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NONLINEAR EFFECTS IN STRONG TURBULENCE IN MAGNETICALLY CONFINED PLASMAS

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Abstract. Strong electrostatic turbulence in magnetically confined plasmas is characterized by trapping or eddying of test particle trajectories produced by the $E \times B$ stochastic drift. Trapping is shown to produce non-standard statistics of trajectories: non-Gaussian distribution, memory effects and quasi-coherence. Two types of effects produced by trapping are analyzed. The first type concerns particle and energy transport and consists in very strong nonlinear modification of the diffusion coefficients. Anomalous diffusion regimes are obtained when the other components of the motion (particle collisions, plasma rotation, the motion along the confining magnetic field) do not destroy trajectory eddying. The second type of effects are evidenced by studying test modes on turbulent plasma. We show that trappyng provides the physical mechanism for the inverse cascade observed in drift turbulence.

Key words: plasma turbulence, statistical approaches, test particle transport, Lagrangian methods.

1. INTRODUCTION

A component of test particle motion in magnetically confined plasmas is the stochastic $\mathbf{E} \times \mathbf{B}$ drift produced by the electric field of the turbulence and by the confining magnetic field. It is well known since many years [1] that, for slowly varying or large amplitude turbulence, the electric drift determines a process of dynamical trapping of the trajectories. It consists of trajectory winding on almost closed paths. Typical particle trajectories show sequences of trapping events and long jumps. Particle motion in a stochastic potential was extensively studied [2–4]. Important progresses in the study of this nonlinear process were recently obtained. New statistical methods were developed [5, 6] that permitted to determine the time dependent (running) diffusion coefficient and even the probability of displacements. It was shown that the trapping process completely changes the statistical properties of the trajectories determining memory effects, quasi-coherent behavior and non-Gaussian distribution [6]. The diffusion coefficients decrease due to trapping and their scaling in the parameters of the stochastic field is modified. A short description of the statistical methods (the decorrelation trajectory method [5] and the nested subensemble method [6]) and of these statistical characteristics of the $\mathbf{E} \times \mathbf{B}$ drift motion is presented in the first part of this paper.

The test particle studies rely on known statistical characteristics of the stochastic field. They are determined from experimental studies or numerical simulations. The main aim of these studies is to determine the diffusion coefficients. The statistics of test particle trajectories provides the transport coefficients in turbulent plasmas without approaching the very complicated problem of self-consistent turbulence that explains the detailed mechanism of generation and saturation of the turbulent potential. The possible diffusion regimes can be obtained by considering various models for the statistics of the stochastic field. Test particle transport in magnetized plasmas was analytically studied only the last decade. The decorrelation trajectory method [5] was developed to include besides the $\mathbf{E} \times \mathbf{B}$ stochastic drift other components of the motion (particle collisions, average flows, motion along the confining magnetic field, etc.). Several anomalous diffusion regimes were found in the presence of trajectory trapping [7–10]. We discuss here the physical reason for this behavior of the diffusion coefficient and the conditions when these anomalous regimes appear.

The semi-analytical methods developed for test particles are extended to test mode evolution in a turbulent magnetized plasmas. Test modes are usually studied for modelling wave-wave interaction in turbulent plasmas [11]. A different perspective is developed here by considering test modes on turbulent plasmas. They are described by nonlinear equation with the advection term containing the stochastic $\mathbf{E} \times \mathbf{B}$ drift with known statistical characteristics of the turbulence. The growth rate of the test modes is determined as function of these statistical parameters. We develop a Lagrangian approach of the type of that introduced by Dupree [12, 13]. The difference is that in Dupree's method the stochastic trapping of trajectory was neglected and consequently the results can be applied to quasilinear turbulence. Our method takes into account the trapping and the non-standard statistics of trajectories that it yields and thus it is able to describe the nonlinear effects appearing in strong turbulence.

The paper is organized as follows. The test particle model is presented in Section 2 and the statistical methods are shortly described in Section 3. The nonlinear effects on test particle transport are presented in Section 4 where the general physical explanation for the anomalous diffusion regimes appearing in the presence of trajectory trapping is discussed. The quasi-coherence of trapped trajectories is analyzed in Section 5. The problem of test modes in turbulent plasmas for the case of drift turbulence is presented in Section 6 and the conclusions are summarized in Section 7.

