Romanian Reports in Physics, Vol. 60, No. 2, P. 353–359, 2008

FERROMAGNET-SUPERCONDUCTOR JUNCTION

OANA ANCUTA DOBRESCU,[1] L. C. CUNE[2], M. APOSTOL[2]

[1] *Department of Physics, University of Bucharest*
[2] *Department of Theoretical Physics, Institute for Atomic Physics, Magurele–Bucharest MG-6, P.O. Box MG-35, Romania*

(Received January 14, 2008)

Abstract. The ferromagnet-superconductor junction is analyzed, as motivated by a previous investigation of a possible magnetization control of the electric flow through such a junction. The formation of a perfect contact at such junctions is characterized, and reasons are given that such a contact may support a ballistic regime of transport for the ferromagnetic sample, while the superconducting sample being subject to a diffusive transport. The size of a perfect contact bewteen two solids is estimated, as well as the length of the spin flip and the length scale of the gap damping at the ferromagnet-superconductor junction.

Key words: solid surface, contacts, junctions, interface.

PACS: 85.30.Hi; 73.25.+i; 31.50.Gh; 68.35.Fx

According to the quasi-classical description of matter aggregation[1] the cohesion of a solid is governed by a self-consistent potential $\varphi(\mathbf{r})$; its simplest form is given by

$$\varphi(\mathbf{r}) = \sum_i \frac{z_i^*}{|\mathbf{r}-\mathbf{R}_i|} e^{-q|\mathbf{r}-\mathbf{R}_i|} \quad , \tag{1}$$

where z_i^* denote effective ionic charges (in atomic units), $\mathbf{R}_i$ are the ionic positions and q is a screening wavevector; the screening wavevector is estimated as $q \simeq 0.77 z^{*1/3}$ (in atomic units), where $z^* = \overline{z_i^*}$ is the average effective charge; one may introduce an average inter-ionic separation a, and the product qa is estimated as $c = qa \simeq 2.73$; the electron density n is related to the self-consistent potential through $4\pi n = q^2\varphi$; the average potential as given by (1) is $\varphi = 4\pi z^*/a^3q^2$. This quasi-classical description is refined in the next step of this theory by so-called quantum corrections, which lead, among others, to a shift of the Fermi level toward

[1] L. C. Cune and M. Apostol, **Metallic Binding**, *apoma*, MB (2000).

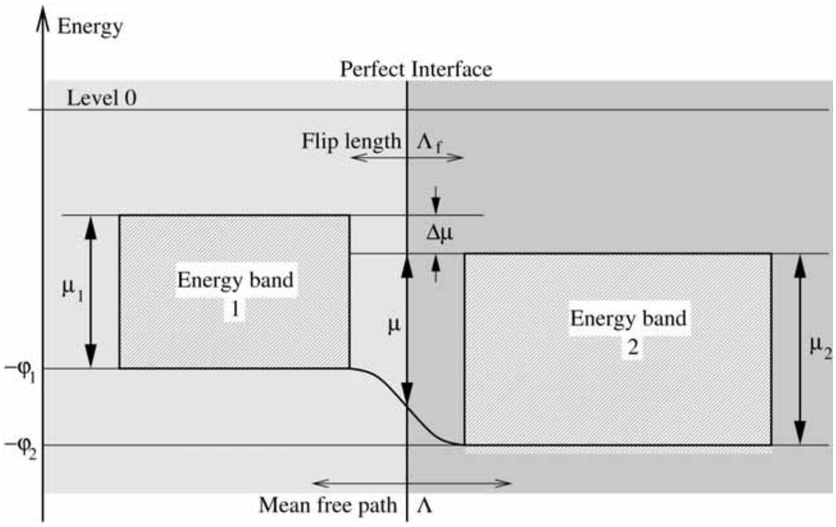

Fig. 1 – Two solids with a perfect contact.

negative values (at this level of approximation the Fermi level is placed at zero energy, while the chemical potential of the electrons is $-\varphi$).

