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STRING COSMOLOGICAL MODEL IN CYLINDRICALLY SYMMETRIC INHOMOGENEOUS UNIVERSE WITH ELECTROMAGNETIC FIELD

ANIRUDH PRADHAN[1], SUNIL KUMAR SINGH[2], LAL JI SINGH YADAV[2]

[1] *Department of Mathematics, Hindu Post-graduate College, Zamania-232 331, Ghazipur, India*
E-mail : pradhan@iucaa.ernet.in
[2] *Department of Physics, S. D. J. Post-graduate College, Chandeswar-276 128, Azamgarh, India*

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Abstract. Cylindrically symmetric inhomogeneous string cosmological model of the universe in presence of electromagnetic field is investigated. We have assumed that F_{12} is the only non-vanishing component of electromagnetic field tensor F_{ij}. The Maxwell's equations show that F_{12} is the function of x alone whereas the magnetic permeability is the function of x and t both. To get the deterministic solution, it has been assumed that the expansion (θ) in the model is proportional to the eigen value $\sigma^1{}_1$ of the shear tensor $\sigma^i{}_j$. Some physical and geometric properties of the model are also discussed.

Key words: cosmic string, electromagnetic field, inhomogeneous universe.

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1. INTRODUCTION

Cosmic strings play an important role in the study of the early universe. These strings arise during the phase transition after the big bang explosion as the temperature goes down below some critical temperature as predicted by grand unified theories [1–5]. It is believed that cosmic strings give rise to density perturbations which lead to formation of galaxies [6]. These cosmic strings have stress energy and couple to the gravitational field. Therefore, it is interesting to study the gravitational effect which arises from strings. The general treatment of strings was initiated by Letelier [7, 8] and Stachel [9]. Magnetic fields are known to have a widespread presence in our Universe, being a common property of the intergalactic medium in galaxy clusters [10], while, reports on Faraday rotation imply significant magnetic fields in condensations at high red-shifts [11]. Studies of large-scale magnetic fields and their potential implications for the formation and the evolution of the observed structures, have been the subject of continuous investigation (see *e.g.* [12–19] for a representative though incomplete list). The origin of these fields, whether of astrophysical or cosmological origin, remains an unresolved issue. Also Harrison [20] has suggested that magnetic field could have a cosmological origin. If magnetism has a cosmological origin, as observation of μG fields in galaxy clusters and high red-shift protogalaxies seem to suggest, it could have affected the evolution of the Universe [21]. As a natural consequences, we should include magnetic fields in the energy-momentum tensor of the early universe. The choice of anisotropic cosmological models in Einstein system of field equations leads to the cosmological models more general than Robertson-Walker model [22]. The presence of primordial magnetic fields in the early stages of the evolution of the universe has been discussed by several authors [23–31]. Melvin [32], in his cosmological solution for dust and electromagnetic field suggested that during the evolution of the universe, the matter was in a highly ionized state and was smoothly coupled with the field, subsequently forming neutral matter as a result of universe expansion. Hence the presence of magnetic field in string dust universe is not unrealistic.

Benerjee *et al.* [33] have investigated an axially symmetric Bianchi type I string dust cosmological model in presence and absence of magnetic field. The string cosmological models with a magnetic field are also discussed by Chakraborty [34], Tikekar and Patel [35, 36]. Patel and Maharaj [37] investigated stationary rotating world model with magnetic field. Ram and Singh [38] obtained some new exact solution of string cosmology with and without a source free magnetic field for Bianchi type I space-time in the different basic form considered by Carminati and McIntosh [39]. Singh and Singh [40] investigated string cosmological models with magnetic field in the context of space-time with G_3 symmetry. Singh [41, 42] has studied string cosmology with electromagnetic fields in Bianchi type-II, -VIII and -IX space-times. Lidsey, Wands and Copeland [43] have reviewed aspects of super string cosmology with the emphasis on the cosmological implications of duality symmetries in the theory. Bali *et al.* [44, 45] have investigated Bianchi type I magnetized string cosmological models.