2. TEST PARTICLE MODEL

We consider in slab geometry an electrostatic turbulence represented by an electrostatic potential $\phi^e(\mathbf{x},t)$, where $\mathbf{x}\equiv(x_1,x_2)$ are the Cartesian coordinates in the plane perpendicular to the confining magnetic field directed along z axis, $\mathbf{B}=B\mathbf{e}_z$. The test particle motion in the guiding center approximation is determined by

$$\frac{d\mathbf{x}(t)}{dt}=v(\mathbf{x},t)\equiv-\nabla\phi(\mathbf{x},t)\times\mathbf{e}_z, \tag{1}$$

where $\mathbf{x}(t)$ represent the trajectory of the particle guiding center, ∇ is the gradient in the (x_1,x_2) plane and $\phi(\mathbf{x},t)=\phi^e(\mathbf{x},t)/B$. The electrostatic potential $\phi(\mathbf{x},t)$ is considered to be a stationary and homogeneous Gaussian stochastic field, with zero average. It is completely determined by the two-point Eulerian correlation function (EC), $E(\mathbf{x},t)$, defined by

$$E(\mathbf{x},t)\equiv\left\langle\phi(\mathbf{x}',t')\,\phi(\mathbf{x}'+\mathbf{x},t'+t)\right\rangle. \tag{2}$$

The average $\langle ...\rangle$ is the statistical average over the realizations of $\phi(\mathbf{x},t)$, or the space and time average over $\mathbf{x}'$ and t'. This function evidences three parameters that characterize the (isotropic) stochastic field: the amplitude $\Phi=\sqrt{E(\mathbf{0},0)}$, the correlation time τ_c, which is the decay time of the Eulerian correlation and the correlation length λ_c, which is the characteristic decay distance. These three parameters combine in a dimensionless Kubo number

$$K=\tau_c/\tau_{fl} \tag{3}$$

where $\tau_{fl}=\lambda_c/V$ is the time of flight of the particles over the correlation length and $V=\Phi/\lambda_c$ is the amplitude of the stochastic velocity.

The diffusion coefficient is determined as (see [14])

$$D_i(t)=\int_0^t d\tau\, L_{ii}(\tau), \tag{4}$$

where

$$L_{ij}(t;t_1)\equiv\left\langle v_i(\mathbf{0},0)\,v_j(\mathbf{x}(t),t)\right\rangle \tag{5}$$

is the correlation of the Lagrangian velocity (LVC). It is obtained using the decorrelation trajectory method, a semi-analytical approach presented below.

3. THE NESTED SUBENSEMBLE APPROACH

Trajectory trapping is essentially related to the invariance of the Lagrangian potential. Thus, a statistical method is adequate for the study of this process if it is compatible with the invariance of the potential. Such methods are presented in [5, 6]. The main idea in our approach is to study the stochastic equation (1) in subensembles of realizations of the stochastic field. First the whole set of realizations R is separated in subensembles ($S1$), which contain all realizations with given values of the potential and of the velocity in the starting point of the trajectories $\mathbf{x} = 0$, $t = 0$:

$$(S1): \quad \phi(\mathbf{0}, 0) = \phi^0, \quad \mathbf{v}(\mathbf{0}, 0) = \mathbf{v}^0. \tag{6}$$

Then, each subensemble ($S1$) is separated in subensembles ($S2$) corresponding to fixed values of the second derivatives of the potential in $\mathbf{x} = 0$, $t = 0$

$$(S2): \quad \phi_{ij}(\mathbf{0}, 0) \equiv \left.\frac{\partial^2 \phi(\mathbf{x}, t)}{\partial x_i \partial x_j}\right|_{\mathbf{x}=0, t=0} = \phi_{ij}^0, \tag{7}$$

where $ij = 11, 12, 22$. Continuing this procedure up to an order n, a system of nested subensembles is constructed. The stochastic (Eulerian) potential and velocity in a subensemble are Gaussian fields but non-stationary and non-homogeneous, with space and time dependent averages and correlations. The correlations are zero in $\mathbf{x} = 0$, $t = 0$ and increase with the distance and time. The average potential and velocity performed in a subensemble depend on the parameters of that subensemble and of the subensembles that include it. They are determined by the Eulerian correlation of the potential (see [6] for details). The stochastic equation (1) is studied in each highest order subensemble (Sn). The average Eulerian velocity determines an average motion in each (Sn). Neglecting the fluctuations of the trajectories, the average trajectory in (Sn), $\mathbf{X}(t; Sn)$, is obtained from

$$\frac{d\mathbf{X}(t; Sn)}{dt} = \varepsilon_{ij} \frac{\partial \Phi(\mathbf{X}; Sn)}{\partial X_j}. \tag{8}$$