Let us consider a semi-infinite solid with a plane free surface at $x = 0$; the corresponding average potential as given by (1) is

$$\begin{aligned}\varphi(x) &= \frac{4\pi z^*}{a^3q^2}\left(1-\frac{1}{2}e^{qx}\right), \quad x<0,\\ \varphi(x) &= \frac{2\pi z^*}{a^3q^2}e^{-qx}, \quad x>0;\end{aligned} \tag{2}$$

one can see that there exists a change

$$\begin{aligned}\delta\varphi(x) &= -\frac{2\pi z^*}{a^3q^2}e^{qx}, \quad x<0,\\ \delta\varphi(x) &= \frac{2\pi z^*}{a^3q^2}e^{-qx}, \quad x>0\end{aligned} \tag{3}$$

in the self-consistent potential at the surface, in comparison with the potential in the bulk, so that a corresponding change $\delta n(x) = q^2\delta\varphi(x)/4\pi$ in the electron density appears at the free surface; the electrons spill over the surface and give rise to a charge double layer; the surface is depleted by $z^*/2a^3q$ electrons per unit area; the electric field arising from the sharp surface is compensated by the dipole field of the surface charge redistribution. The energy (per unit area of the surface) associated with this double layer can be estimated as

$$\delta E = -\frac{1}{2}\int dx\cdot\delta\varphi\delta n = -\frac{q^2}{8\pi}\int dx\cdot(\delta\varphi)^2 = \\ = \frac{q^2}{4\pi}\int dx\cdot x\delta\varphi(\partial\delta\varphi/\partial x) = -\int dx\cdot x\delta nE = -\frac{\pi z^{*2}}{2a^6q^3}\ , \tag{4}$$

and one can see that it originates in the dipole field of intensity $E = -\partial\delta\varphi/\partial x$; it is a surface energy. It is also worth noting that the surface double layer extends over distances of the order of the atomic distances a (~ $1/q$), and the surface ions relax by $\delta a \sim 1/4q \sim 0.1a$, for the surface energy given above. The surface energy (4) is a second-order contribution in the potential (and electron density) change, and, consequently, it is comparable to the uncertainy in quasi-particle energy; it contributes therefore to the surface-scattering lifetime of the quasi-particles. Indeed, the surface energy (4) can also be written as $\delta E = -\pi n^2/2q^3$, where $n = z^*/a^3$ is the electron concentration, or $\delta\varepsilon = \pi n/2q^3d$ an energy per electron, where d denotes the length of the sample; it can be compared with the Fermi energy $\mu \sim \varphi$ of the electron liquid, leading to $\delta\varepsilon/\mu = (1/8c^2)(a/d)$, and a surface-scattering lifetime $\tau_s \sim (\hbar/\mu)(d/a)$; this is Casimir's finite-size (boundary scattering) lifetime of the electron quasi-particles.[2] The so-called Sharvin resistance of a micro-bridge is Casimir's resistance in a disguised form.[3] Typically, for conductors, the electron-electron uncertainty in electron energy is of the order of $\delta\varepsilon \sim (T/\mu)^2\mu$ (or $(\Delta\varepsilon/\mu)^2\mu$), while the electron-phonon uncertainty in electron energy can be represented as $\delta\varepsilon \simeq T/F$, $F = (M/m)(\hbar\omega_D/\mu)^2$, where M is the ionic mass and ω_D is the Debye frequency.[4] Except for very low temperatures, or atomic-size samples, the boundary scattering lifetime is very long, and, therefore, it contributes little to transport. It is also worth noting the work function of the solid

$$W = -\varphi(+\infty) + \varphi(-\infty) = -\int dx\cdot\partial\delta\varphi/\partial x = \\ = \int dx\cdot x(\partial^2\delta\varphi/\partial x^2) = -4\pi\int dx\cdot x\delta n = \\ = q^2\int dx\cdot x\delta\varphi = 4\pi z^*/a^3q^2 = \varphi , \tag{5}$$

as expected, where Poisson's equation $\partial^2\delta\varphi/\partial x^2 = 4\pi\delta n = q^2\delta\varphi$ is employed. A similar exponential decay at the surface is suffered by the quantum corrections to the electron energy levels, in particular by the energy band structure.

