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WEBER'S COHERENT SCATTERING AND NEUTRINO DETECTION

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Abstract. Weber's proposal of detecting neutrinos by their coherent scattering on a stiff crystal is analyzed and the feasibility of the concept is assessed.

Key words: neutrino cross-section, coherent scattering, crystal detector.

One of the most intriguing and controversial isssue in modern physics is Weber's proposal of detecting neutrinos by their coherent scattering on a stiff crystal [1, 2]. We give here a brief review of this issue which may point towards the feasibility of such a detection procedure.

Neutrinos. Neutrinos (actually antineutrinos) can be obtained for instance from a tritium source at energy $\sim 10\,\mathrm{keV}$ with an activity of $3000\,\mathrm{Ci}$ ($1\,\mathrm{Ci} = 3.7\times 10^{10}\,\mathrm{Bq}$, $1\,\mathrm{Bq}$ = one decay per second). The nuclear reactors may generate antineutrinos with energies in the range of $1\,\mathrm{MeV}$ and flows (at near distance) of cca $10^{12}/\mathrm{cm}^2\cdot\mathrm{s}$. Another source is provided by Sun which gives $10^{11}/\mathrm{cm}^2\cdot\mathrm{s}$ neutrinos (in all flavours) with an average energy $\sim 300\,\mathrm{keV}$. Such sources have been used in Weber's experiments.

Notoriously, neutrinos interact extremely weakly with matter. For instance their cross-section with a nucleon is of the order of $\sigma_{nucl} \sim G^2E^2/\hbar^4c^4$, where $G \sim 10^{-4}\,\mathrm{MeV}\cdot\mathrm{fm}^3$ is Fermi's weak-interaction coupling constant ($1\,\mathrm{fm} = 10^{-15}\,\mathrm{m}$). For an energy $E = 1\,\mathrm{MeV}$ this cross-section is $\sim 10^{-44}\,\mathrm{cm}^2$.[1] Being given the available flows of neutrinos, their detection is extremely difficult. Large amounts of suitable fluids are used in order to detect them.[2] A more efficient method of neutrino detection is highly desirable.

[1] For comparison, nuclear cross-sections are of the order of $1\ \mathrm{barn} = 10^{-24}\ \mathrm{cm}^2$.
[2] Neutrinos were discovered by such methods [3, 4].

A first hint toward such a method is provided by the nuclear physics. The nucleons are held together by strong forces in a nucleus, so it is conceivable that if a certain amount of energy is transmitted by a neutrino to a nucleon, then it is in fact transferred to the whole nucleus which will suffer the collision as a whole. The cross-section is the square of the sum of the scattering amplitudes arising from all the nucleons, so it is conceivable that they add coherently to give a cross-section $\sigma_0 \sim (G^2E^2/\hbar^4c^4)A^2$, where A is the mass number of the nucleus. Actually, the proton scattering amplitude are of opposite sign with respect to the neutron scattering amplitude and it is also diminished by the (Weinberg) mixing angle θ (~ 0.2). The nuclear cross-section reads then $\sigma_0 \sim (G^2E^2/\hbar^4c^4)[N - Z(1-4\sin^2\theta)]^2$, where N stands for the number of neutrons and Z for the number of protons. The cross-section goes like the square of the number of scatterers which enhances it appreciably in comparison with an incoherent scattering where the cross-section is thought to go like the first power of the number of scatterers [1, 2]. The possibility of a nuclear coherent scattering is here supported by the neutrinos' wavelength which is much larger than the nuclear dimensions (but it could be much shorter than atomic distances).

This observation opens up the way of investigating the coherent scattering of neutrinos on a larger scale, like in a stiff crystal. It is worth noting that the only condition for coherent scattering seems to be the stiffness of the target crystal.

