlation lengths and of the spectral indices leads to a strong anisotropy of the collision integral as we will show further. To do this we will adopt $s_y = 1/2$, $s_{\perp y} = 3/2$, $k_{\min y}^{-1} = 0.25 k_{\min \perp y}^{-1}$.

A proper study of the behavior of the collision integral requires spatial intervals of the order of the correlation length. Therefore we will perform a computation involving $k_{\min,\perp y}^{-1} = \mathrm{v}_{\perp \mathrm{y}}\tau$. In our paper we proposed a model of a random synchrotron emission of moving charged particles, this emission being also the random magnetic field that accelerates them. Hence one can adopt for relativistic particles the small cone of the emission which leads to the ratio of the velocities

$$\frac{\mathrm{v}_y}{\mathrm{v}_{\perp y}} \cong \gamma^{-1} \tag{16}$$

where v_y and $\mathrm{v}_{\perp y}$ are the velocities along y axis, respectively along a perpendicular direction to the y axis.

According to the assumptions made above the collision integral becomes

$$\begin{aligned}
< \widehat{M}f^{(1)} > = & -\frac{0.261\pi e^2 c^4 \langle \mathrm{E}_{\mathrm{st}}^2 \rangle i k_y}{\varepsilon^2} [k_{\min,y} \mathrm{v}_\beta \tau]^{1/2} K_{1/2} [k_{\min,y} \mathrm{v}_\beta \tau] \times \\
& \times \Big\{ -0.131 k_{\min y} \int \tau^2 d\tau [a^{-1/2} K_{1/2}(a) - a^{1/2} K_{3/2}(a)] f + \\
& +0.089 k_{\min \perp y} \int \tau^2 d\tau \left[3\frac{\sqrt{2\pi}}{2.71} - \sqrt{\frac{\pi}{2}} \frac{7}{2.71} \right] f \Big\} + \\
& + \frac{0.179\pi e^2 c^4 \langle \mathrm{E}_{\mathrm{st}}^2 \rangle i k_{\perp y}}{\varepsilon^2} \frac{\sqrt{2\pi}}{2.71} \times \\
& \times \Big\{ 0.089 k_{\min,\perp y} \int \tau^2 d\tau \left[3\frac{\sqrt{2\pi}}{2.71} - \sqrt{\frac{\pi}{2}} \frac{7}{2.71} \right] f + \\
& - 0.131 k_{\min,y} \int \tau^2 d\tau [a^{-1/2} K_{1/2}(a) - a^{1/2} K_{3/2}(a)] f \Big\}
\end{aligned} \tag{17}$$

A final form of the equation (17) may be obtained if one takes into account the form of the modified Bessel function of the second order

$$K_{1/2}(a)=\sqrt{\frac{\pi}{2}}\frac{\exp(-a)}{\sqrt{a}} \tag{18}$$

$$K_{3/2,\alpha}(a)=\sqrt{\frac{\pi}{2}}\frac{(a+1)\exp(-a)}{a^{3/2}} \tag{19}$$

But for its physical significance it is more interesting to evaluate the ratio of the two components of the integral collision

$$<\hat{M}f^{(1)}>_{\perp y}/<\hat{M}f^{(1)}>_{y}\leq 0,182 \tag{20}$$

This ratio depends exponentially on the Lorentz factor as

$$<\hat{M}f^{(1)}>_{\perp y}/<\hat{M}f^{(1)}>_{y}=0.494\exp(\frac{\sqrt{1-\gamma^{-2}}-0.25c^{-1}\mathrm{v}_{\perp y}-0.75}{0.25c^{-1}\mathrm{v}_{\perp y}-0.25} \tag{21}$$