This approximation consists in neglecting the fluctuations of the trajectories in the subensemble (Sn). It is rather good because it is performed in the subensemble (Sn) where the trajectories are similar due to the fact that they are super-determined. Besides the necessary and sufficient initial condition $\mathbf{x}(0) = 0$, they have supplementary initial conditions determined by the definition (6-7) of the subensembles. The strongest condition is the inial potential $\phi(0, 0) = \phi^0$ that is a conserved quantity in the static case and determines comparable sizes of the trajectories in a subensemble. Moreover, the amplitude of the velocity fluctuations in (Sn), the source of the trajectory fluctuations, is zero in the starting point of the trajectories and reaches the value corresponding to the whole set of realizations only asymptotically. This reduces the differences between the trajectories in (Sn) and thus their fluctuations.

The statistics of trajectories for the whole set of realizations (in particular the LVC) is obtained as weighted averages of these trajectories $\mathbf{X}(t; Sn)$. The weighting factor is the probability that a realization belongs to the subensemble (Sn); it is analytically determined.

Essentially, this method reduces the problem of determining the statistical behavior of the stochastic trajectories to the calculation of weighted averages of some smooth, deterministic trajectories determined from the EC of the stochastic potential. This semi-analytical statistical approach (the nested subensemble method) is a systematic expansion that satisfies at each order $n > 1$ all statistical conditions required by the invariance of the Lagrangian potential in the static case. The order $n = 1$ corresponds to the decorrelation trajectory method introduced in [5]. In this case only the average potential is conserved.

The nested subensemble method is quickly convergent. This is a consequence of the fact that the mixing of periodic trajectories, which characterizes this nonlinear stochastic process, is directly described at each order of our approach. The results obtained in first order (the decorrelation trajectory method) for $D(t)$ are practically not modified in the second order [6]. Thus, the decorrelation trajectory method is a good approximation for determining diffusion coefficients. The second order nested subensemble method is important because it provides detailed statistical information on trajectories: the probability of the displacements and of the distance between neighboring trajectories in the whole ensemble of realizations and also in the subensembles ($S1$). A high degree of coherence is so evidenced in the stochastic motion of trapped trajectories.

4. TRANSPORT COEFFICIENTS

Test particle studies connected with experimental measurements of the statistical properties of the turbulence provide the transport coefficients with the condition that there is space-time scale separation between the fluctuations and the average quantities. Particle density advected by the stochastic $\mathbf{E} \times \mathbf{B}$ drift in turbulent plasmas leads in these conditions to a diffusion equation for the average density. Recent numerical simulations [15] confirm a close agreement between the diffusion coefficient obtained from the density flux and the test particle diffusion coefficient. Experiment based studies of test particle transport permit to strongly simplify the complicated self-consistent problem of turbulence and to model the transport coefficients by means of test particle stochastic advection. The running diffusion coefficient $D(t)$ is defined as the time derivative of the mean square displacement of test particles and is determined according to Eq. (4) as the time integral of the Lagrangian velocity correlation (LVC). Thus, test particle approach is based on the evaluation of the LVC for given Eulerian correlation (EC) of the fluctuating potential.

The turbulent transport in magnetized plasmas is a strongly nonlinear process. It is characterized by the trapping of the trajectories, which determines a strong influence on the transport coefficient and on the statistical characteristics of the trajectories. The transport induced by the $\mathbf{E} \times \mathbf{B}$ stochastic drift in electrostatic turbulence [16] (including effects of collisions [7], average flows [8], motion along magnetic field [10], effect of magnetic shear [17]) and the transport in magnetic turbulence [18, 19] were studied in a series of papers using the decorrelation trajectory method. It was also shown that a direct transport (an average velocity) appears in turbulent magnetized plasmas due to the inhomogeneity of the magnetic field [20–22]. This statistical method was developed for the study of complex processes as the zonal flow generation [23, 24]. The results of all these studies are rather unexpected when the nonlinear effects are strong. The diffusion coefficients are completely different of those obtained in quasilinear conditions. A rich class of anomalous diffusion regimes is obtained for which the dependence on the parameters is completely different compared to the scaling obtained in quasilinear turbulence. All the components of particle motion (parallel motion, collisions, average flows, etc.) have strong influence on the diffusion coefficients in the non-linear regimes characterized by the presence of trajectory trapping.