Cross-section. With usual notations the rate of transition is given by the well-known Fermi's "golden rule" (Born approximation)

$$dw = (2\pi/\hbar)|H_1|^2\delta(E - E')d\nu' , \tag{1}$$

where $d\nu' = V p'^2 dp' d\Omega/(2\pi\hbar)^3$. H_1 in equation (1) denotes the matrix element

$$H_1 = \frac{1}{V}\int d\mathbf{r}\cdot v(\mathbf{r})e^{i(\mathbf{p}-\mathbf{p}')\mathbf{r}/\hbar} . \tag{2}$$

of the interaction $v(\mathbf{r})$. We assume that the final state is that of the scattered particle with $E' = cp$, as for elastic scattering, and integrate over final energies.[3] With the same usual notation we get

$$dw = \frac{2\pi}{Vc^3\hbar(2\pi\hbar)^3}\left|\int d\mathbf{r}\cdot v(\mathbf{r})e^{i(\mathbf{p}-\mathbf{p}')\mathbf{r}/\hbar}\right|^2 E'^2 d\Omega . \tag{3}$$

Now we assume further that the interaction is given by a periodic array of δ-functions,

$$v(\mathbf{r}) = ua^3\sum_i \delta(\mathbf{r} - \mathbf{r}_i), \tag{4}$$

[3] Or notice that $dp'\delta(E - E') = dp'/dE' = 1/c$.

as for a crystal, where u is the strength of the interaction and a is a characteristic length of the elementary cell, and get

$$dw = \frac{2\pi u^2 a^6}{Vc^3\hbar(2\pi\hbar)^3}\left|\sum_i e^{i(\mathbf{p}-\mathbf{p}')\mathbf{r}_i/\hbar}\right|^2 E'^2 d\Omega, \tag{5}$$

with the well-known form-factor

$$F(\mathbf{p}-\mathbf{p}') = \sum_i e^{i(\mathbf{p}-\mathbf{p}')\mathbf{r}_i/\hbar} \tag{6}$$

of the crystal in the modulus brackets. The flow of the incoming particle is $\Phi = c/V$ (for one particle in volume V), so the differential cross-section $d\sigma = dw/\Phi$ is given by

$$d\sigma = \frac{2\pi u^2 a^6}{c^4\hbar(2\pi\hbar)^3}\left|F(\mathbf{p}-\mathbf{p}')\right|^2 E'^2 d\Omega\,. \tag{7}$$

We can write $\sigma_0 = [2\pi u^2 a^6 E'^2/c^4\hbar(2\pi\hbar)^3]$ as coming from a single cell and

$$d\sigma = \sigma_0\left|F(\mathbf{p}-\mathbf{p}')\right|^2 d\Omega\,. \tag{8}$$

For neutrinos the interaction potential is replaced by relativistic currents and, up to numerical factors, the matrix element of the interaction amounts to replacing ua^3 in (7) by Fermi's weak-interaction coupling constant G. We get for σ_0 the estimation $\sigma_0 \sim G^2E'^2/c^4\hbar^4$ as given above for one nucleon (or one nucleus, or one elementary cell in the crystal, with suitable modifications).

The rate of collisions $dw = (1/\tau)d\Omega$, where τ is the collison time, is obtained from (1) and (8) as

$$dw = \Phi d\sigma = \Phi\sigma_0\left|F(\mathbf{p}-\mathbf{p}')\right|^2 d\Omega\,. \tag{9}$$

For one particle the flow Φ in (9) is numerically equal to unity. One can see that the collision time can be extremely long, for instance of the order of $\tau \sim 1/\sigma_0 \simeq 10^{44}$ s for the elementary cross-section σ_0, which shows again that the detection of the neutrinos is extremely difficult indeed.

Evaluation of the form factor. The form factor in equation (6) consists of three components: $F(\mathbf{p}-\mathbf{p}') = F_1(p_1-p_1')F(p_2-p_2')F(p_3-p_3')$, where 1, 2, 3 denote the axes of coordinates. Their evaluation is straightforward. We get

$$F_1 = \sin[(p_1-p_1')N^{1/3}a/\hbar]/\sin[(p_1-p_1')a/\hbar]\,, \tag{10}$$

and similar expressions for $F_{2,3}$, where N is the number of cells in the crystal.[4] As is well-known, in the limit of large N the form factor given by (10) reads

$$F = \pi \sum_G \delta[(\mathbf{p} - \mathbf{p}')/\hbar + \mathbf{G}], \tag{11}$$

where $\mathbf{G}$ are the reciprocal vectors of the lattice. This is the well-known Laue's diffraction on the lattice. Equation (11) tells that summation are extended over those $\mathbf{G}$'s which satisfy Bragg's condition $\mathbf{p}' = \mathbf{p} + \hbar\mathbf{G}$, $p = p'$, *i.e.* $\hbar G = 2p\sin\theta/2$, where θ is the scattering angle between $\mathbf{p}$ and $\mathbf{p}'$; they are given by the well-known Ewald-sphere's construction. For neutrinos they range from a central peak to a great number of by-side diffraction spots, according to the extent of the $p = p'$, *i.e.*, of neutrino's energy. For high energies this number of peaks is given approximately by $(p/\hbar g)^2 \sim (ap/\hbar)^2$, where $g \sim 1/a$ is the elementary reciprocal vector.