[2] H. B. G. Casimir, Physica **5** 495 (1938).
[3] Y. V. Sharvin, ZhETF **48** 984 (1965) (Sov. Phys.-JETP **21** 655 (1965).
[4] M. Apostol, **Transport Theory**, *apoma*, MB (2001) (a).

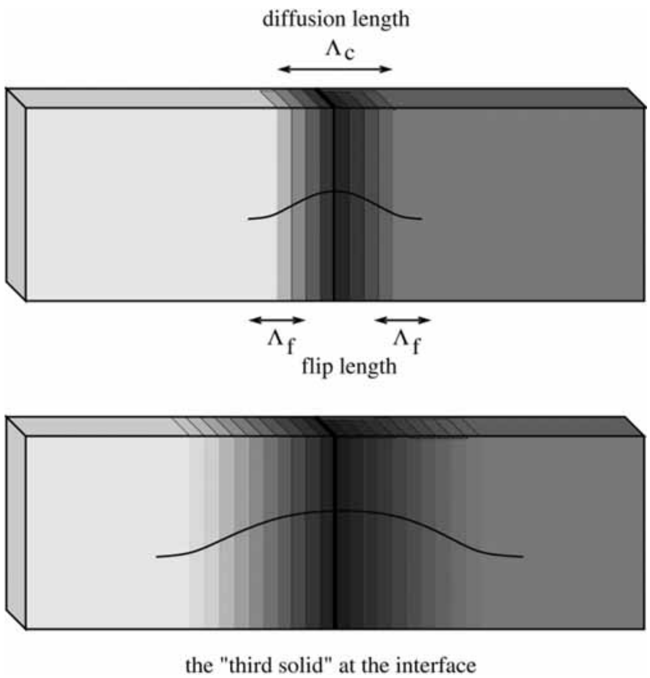

Fig. 2 – Two solids in contact.

Let us consider now two distinct solids labelled by 1 and 2, respectively, with a plane interface at $x = 0$; solid 1 extends from $x = -\infty$ to $x = 0$ and solid 2 extends from $x = 0$ to $x = +\infty$. The Fermi energies, *i.e.* the extension in energy from the bottom of the energy bands to the top of the Fermi seas (the top not necessarily placed at zero energy) are denoted by $\mu_{1,2}$; the bottom of the energy bands are placed at $-\varphi_{1,2}$. When put in contact the interface ions are separated by a potential barrier of width ~ a and height $\sim z^*\varphi$, where a is an average inter-ionic separation and $z^*\varphi$ denotes an average potential energy; if the two solids are similar, *i.e.* the difference Δz^* in their effective charges is very small, they form up a perfect contact; otherwise, the ions are perturbed by $\sim z^*\Delta\varphi \sim z^*\Delta z^* e^2/a$, and tunnel through across the top of the barrier; the well-known transmission coefficient

$$T^2 = \frac{4k^2\kappa^2}{(k^2+\kappa^2)^2 \sinh^2 a\kappa + 4k^2\kappa^2} \tag{6}$$

of a rectangular barrier, where $\kappa^2 \ll k^2 = (2M/\hbar^2) z^*\Delta\varphi \sim (2M/\hbar^2) z^*\Delta z^* e^2/a$, becomes $T^2 = 4/(a^2k^2+4)$, and, since $a^2k^2 \sim (M/m) z^*\Delta z^*(a/a_H)$, one gets

$$T^2 \simeq \frac{m}{M}\frac{1}{z^*\Delta z^*}\frac{a_H}{a}, \tag{7}$$

where M is the ionic mass and $a_H = \hbar^2/me^2$ is Bohr's radius. The distance covered by the ion is[5] $\Lambda_c \sim -a/\ln R$, where R is the reflection coefficient, $R^2 = 1 - T^2$; one obtains

$$\Lambda_c \sim a\frac{M}{m}z^*\Delta z^*(a/a_H)\,; \tag{8}$$

this length Λ_c is a measure of the contact width, but the two solids are not at equilibrium, and it is in fact a diffusion length of one solid into another (with a decreasing velocity); the diffusion velocity

$$v = \hbar k/M \sim \sqrt{m/M}\cdot\sqrt{z^*\Delta z^*(a/a_H)}\cdot v_F \tag{9}$$