The reason for these anomalous transport regimes can be understood by analyzing the shape of the correlation of the Lagrangian velocity for particles moving by the $\mathbf{E} \times \mathbf{B}$ drift in a static potential. In the absence of trapping, the typical LVC for a static field is a function that decay to zero in a time of the order $\tau_{fl} = \lambda_c / V$. This leads to Bohm type asymptotic diffusion coefficients $D_B = V^2 \tau_{fl} = V\lambda_c$. Only a constant c is influenced by the EC of the stochastic field and the diffusion coefficient is $D = cD_B$ for all EC's. In the case of the $\mathbf{E} \times \mathbf{B}$ drift, a completely different shape of the LVC is obtained for static potentials due to trajectory trapping. A typical example of the LVC is presented in Fig. 1. This function decays to zero in a time of the order τ_{fl} but at later times it becomes negative, it reaches a minimum and then it decays to zero having a long, negative tail. The tail has power law decay with an exponent that depends on the EC of the potential [16]. The positive and negative parts compensate such that the integral of $L(t)$, the running diffusion coefficient $D(t)$, decays to zero. The transport in static potential is thus subdiffusive. The long time tail of the LVC shows that the stochastic trajectories in static potential have a long time memory.

This stochastic process is unstable in the sense that any weak perturbation produces a strong influence on the transport. A perturbation represents a decorrelation mechanism and its strength is characterized by a decorrelation time τ_d. The weak perturbations correspond to long decorrelation times, $\tau_d > \tau_{fl}$. In the absence of trapping, such a weak perturbation does not produce a modification of the diffusion coefficient because the LVC is zero at $t > \tau_{fl}$. In the presence of trapping, which is characterized by long time LVC as in Fig. 1, such perturbation influences the tail of the LVC and destroys the equilibrium between the positive and the negative parts. Consequently, the diffusion coefficient is a *decreasing function of* τ_d. It means that when the decorrelation mechanism becomes stronger (τ_d decreases) the transport increases. This is a consequence of the fact that the long time LVC is negative. This behavior is completely different of that obtained in stochastic fields that do not produce trapping. In this case, the transport is stable to the weak perturbations. An influence of the decorrelation can appear only when the later is strong such that $\tau_d < \tau_{fl}$ and it determines the increase of the diffusion coefficient with the increase of τ_d. This inverse behavior appearing in the presence of trapping is determined by the fact that a stronger perturbation (with smaller τ_d) liberates a larger number of trajectories, which contribute to the diffusion.

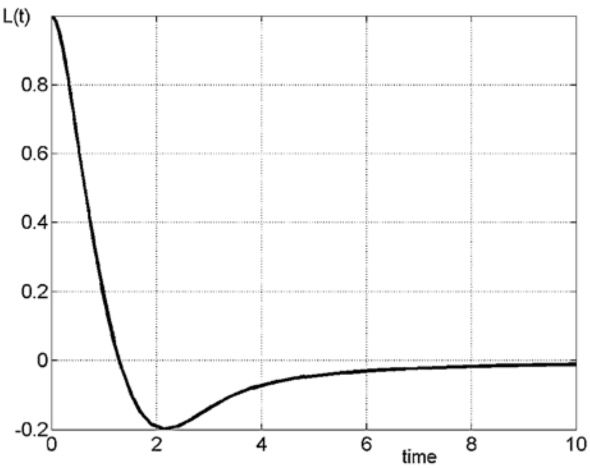


Fig. 1 – Typical Lagrangian velocity correlation in static potential.

The decorrelation can be produced for instance by the time variation of the stochastic potential, which produces the decay of both Eulerian and Lagrangian correlations after the correlation time τ_c. The decorrelation time in this case is τ_c and it is usually represented by a dimensionless parameter, the Kubo number defined by Eq. (3). The transport becomes diffusive with an asymptotic diffusion coefficient that scales as $D_{tr} = cV\lambda_c K^{\gamma}$, with γ in the interval [–1, 0] (trapping scaling [16]). The diffusion coefficient is a decreasing function of τ_c in the nonlinear regime $K > 1$.