According to (10), the maximum value of the form-factor in a diffraction peak is $F_{max} = N$. Therefore, the corresponding maximum cross-section is $d\sigma_{max} = \sigma_0 N^2 d\Omega$. The N^2-dependence is a general characteristic of the diffraction which makes it to be called coherent diffraction, or coherent scattering. This dependence is not affected by phonons, or deffects, or additional modulations of the lattice, etc, though the diffraction figure may be affected correspondingly in such cases; long-range phonons brings an additional contribution, called the diffuse scattering, which is proportional to N; but the scattering at the centre of the diffraction spot remains proportional to N^2. For a dispersion of the incoming particles either in energy or direction or both the by-side peaks of diffraction decrease in intensity with respect to the main peak [5].

One can see from (10) that the spread of a diffraction peak has a linear size $\delta p_1' \sim \hbar/aN^{1/3}$, so it shows an area $\sim \hbar^2/a^2N^{2/3}$ within a solid angle $\Delta\Omega \sim \hbar^2/p^2a^2N^{2/3}$. It follows that the cross-section for one peak of diffraction is

$$\sigma_{one-peak} \simeq \sigma_0 N^2 \cdot (\hbar^2/p^2a^2N^{2/3}) \sim N^{4/3}. \tag{12}$$

This cross-section must be multiplied by the number of total peaks, which is $\sim (pa/\hbar)^2$, as discussed above. It follows that the total cross-section can be estimated as

$$\sigma \simeq \sigma_0 N^{4/3}. \tag{13}$$

It would appear that for large N this cross-section would increase indefinitely. However, it is subjected to the condition that it must be much smaller than the area

[4] We assume the crystal spatially uniform in size; for special geometries, the form-factor is different from the one given by (10), as expected.

of the sample perpendicular to the incident particles; the latter goes like $N^{2/3}$, so N is in fact limited by this condition, though fairly weakly ($N \ll (a^2/\sigma_0)^{3/2}$). Otherwise, the perturbation theory employed in deriving the cross-section wouldn't hold anymore. It is sometimes said that σ goes like N; this is true, however, for two-dimensional samples, where the spread of the spot is given by $\delta p_1' \sim \hbar/aN^{1/2}$.

For an amorphous solid (or for a gas, a liquid) the form-factor in equation (6) reduces to the central peak corresponding to **G** = 0. The total cross-section is then given by (12)

$$\sigma_{amorphous} \simeq \sigma_0 \cdot (\hbar/pa)^2 N^{4/3}, \tag{14}$$

and one can see that it is reduced by the square of the factor $\hbar/pa$ in comparison with a crystal.

Giving a momentum to the crystal. If the scatterers are not fixed, they receive a momentum and an energy from the incoming particles, the collision is inelastic, and these final states of the scatterers must be included explicitly in equation (1). Suppose that in an elementary act of collision between a particle and an atom (nucleus, elementary cell) in the crystal the latter receives a momentum $\delta\mathbf{p}$ and an energy $\delta\varepsilon$, such that $\mathbf{p}' = \mathbf{p} - \delta\mathbf{p}$ and $\delta\varepsilon = c(p' - p)$. The incoming particle loses correspondingly an energy $\delta\varepsilon$ and a momentum $\delta\mathbf{p}$. This momentum and energy transfer is shared by various motions in the target. A solid is a collection of interacting atoms. Mainly, its motion is governed by the relative coordinates of the interacting atoms and the coordinates of the center of mass of the sample (we leave aside the rotations of the sample, as well as other possible internal motions). The former give the vibrations of the solid, *i.e.*, the phonons, the latter give the motion of the sample as a whole. The motion of the sample as a whole proceeds in a well-defined mechanical state, and we may consider that its states weight in the collision rate is unity; *i.e.* the initial position $\mathbf{r}_0$ and momentum $\mathbf{p}_0$ of the solid are such that $\delta\mathbf{r}_0\delta\mathbf{p}_0/(2\pi\hbar)^3 = 1$. Similar statistical weights must be considered for the excitation of the phonons. Equation (7) is then maintained, for the scattering of the incoming particle, except for $\mathbf{p} - \mathbf{p}'$ being replaced by $\mathbf{p} - \mathbf{p}' + \delta\mathbf{p}$ and $E' = E - \delta\varepsilon$. It remains therefore to estimate the form-factor

$$F = \sum_i e^{i(\mathbf{p}-\mathbf{p}'+\delta\mathbf{p})\mathbf{r}_i/\hbar} \tag{15}$$

and to average over phonon states.