is much lower than the Fermi velocity v_F of the electrons,[6] but the diffusion takes a longer time, so that the width of the contact is much larger than the lattice constant. Typical values for Λ_c are of the order of 10^2–10^3 Å, while the mean-free path of the electron quasi-particles, as given by $\Lambda \sim v_F\hbar\mu/T^2$, for instance, (for electron-electron interaction in conductors[7]) are of the order of 10^3–10^4 Å at room temperature. One can see that Λ_c is shorter than Λ, and at low temperatures Λ may increase appreciably.[8] Over the contact width Λ_c the electron energy levels vary smoothly; for instance, the self-consistent potential across the interface reads

$$\begin{aligned} \varphi(x) &= \varphi_1 + \frac{1}{2}\Delta\varphi e^{x/\Lambda_c}, \quad x<0\,, \\ \varphi(x) &= \varphi_2 - \frac{1}{2}\Delta\varphi e^{-x/\Lambda_c}, \quad x>0\,, \end{aligned} \tag{10}$$

and one can see that the relative work function $-\int dx\cdot(\partial\varphi/\partial x) = -\Delta\varphi = \varphi_1 - \varphi_2$ is the contact potential between the two solids, as expected. For large contact widths, the interface brings its own contribution (as a third solid in-between the junction of the two), for instance to transport coefficients.[9]

[5] M. Apostol, J. Theor. Phys. **74** 96 (2001).

[6] It may be increased by external perturbations, like an electric field, for instance, or raising the temperature, which also helps bringing the two solids in atomic contact.

[7] And similarly for electron-phonon interaction (M. Apostol, *loc cit*, (a)).

[8] In classical semiconductors the contact width is much narrow, of the order of 10 A , as a consequence of the drastic reduction in the effective charges z^* (and their differences), while the mean-free path of the charge carriers is longer (~ 100 A); see, for instance, M. Apostol, *loc cit* (a).

[9] It is worth noting in this connection that the contacts discussed here are those appearing naturally and freely between two solids, and not contacts realized by a limited deposition or growth of an additional, external solid in-between, like metal-oxide-metal, semi- or superconductor, where tunneling currents are measured through the oxide potential barrier (see, for instance, I. Giaever, Revs. Mod. Phys. **46** 245 (1974)).

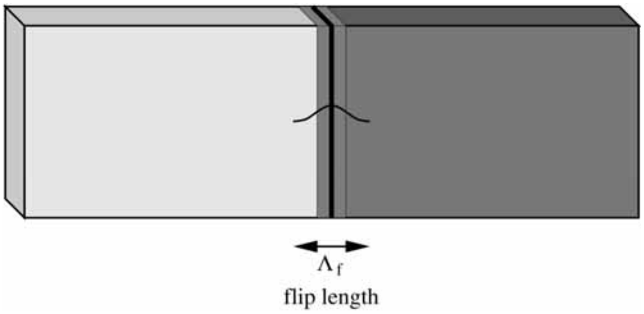

Fig. 3 – A perfect contact.

We assume a perfect interface between two conductors, and focus on the electron quasi-particles transport; the difference $\Delta\varphi$ between the two chemical potentials is small in comparison with the cohesion scale φ, and placed at the bottom of the bands; the quantal corrections bring even smaller contributions; it follows that the difference $\Delta\mu$ between the two Fermi energies (*i.e.*, the extent in energy from the bottom of the bands to the Fermi surface) is small in comparison with the Fermi energy μ, so that the quasi-particles have in fact a common Fermi level μ and a change $\Delta\mu$ in energy on passing across the interface.[10] It follows that the electron energy uncertainty $\hbar/\tau \sim (\delta\mu)^2/\mu$ (or $\hbar/\tau \sim T^2/\mu$) suffers a change $\Delta(\hbar/\tau) \sim ((\delta\mu)^2/\mu)(\Delta\mu/\mu)^2 = (\hbar/\tau)(\Delta\mu/\mu)^2$, and a similar change occurs for the electron-phonon uncertainty in the quasi-particle energy; therefore, the quasi-particle lifetime τ suffers a relative change according to

$$\Delta(1/\tau)/(1/\tau) = (\Delta\mu/\mu)^2 \tag{11}$$

on passing through the interface. All the transport coefficients are affected by a similar relative change due to the presence of the interface; in particular the electric conductivity σ undergoes a decrease $\Delta\sigma/\sigma = -(\Delta\mu/\mu)^2$, and the electric resistivity ρ increases by $\Delta\rho/\rho = (\Delta\mu/\mu)^2$; this is Kapitza's contact resistance;[11] a relative voltage drop $\Delta U/U = (\Delta\mu/\mu)^2$ occurs at the interface.