The equation (20) establishes a relation between the two components of the collision integral. In order to accomplish our task, one can also estimate the collision integral by fitting the theoretical spectrum of the synchrotron emission

$$\begin{aligned}P(\omega)=&-\frac{e^2}{4\pi^2c^3}K_e\times\frac{\sin\int d\tau[\omega-\mathbf{kv}+\omega\varepsilon^{-2}<Nf^{(1)}/i>_{\mathbf{k},\omega,\mathbf{1}}]}{\varepsilon^2\omega\int d\tau<Nf^{(1)}/i>_{\mathbf{k},\omega,\mathbf{1}}}-\frac{e^2}{4\pi^2c^3}K_e\times\\&\times\frac{\cos\int d\tau[\omega-\mathbf{kv}+\omega\varepsilon^{-2}<Nf^{(1)}/i>_{\mathbf{k},\omega,\mathbf{1}}]}{\omega^2\left(\int d\tau<Nf^{(1)}/i>_{\mathbf{k},\omega,\mathbf{1}}\right)^2}\end{aligned} \tag{22}$$

to the data[6] , where

$$<Nf^{(1)}/i>_{\mathbf{k},\omega,\mathbf{1}}=\frac{\varepsilon^2<\hat{M}f^{(1)}/i>_{\mathbf{k},\omega,\mathbf{1}}}{\omega} \tag{23}$$

and $<\hat{M}f^{(1)}/i>_{\mathbf{k},\omega,\mathbf{1}}$ is the collision integral where we formally replaced f by 1 and pointed out its dependence on ω and $\mathbf{k}$.

In the equation (22) K_e is the constant of the power law spectrum

$$dN=K_e\varepsilon^{-\zeta}d\varepsilon \tag{24}$$

of the number of the particles. We performed a "visual" fitting to the averaged spectrum of Mkn 501, observed in April 1997, using the function

$$f(x)=\log[-10^{-x+1}\sin(10^{x+0.008})-10^{-2x+2}\cos(10^{x+0.008})] \tag{25}$$

where $x=10^{\log\omega}$ Hz.

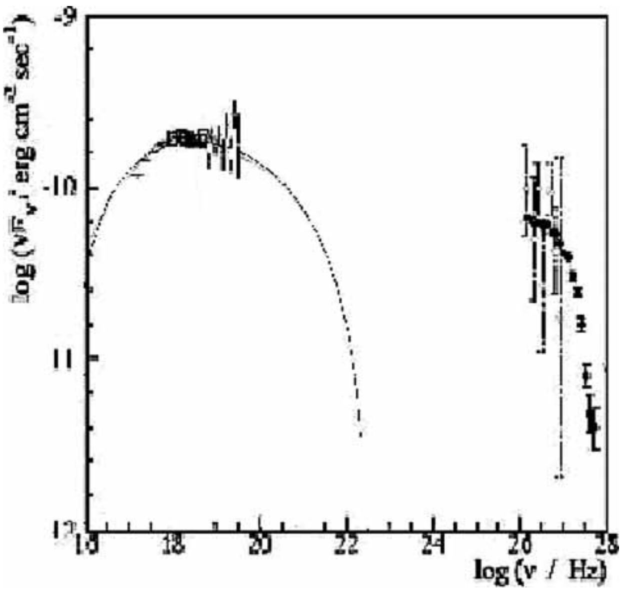

Fig. 1 – Our model (dotted line) fitting BeppoSAX (asterisks), RXTE (squares) and HEGRA (open circles) observations of April 7, 1997 (MJD 50545) and the HEGRA 1997 time-averaged spectrum scaled according to the detection rate of this day (full circles) [7].