The question then arises: under what conditions such a momentum and energy transfer is possible to the solid as a whole? According to the quantum mechanics, as Weber explains, [1] this may be quite a legitimate channel of reaction, under certain conditions. For *X*-rays diffraction, for instance (or similar

incoming radiation), the electrons acted by the *X*-rays do not interact directly with each other (they radiate instead); they are indepedent scatterers and the momentum transferred to one of them is not shared with the others. These scatterers are incoherent and the motion of the atomic nuclei affects the *X*-ray scattering in a peculiar way, as discussed above. On the contrary, since neutrinos interact with the nuclei (or their quark constituents), these nuclei may move coherently, providing the crystal has a high stiffness, like the strong-force interacting nucleons in the atomic nucleus under the same action of the neutrinos.

First, we note that the energy transfer must be written as $\delta\varepsilon = \varepsilon_0 + \varepsilon_{ph}$, where ε_0 is the energy transferred to the target as a whole and ε_{ph} is the energy of the excited phonons. The incoming particle can give a maximum momentum $2p$ to the target as a whole, assuming that there is no phonon excitation. Consequently, the target may take an maximum energy $\varepsilon_0 = 2p^2/M$, where M is the mass of the target. This makes $\varepsilon_0 = 2(cp)^2/Mc^2 = 2E_{inc}^2/Mc^2$, where $E_{inc} = cp$ is the energy of the incident particle. It is an extremely small energy, so we may neglect it in equation $\delta\varepsilon = \varepsilon_0 + \varepsilon_{ph} \simeq \varepsilon_{ph}$, where the average energy transferred to the phonons $\left\langle \varepsilon_{ph} \right\rangle$ is of the order of the Debye temperature Θ of the solid. One of the highest values of Debye temperatures is $\Theta \sim 2000$ K (0.2 eV), for diamond. The transfer of energy to the excited phonons is a statistical proces, so it takes place with the maximization of the entropy. Therefore, the probability distribution for such a process is of the form $\sim e^{-\lambda\varepsilon_{ph}}$. The constant λ is obtained by requiring $\left\langle \varepsilon_{ph} \right\rangle = \Theta$, so we get the probability distribution $\Theta^{-1}e^{-\varepsilon_{ph}/\Theta} \simeq \Theta^{-1}e^{-\delta\varepsilon/\Theta}$. This is the statistical weight which must be included in the transition rate in order to account for the energy and momentum transfer to the target.[5] However, since the dependence on $\delta\varepsilon$ of this transition rate is very weak, the effect of averaging with this distribution is practically irrelevant. The analogy with the Mossbauer effect is obvious. There, the energy transfer is written usually $\delta\varepsilon = mv^2/2$, where m is the atomic mass. Indeed, this is the energy transferred to the excited phonons in an elementary act of impact. We can see that for an infinite stiffness $(\Theta \to \infty)$ this probability tends to unity.

From the conservation of the energy $\delta\varepsilon = c(p - p')$ and $\delta\varepsilon/cp \sim \Theta/cp \ll 1$, it follows that the modulus of $\mathbf{p}$ is practically unchanged in such a collision process. This amounts to $\delta p^2 - 2\delta\mathbf{p}\cdot\mathbf{p} = 0$. It follows that the momentum transfer runs from

[5] A correction term can be included in this probability, which gets the form $\sim \exp[-(\delta\varepsilon/\Theta)(1 + \Delta\varepsilon/\Theta)]$ for $\Delta\varepsilon/\Theta \ll 1$, where $\Delta\varepsilon$ comes from the change in the thermal equilibrium; since $\delta(\Delta\varepsilon) = T\delta S$, where T is the temperature and δS is the entropy, and since $\delta S \sim \delta T$ during this change of equilibrium, we get $\Delta\varepsilon \simeq T^2/\Theta$, up to some numerical factors.

zero to $2p$, with an average value $\delta p \sim p$. The momentum transferred to the phonons packet during the elementary act of impact is of the order of $\hbar/a$ (for a crystal the elementary reciprocal vector). This momentum is much smaller than the total average momentum transfer $\delta p \sim p$, so the latter goes almost entirely to the target as whole, *i.e.* $\delta p \sim p \sim p_0$. At the same time, the average momentum imparted to the phonons is vanishing. Contrary to the energy transfer, which goes mainly to the phonons, the momentum transfer goes mainly to the solid target.