For other types of perturbations, their interaction with the trapping process produces more complicated nonlinear effects. For instance, particle collisions lead to the generation of a positive bump on the tail of the LVC [7] due to the property of the 2-dimensional Brownian motion of returning in the already visited places. Other decorrelation mechanisms appearing in plasmas are average component of the velocity like poloidal rotation [8] or the parallel motion that determines decorrelation when the potential has a finite correlation length along the confining magnetic field.

5. TRAJECTORY TRAPPING AND STATISTICAL COHERENCE

Detailed statistical information about particle trajectories was obtained using the nested subensemble method [6]. This method determines the statistics of the trajectories that start in points with given values of the potential. This permits to evidence the high degree of coherence of the trapped trajectories.

The trapped trajectories correspond to large absolute values of the initial potential while the trajectories starting from points with the potential close to zero perform long displacements. These two types of trajectories have completely different statistical characteristics [6]. The trapped trajectories have a quasi-coherent behavior. Their average displacement, dispersion and probability distribution function saturate in a time τ_s. The time evolution of the square distance between two trajectories is very slow showing that neighboring particles have a coherent motion for a long time, much longer than τ_s. They are characterized by a strong clump effect with the increase of the average square distance that is slower than the Richardson law. These trajectories form structures, which are similar with fluid vortices and represent eddying regions. The statistical parameters of these structures (size, build-up time, dispersion) are determined. The dispersion of the trajectories in such a structure is of the order of its size. The size and the built-up time depend on the value of the initial potential. Trajectory structures appear with all sizes, but their characteristic formation time increases with the size. These structures or eddying regions are permanent in static stochastic potentials. The saturation time τ_s represents the average time necessary for the formation of the structure. In time dependent potentials the structures with $\tau_s > \tau_c$ are destroyed and the corresponding trajectories contribute to the diffusion process. These free trajectories have a continuously growing average displacement and dispersion. They have incoherent behavior and the clump effect is absent. The probability distribution functions for both types of trajectories are non-Gaussian.

The average size of the structures $S(K)$ in a time dependent potential is plotted in Fig. 2. One can see that for $K < 1$ the structures are absent ($S \cong 0$) and that they appear for $K > 1$ and continuously grow as K increases. The dependence on K is a power low with the exponent dependent on the EC of the potential. The exponent is 0.19 for the Gaussian EC and 0.35 for a large EC that decays as $1/r^2$.

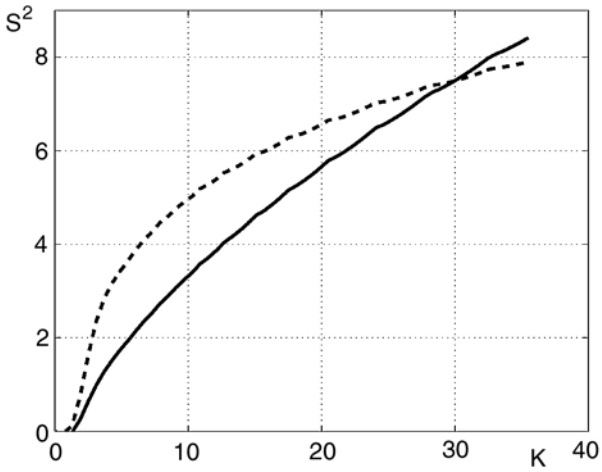


Fig. 2 – The average size of the trajectory structures for Gaussian EC (dashed line) and for an EC that decays as $1/r^2$ (continuous line).

6. TEST MODES ON DRIFT TURBULENCE

Test particle trajectories are strongly related to plasma turbulence. The dynamics of the plasma basically results from the Vlasov-Maxwell system of equations, which represents the conservation laws for the distribution functions along particle trajectories. Studies of plasma turbulence based on trajectories were initiated by Dupree [12, 13] and developed especially in the years seventies (see the review paper [11] and references there in). These methods do not account for trajectory trapping and thus they apply to the quasilinear regime or to unmagnetized plasmas. A very important problem that has to be understood is the effect of the non-standard statistical characteristics of the test particle trajectories on the evolution of the instabilities and of turbulence in magnetized plasmas.