The relative jump $(\Delta\mu/\mu)^2$ at the interface affects all the transport properties of the quasi-particles, in particular their lifetime τ and mean-free path Λ.[12] The fraction $\tau_f = (\Delta\mu/\mu)^2\tau$ is the flip time of the quasi-particles on passing through the

[10] It is worth noting that such perfect contacts appears in fact at the separation between each of the two solids and their common, extended contact (the third solid), as described above.

[11] P. L. Kapitza, ZhETF **11** 1 (1941).

[12] It is also worth noting that the frequency $\Delta(1/\tau)$ gives the number of transitions per unit time for a superconducting pair from one superconductor into another in Josephson junctions, *i.e.*, $i\hbar\partial\psi_{1,2}/\partial t = \hbar\Delta(1/\tau)\psi_{2,1}$, where $\psi_{1,2}$ are the condensate wavefunctions.

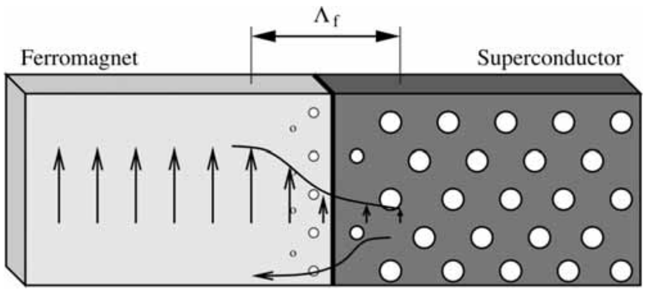

Fig. 4 – Spin-flip and gap damping at the interface.

interface, and $\Lambda_f = (\Delta\mu/\mu)^2 \Lambda$ is the corresponding flip path, or penetration length. Accordingly, at the ferromagnet-conductor interface the quasi-particle spin flips over the length Λ_f, so the magnetization is gradually destroyed over the length Λ_f into the ferromagnet, and it penetrates similarly over a distance Λ_f into the normal conductor (the difference in the two Fermi velocities in the magnetic state bears no relevance in estimating the penetration length; a more suggestive measure of two competing flip lengths $\Lambda_{1,2}$ is $(\Lambda_1\Lambda_2)^{1/2}$).[13] At the superconductor-normal conductor interface the superconducting gap extends up to Λ_f into the normal conductor, and is destroyed gradually over the same length into the superconductor; the Fermi velocity in the superconductor is that corresponding to the normal state of the superconductor, while the quasi-particle lifetime is formally affected by the superconducting gap.[14] The same description applies also to a conductor-insulator interface, by noting that the insulating gap is a quantal correction in terms of the present approach, so the insulating gap is destroyed over Λ_f length at the interface, and penetrates over a similar length into the conductor.[15] It follows, according to the present description, that the charge carriers in a (perfect contact) ferromagnet-superconductor junction move close to the same common Fermi level μ, with an almost common (normal state) Fermi velocity, and possess an additional, small contribution to their lifetime due to the presence of the interface; both the magnetization and the superconductiong gap vanishes over a penetration length Λ_f across the interface, as determined above. It is easy to see that for a perfect contact Λ_f is comparable with Λ_c.

[13] See the previous contribution on the electric flow through a ferromagnet-superconductor junction by the same authors in this journal.

[14] See, for instance, R. J. Schrieffer, **Theory of Superconductivity**, *Benjamin*, NY (1964).

[15] At the interface between a conductor and a semiconductor (of narrow band) an extended contact is built up (except for a limited growth or deposition).