It is worth noting that the elastic scattering is also included in such a description, though its contribution is extremely small, precisely due to the fact that the target may move as a whole.

There is also a certain process related to the disruption of an atom, or group of atoms, from the target, under the action of the incoming particle. The occurence of such a process (which produces damages to the solid target) is related to the resonance movements of the atoms in the target. The characteristic frequency of such an individual movement is of the order $\Theta/\hbar$; the characteristic frequency of the collisions is w, as given by equation (9).[6] The disruption of individual atoms occurs therefore only for $w \sim \Theta/\hbar$, a process with a very low rate due to the mismatch between the two frequencies. Actually, the impact force of the incoming particle contains in fact all the frequencies,[7] so there is always a certain probability of producing such disruptions. For an infinite stiffness $(\Theta \to \infty)$ such processes are circumvented. Therefore, it is not necessarily to have a very stiff target (nor a perfect one), but, if softer, the collisions will degrade it sooner.

Coherent form factor. The form-factor given by (15) can be evaluated as the one in equation (10). Its maximum value is still $F_{max} = N$. Ewald's sphere does not change too much, since the energy transfer is extremely small in comparison with the incident energy $(p \sim p')$. The momentum transfer to the crystal is $\mathbf{p}_0$, which may have various values such as $p_0 \sim p$ on the average. Therefore $\mathbf{p}'$ must be so as to satisfy first $\mathbf{p}' = \mathbf{p} - \mathbf{p}_0$ and $p' \sim p$. This is a certain point on Ewald's sphere. Then, we must allow any $\mathbf{p}'$ on this sphere to move by reciprocal vectors $\hbar\mathbf{G}$, such as the new $\mathbf{p}'$ to remain on the sphere. This amounts to changing the vector $\mathbf{p}_0$ by crystal momenta $\hbar\mathbf{G}$. It follows that the $\hbar\mathbf{G}$-momentum of the crystal is already incorporated in the crystal momentum $\mathbf{p}_0$, as it is well-known. Indeed, we know that the crystal wavefunction $\exp(i\mathbf{p}_0 \sum_i \mathbf{r}_i/\hbar)$ is invariant under a $\hbar\mathbf{G}$-change in the crystal momentum $\mathbf{p}_0$. Therefore, we may conclude that we have a continuous distribution of peaks of magnitude $F_{max} = N^2$ which subtend now the

[6] It is given by $w = \Phi\sigma$, where Φ is numerically set equal to unity.
[7] Its frequency content depends on the shape of the temporal pulse.

solid angle $\Delta\Omega \simeq 1$. Indeed, the dispersion in (15) is $\delta p_1' \sim (p_0)_1$, etc, so the solid angle is given by $\delta p'^2/p^2 \sim p_0^2/p^2 \sim 1$. It follows that the total cross-section is given now by its coherent form

$$\sigma \simeq \sigma_0 N^2 . \tag{16}$$

This cross-section is appreciably enhanced by the coherence factor N^2, in comparison with the elastic cross-section given by (13).[8] It may be called the coherent cross-section.

This is not so anymore for an amorphous solid. In that case we have only a central peak, and the above cross-section must be reduced by the number $(ap/\hbar)^2$ of absent peaks, where a is the mean inter-particle spacing.[9]

The same result can be obtained by a different reasoning. According to (10) and (15), for a fixed $\delta\mathbf{p} = \mathbf{p}_0$, the diffraction figure consists of $(ap'/\hbar)^2$ peaks, as many as allowed reciprocal vectors, each of solid angle $(\hbar/ap')^2/N^{2/3}$ and maximum value N^2. Therefore, the corresponding cross-section is $\sim N^2/N^{2/3}$. However, the crystal moves around, and $\mathbf{p}_0$ changes; the "diffraction figure" is now smeared out over a range $\sim p_0$; for $p_0 \sim p$, p' the vector $\mathbf{p}_0$ acquires $\sim N$ positions in space, and $N^{2/3}$ in the subtended solid angle.[10] Therefore, the total coherent cross-section is $\sim (N^2/N^{2/3})N^{2/3} \sim N^2$. For only one peak, as for an amorphous solid, it is reduced by number $\sim (ap'/\hbar)^2 \sim (ap/\hbar)^2$. In general, these cross-sections must contain a factor $(p_0/p)^2$, which is unity in this case.