We extend the Lagrangian methods of the type of [13, 25] to the nonlinear regime characterized by trapping. We study linear modes on turbulent plasma with the statistical characteristics of the turbulence considered known. We determine the dispersion relation for such test modes. We consider the drift instability in slab geometry with constant magnetic field. The combined effect of the parallel motion of electrons (non-adiabatic response) and finite Larmor radius of the ions destabilizes the drift waves. The perturbations of the electron and ion distribution functions are obtained from the gyrokinetic equation as integrals along test particle trajectories of the source terms determined by the average density gradient. The gyrokinetic equations are not linearized around the unperturbed state as in the linear theory but around a turbulent state with known spectrum.

The background turbulence produces two modifications of the equation for the linear modes. One consists in the stochastic $\mathbf{E}\times\mathbf{B}$ drift that appears in the trajectories and the other is the fluctuation of the diamagnetic velocity. Both effects are important for ions while the response of the electrons is approximately the same as in quiescent plasma.

The average propagator of the modes is evaluated using the above results on trajectory statistics. In the first order it depends on the size $S(K)$ of the structures. The solution of the dispersion relation for a mode with frequency ω and wave number components k_1, k_2 is obtained as

$$\omega = \omega_{*e} \frac{\Gamma_0\left(\frac{k_\perp^2\rho_L^2}{2}\right)\exp\left(-\frac{1}{2}k_i^2 S^2\right)}{2-\Gamma_0\left(\frac{k_\perp^2\rho_L^2}{2}\right)}, \tag{9}$$

$$\gamma = \sqrt{\pi}\,\frac{\omega\left(\omega_{*e}-\omega\right)}{2-\Gamma_0}\frac{1}{|k_z|v_{Te}} - k_i^2\mathcal{D}_i\cos(\omega\tau_c) + k_i k_j R_{ij}\omega/k_2\,, \tag{10}$$

where V_{*e} is the diamagnetic velocity, $\omega_{*e} = k_2 V_{*e}$ is the diamagnetic frequency, ρ_L is the ion Larmor radius and $\Gamma_0(b) = exp(-b)I_0(b)$. The tensor R_{ij} has the dimension of a length and is defined by

$$R_{ji}(\tau,t) \equiv \int_\tau^t d\theta' \int_{-\infty}^{\tau-\theta'} d\theta M_{ji}\left(|\theta|\right), \tag{11}$$

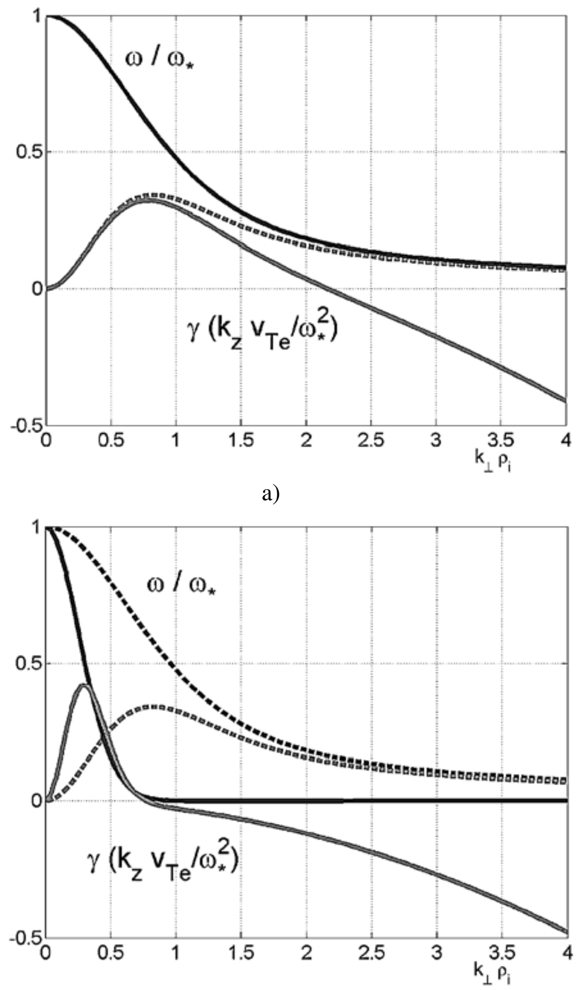
where M_{ij} is the Lagrangian correlation

$$M_{ji}\left(|\theta'-\theta|\right) \equiv \left\langle v_j\left(\mathbf{x}^i(\theta'),z,\theta'\right)\partial_2 v_i\left(\mathbf{x}^i(\theta),z,\theta\right)\right\rangle, \tag{12}$$

and v_j is the $\mathbf{E}\times\mathbf{B}$ drift velocity.