In the case chosen in Fig. 1 ($\zeta = 5$), a crude estimation obtained by numerical simulation leads to $K_e \cong 2.76\times 10^{21}$ for energy in the range $\varepsilon = 10^{15}\,\text{eV}$. This corresponds also to

$$\int d\tau < Nf^{(1)} / i >_{\mathbf{k},\omega,\mathbf{1}} = 25.6 \quad \text{erg}^2 \cdot \text{s}. \tag{26}$$

Therefore, if we choose $\omega = \tau^{-1}$, which is the case for spatial “e-folding”, then

$$< \widehat{M} f^{(1)} / i >_{\mathbf{k},\omega,\mathbf{1}} \cong 10^{-5}.$$

4. DISCUSSIONS AND CONCLUSIONS

Although the spectrum of the synchrotron emission inferred in our model is not a strictly power law spectrum, still at high frequencies, it is a likely one. As we show in the present paper the time collision integral of the equation (9) plays an important role in order to establish the shape of the spectrum. Adopting the appropiate Lorentz factor in the collision integral one can fit the present data concerning gamma ray emissions of TeV blazars. This leads us to estimate the collision integral of a certain source and hence, using our model, to estimate the efficiency of the injection. If one can prove the efficiency of the injection, then our model might be a solution to the issue of the electromagnetic scenario of producing gamma ray bursts or high energy gamma rays in blazars.

The derivation and the estimations made to the collision integral in this paper proved the strong anisotropy of the mean free path when assuming the anysotropic correlation tensor of the random magnetic field. We applied our computation to the chaotic gun effect [6]. Here the particles are accelerated by usual free-space transverse modes and our result matches those cases where the astrophysical plasmas can be neglected. This might be for instance the case of the plasma compound of low energy electrons emitting in the radio frequencies domain and of high energy electrons emitting in the gamma ray frequencies domain. The code used by Argyris and Ciubotariu emphasized an unphysical behavior of the emitted radiation if one supposed that the same radiation is the engine of the acceleration: a linear polarization of the transverse field. The present paper proves the way to fit the analytical picture of the chaotic gun to the physical behavior of the emission of a charged particle. We allow the emission towards two directions perpendicular to the magnetic field and also to each other. But the emissions are of different strengths and one of them is much larger than the other one.

The estimations performed in this paper provide one of the factors of the distribution function of the charged particles involved in a chaotic motion. This function can contribute to computing the efficiency of the injection of the charged particles across the shock front of astrophysical plasma.

The efficiency is the ratio between the particles which cross the shock front and those which attempt to cross it. The latter ones obey the regular motion in the magnetic field. They are at thermal equilibrium or are accelerated by the chaotic gun effect, but do not cross the shock front. Hence a formal expression of the efficiency of injection is

$$\eta = \frac{n\int f_{\mathbf{k},\omega} dk_{\perp y}}{n\int f_{\mathbf{k},\omega} dk_{\perp y} + n\int f_{\varepsilon} d\varepsilon} = \frac{\int f_{\mathbf{k},\omega} dk_{\perp y}}{\int f_{\mathbf{k},\omega} dk_{\perp y} + \int f_{\varepsilon} d\varepsilon} \tag{27}$$

where n is the averaged number of particle in unit volume, $f_{\mathbf{k},\omega}$ is the distribution function which is the solution of the equation (9) and f_ε the Maxwell distribution.

Our result does not take into account the reconnection process and the absorption process due to pair production which may affect our results.

REFERENCES

1. I. N. Toptygin, *Cosmic rays in interplanetary magnetic fields*, D. Reidel Publishing Company, Doordrecht, Holland, p. 110–134, p. 84–91, p. 136 (1985).
2. Yu. P. Mel'nikov, 29th International Cosmic Ray Conference Pune) **00**, 101–104 (2005).
3. Y.-P. Zhao, G.-C. Wang, T.-M. Lu, *Diffraction from anisotropic random rough surfaces*, Phys. Rev. B, Vol. 58, No. 11 (1998).
4. J. Argyris, C. Ciubotariu, *Chaos, Solitons and Fractals*, 11, 1001 (2000).
5. V. L. Ginzburg, S. I. Syrovatskii, *Cosmic magnetobremsstrahlung*, ARA&A 3 279 (1965).
6. G. Dumitrescu, *Astrophysical Chaotic Gun Effect*, submitted to A&A.
7. H. Krawczinsky *et. al.*, astro-ph/0204229 A&A.