For an incident energy $cp = 1$ MeV, $\sigma_0 \sim 10^{-44}\,\mathrm{cm}^2$ and $N \sim 10^{23}$ for one mol, we get $\sigma \simeq 10^2\ \mathrm{cm}^2$, which represents an appreciable increase in the cross-section. It may even overpass the condition of applicability of the perturbation theory regarding the area of the crystal, as discussed above. However, it is conceivable that the collision processes are limited to the first layers of the crystal

[8] Though it is subjected to the same limitation $\sigma \ll S$, where S is the area of the crystal perpendicular to the incoming momentum $\mathbf{p}$, as discussed above. This condition amounts to $N \ll (a^2/\sigma_0)^{3/4}$ for a crystal spatially uniform in size. As expected, this is a more restrictive condition than the one implied by the elastic scattering. For an elementary cross-section $\sigma_0 = 10^{-44}\ \mathrm{cm}^2$ it amounts to a maximally allowed value of $N \sim 10^{21}$, which restricts, practically, the use of these formula to cca one tenth of mol.

[9] For an energy $cp = 1$ MeV this factor amounts to $(ap/\hbar)^2 = (a/\lambda)^2 \sim 10^{-4}$, where $\lambda \sim 10^{-2}$ Å (10^3 fm) is the wavelength of the incident radiation.

[10] In the crystal wavefunction $\exp(i\mathbf{p}_0 \sum_i \mathbf{r}_i/\hbar)$ the crystal momentum $\mathbf{p}_0$ is given by $(p_0)_1 = (2\pi\hbar/N^{1/3}a) \times integer$, etc, such that the wavefunction is invariant under a translation $(r_i)_1 \to (r_i)_1 + a$, etc. Note that under such a translation $\left(\sum r_i\right)_1$ changes by $N^{1/3}a$.

facing the incoming particles, so a reduction factor of $\sim N^{1/3}$ is reasonable in equation (12) (though we neglect the multiple scattering).[11] We get then $\sigma \simeq 10^{-6}$ cm^2 (for an incident energy 1 MeV). For 0.1 mol (which seems to be the mass of the crystal used in Weber's experiments; its area was ~ 25 cm^2) we get from (16) $\sigma \sim 1$ cm^2, which agrees with the measured value.[12]

Force acting upon the crystal. The force acting upon the crystal is the rate of the momentum transfer; the latter is of the order $p_0 \simeq p$, and the rate of collisions is given by $\Phi\sigma$, where Φ is the incoming flow of particles. We get therefore for the average force

$$F \sim p\Phi\sigma. \tag{17}$$

For an incident energy 1 MeV the momentum is $p \simeq 10^{-17}\,\mathrm{dyn}\cdot\mathrm{s}$. For a flow $\Phi \simeq 10^{12}/\mathrm{cm}^2\cdot\mathrm{s}$ the force may range from $F \sim 10^{-3}\,\mathrm{dyn}$ ($\sigma \sim 10^2\,\mathrm{cm}^2$) to $F \sim 10^{-9}\,\mathrm{dyn}$ (($\sigma \sim 10^{-4}\,\mathrm{cm}^2$). For comparison we give the gravitational force acting between two masses each of 100 g placed at distance 1 cm apart from each other: $F_{grav} \simeq 10^{-6}\,\mathrm{dyn}$. Weber's experiments report a force $\sim 10^{-5}\,\mathrm{dyn}$ for approximately 0.1 mol of crystal (saphire, $\Theta \sim 1000$ K) for all types of neutrinos (10 keV, 300 keV, 1 MeV), which is rather strange as both the cross-section and the force vary with energy (a factor $\sim E^3$ in (17)). The flow, the superficial scattering and, especially, the variations in the crystal masses employed may be so as to correspond to such results.[13] For $\sigma = 1$ cm^2 the force given by (17) is $F = 10^{-5}$ dyn for an incident energy 1 MeV.