The trajectory trapping has a complex influence on the mode. The quasi-coherent component of ion motion (the stochastically trapped ions) determines the S-dependent exponential factor in the frequency ω. Its effect is the displacement of the unstable k-range toward small values. The random component in the ion motion (the free ion trajectories) determines a diffusive damping term in the growth rate γ that produces the stabilization of the large wave numbers. It is similar with the term obtained in [13], but with the diffusion coefficient influenced by trapping. The fluctuations of the diamagnetic velocity determines the last term in the growth rate (10). The tensor R_{ij} contributes to the growth of the modes.

The test mode growth rate γ strongly depends on the characteristics of the background turbulence represented by the Kubo number as seen in Fig. 3 where ω and γ are plotted for a quiescent background plasma (dashed lines) and for a turbulent one (continuous lines). At small Kubo numbers (Fig. 3a), the results of Dupree are obtained. The turbulence determines the stabilization of the large $k_\perp$ modes due to ion trajectory diffusion and it does not influence the frequency. At large Kubo numbers (Fig. 3b), a strong decay of the frequency appears due to the quasi-coherent component of ion motion [$S(K) \neq 0$ in Eq. (9)] and the maximum of the growth rate is displaced to smaller wave numbers.



a)

b)

Fig. 3 – The wave number and the growth rate as function of the wave number for unperturbed plasma (dashed lines) and for turbulent plasma (continuous lines). The Kubo number of the background turbulence in: a) $K = 0.1$; b) $K = 10$.

The dynamics of the drift turbulence is determined starting from a large spectrum of modes with very small amplitudes (thermal bath). In this quasilinear regime only the diffusion of the ions influences the modes producing the damping of the modes with large wave numbers, $k_\perp \rho_L \gg 1$. The amplitude of the stochastic potential increases continuously while the correlation length remains comparable with ρ_L and the correlation time is $\tau_c = 1/\omega_{*e} \sim \rho_L / V_{*e}$. This amplitude increase eventually leads to $K > 1$ and produces trajectory trapping and coherent motion for a part of the ions. Small trajectory structures are formed and persist during the correlation time of the potential. This ordered motion of the ions acts similarly with the cyclotron gyration: it decreases the frequency of the modes and displaces the maximum of the spectrum toward smaller wave numbers. In this stage of the evolution the amplitude of the $\mathbf{E} \times \mathbf{B}$ velocity remain approximately constant, while the correlation length and the correlation time are slowly increasing. The spectrum is continuously displaced toward small wave numbers and narrowed due to the increase of the diffusion coefficient. Thus the energy taken by the instability from the electrons produces a motion of the ions with increasing coherent component. The size of the trajectory structures increases and is reflected in the turbulence that looses the random aspect: large ordered potential cells are produced. In the same time, as the Kubo number increases, the term determined by the fluctuation of the diamagnetic velocity is growing due to the fact that the turbulence becomes anisotropic.

A different perspective on the inverse cascade is thus obtained. It does not appear as wave-wave interaction but as the consequence of ion $\mathbf{E} \times \mathbf{B}$ motion in the potential of the turbulence that strongly influences the test mode stability. Namely, the quasi-coherent motion of the trapped ions produces the destabilization of the modes with wave lengths of the order of the average size of the trajectory structures. These decreases the frequency of the modes and produces the increase of the size S of the trajectory structures leading to further decrease of the wave numbers of the unstable modes.

7. CONCLUSIONS

We have discussed the problem of stochastic advection of test particles by the $\mathbf{E} \times \mathbf{B}$ drift in turbulent plasmas. We have shown that the nonlinear effects are very strong in the case of static potentials. The trajectories are non-Gaussian, there is statistical memory and coherence, and the scaling laws are dependent on the Eulerian correlation of the stochastic potential.

These properties persist if the system is weakly perturbed by time variation of the potential or by other components of the motion (collisions, poloidal rotation, parallel motion). The memory effect (long tail of the LVC) determines anomalous diffusion regimes.

These non-standard statistical properties of the trajectories are shown to be associated with order and structure formation in turbulent magnetized plasmas. Particle trajectories have a high degree of coherence when the perturbations are weak. The trajectory structures determine the evolution of the drift turbulence toward large scales (inverse cascade).

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