The rate of collisions for one unit of incident flow is $w \sim 10^2 - 10^{-6}\,s^{-1}$ (σ for 1 MeV); the disruption rate of an atom in crystal is $\sim \Theta/\hbar \simeq 10^{14}\,\mathrm{s}^{-1}$ (for $\Theta = 1000$ K). As one can see, the impact is practically adiabatic, which makes more likely the momentum transfer to the crystal as a whole.

Criticism. Weber's theory and has been criticized by various authors [6–11].

The criticism made by Ho [6] focuses on Weber's argument that a necessary condition for the coherent scattering would be a particle wavelength larger than the Debye-Waller wavelength of the crystal. Ho notes then correctly that such a condition can also be met for neutrons, where a coherent scattering does not seem to have been observed. However, Weber's argument is valid for a rigid array of

[11] As if the layers scattered independently, which is reasonable for short wavelengths. A similar reduction in the incoherent elastic scattering takes the cross-section from $\sigma \sim N^{4/3}$ into $\sigma \sim N$, hence the popular N-dependence of the incoherent scattering.

[12] Weber's estimation is somewhat larger.

[13] In addition, we must note that we use σ_0 for one nucleon; we have, in fact, to employ the value coresponding to the elementary cell of the crystal, which may exhibit an appreciable enhancement.

scatterers, where the particle wavelength is large compared with the lattice spacing. In order to have a coherent scattering in crystals the collision time must be much longer than the time needed for phonons to attain the statistical equlibrium (adiabatic scattering). Under these conditions the transferred energy can be shared by all the scatteres and full advantage can be taken of the stiffness of the crystal. The weak forces present in the neutrino interaction can ensure such a long collision time, in contrast with neutron scattering where the strong interaction force gives rise to a more rapid transfer of energy and momentum which may lead to an incoherent scattering.

The criticism made by Bertsch and Austin [7] is based on the correct observation that the (linear) width of a Bragg peak goes like $N^{-1/3}$. These authors conclude then that one cannot have a large-angle diffraction far away from the Bragg peaks. This is valid for elastic scattering. For inelastic scattering on a stiff crystal (coherent scattering) the Bragg peaks coalesce (are smeared out) in one big peak of a finite solid angle ~ 1, because the momentum transferred to crystal in this case is determined up to a reciprocal vector of the crystal. Further on, Bertsch and Austin bring into discussion the scatering of long-wavelength *X*-rays by atomic nuclei, which would meet Weber's criterion for coherent scattering (Born approximation and recoiless scattering), and still no special enhancement is observed in their cross-section. However, the main objection to this criticism would be that the electromagnetic interaction gives rise to radiation from the individual scatterers, which would prevent the stiffness of the crystal to work.

Butler [8] criticized Weber's reasoning on a technical aspect regarding the construction of the excitation oeprator of the final state of the crystal. The point in controversy is whether it is legitimate to include the momentum transfer for the crystal as a whole, as Weber did, or to use the momentum transfer for all the atoms viewed as independent scatterrers. If we accept the possibility of a coherent scattering underwent by a stiff crystal then it is reasonable to use the momentum transfer for the crystal as a whole.

Another criticism was brought forward by Casella [9] and by Aharonov *et al.* [10], who pointed out that the force acting upon the crystal in scattering is proportional to the number of scatterers N and not N^2. This force is proportional to $\delta p \cdot \sigma$, where δp is the momentum transfer and σ is the cross-section. At its turn, the cross-section is proportional to the maximum value ~ N^2 of the scattering peak multiplied by the solid angle $\Delta\Omega$. In an incoherent scattering $\Delta\Omega \sim N^{-2/3}$ so the cross-section goes like $\sigma \sim N^{4/3}$. Similarly, the momentum transfer (related to the solid angle) is proportional to $N^{-1/3}$, so the force goes indeed like N. Aharonov *et al.* [10] proved this technically by deriving a sum rule for the Born scattering. For a coherent scattering however $\delta p \sim \Delta\Omega \sim 1$, so the force goes like N^2 and the above arguments do not apply.

Lipkin [11] reiterated the basic theoretical structure of the incoherent scattering and refuted Weber's arguments on this basis. In particular, he emphasized the requirement of having a particle wavelength much longer than the bulk size of the scatterer in order to have a coherent scattering (which is not the case in Weber's experiments). As said above this is not a necessary condition for inelastic scattering on stiff crystals.

In any case, the coherent scattering is an intriguing process, just on the limit of applicability of the perturbation calculation, as we have seen, and it deserves to be pursued experimentally.

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