Cylindrically symmetric space-time play an important role in the study of the universe on a scale in which anisotropy and inhomogeneity are not ignored. Inhomogeneous cylindrically symmetric cosmological models have significant contribution in understanding some essential features of the universe such as the formation of galaxies during the early stages of their evolution. Bali and Tyagi [17] and Pradhan *et al.* [18, 19] have investigated cylindrically symmetric inhomogeneous cosmological models in presence of electromagnetic field. Barrow and Kunze [46, 47] found a wide class of exact cylindrically symmetric flat and open inhomogeneous string universes. In their solutions all physical quantities depend on at most one space coordinate and the time. Recently Baysal *et al.* [48], Kilinc and Yavuz [49] have investigated some string cosmological models in cylindrically symmetric inhomogeneous universe. Kilinc and Yavuz [50] have also obtained string cosmology with magnetic field in cylindrically symmetric space-time. The case of cylindrical symmetry is natural because of the mathematical simplicity of the field equations whenever there exists a direction in which the pressure equal to energy density.

Motivated by the situation discussed above, in this paper, we have obtained a new solution of Einstein's field equations in cylindrically symmetric space-time in presence of electromagnetic field and string as a source. We have taken string and electromagnetic field together as the source gravitational field as magnetic field are anisotropic stress source and low strings are one of anisotropic stress source as well. This paper is organized as follows: The metric and field equations are presented in Section 2. In Section 3, we deal with the solution of the field equations in presence of electromagnetic field with perfect fluid distribution. In Section 3, we deal with some physical and geometric properties of model. Concluding remarks are given in Section 4.

2. THE METRIC AND FIELD EQUATIONS

We consider the metric in the form

$$ds^2 = A^2(dx^2 - dt^2) + B^2dy^2 + C^2dz^2, \tag{1}$$

where A, B and C are functions of x and t. The energy momentum tensor for the string with electromagnetic field has the form

$$T_i^j = \rho u_i u^j - \lambda x_i x^j + E_i^j, \tag{2}$$

where u_i and x_i satisfy conditions

$$u^i u_i = -x^i x_i = -1, \tag{3}$$

and

$$u^i x_i = 0. \tag{4}$$

Here ρ being the rest energy density of the system of strings, λ the tension density of the strings, x^i is a unit space-like vector representing the direction of strings so that $x^1 = 0 = x^2 = x^4$ and $x^3 \neq 0$, and u^i is the four velocity vector satisfying the following conditions

$$g_{ij} u^i u^j = -1. \tag{5}$$

In Eq. (2), E_i^j is the electromagnetic field given by Lichnerowicz [51]

$$E_i^j = \bar{\mu}\left[h_l h^l \left(u_i u^j + \frac{1}{2} g_i^j\right) - h_i h^j\right], \tag{6}$$

where $\bar{\mu}$ is the magnetic permeability and h_i the magnetic flux vector defined by

$$h_i = \frac{1}{\bar{\mu}} {}^*F_{ji} u^j, \tag{7}$$

where the dual electromagnetic field tensor ${}^*F_{ij}$ is defined by Synge [52]

$${}^*F_{ij} = \frac{\sqrt{-g}}{2} \epsilon_{ijkl} F^{kl}. \tag{8}$$

Here F_{ij} is the electromagnetic field tensor and ϵ_{ijkl} is the Levi-Civita tensor density.

In the present scenario, the comoving coordinates are taken as

$$u^i = \left(0, 0, 0, \frac{1}{A}\right). \tag{9}$$

We choose the direction of string parallel to z-axis so that

$$x^i = \left(0, 0, \frac{1}{A}, 0\right). \tag{10}$$

We consider the current as flowing along the z-axis so that F_{12} is the only non-vanishing component of F_{ij}. Maxwell's equations

$$F_{[ij;k]} = 0, \tag{11}$$

$$\left[\frac{1}{\bar{\mu}} F^{ij}\right]_{;j} = J^i, \tag{12}$$

require that F_{12} is the function of x-alone and the magnetic permeability is the functions of x and t both. The semicolon represents a covariant differentiation.

The Einstein's field equations (with $\frac{8\pi G}{c^4} = 1$)

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j, \tag{13}$$

for the line-element (1) lead to the following system of equations:

$$\begin{aligned} &\frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - \frac{A_1}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right) - \frac{B_1 C_1}{BC} + \frac{B_4 C_4}{BC} = \\ &= \left[\lambda + \frac{F_{12}^2}{2\bar{\mu} A^2 B^2}\right] A^2, \end{aligned} \tag{14}$$

$$\left(\frac{A_4}{A}\right)_4 - \left(\frac{A_1}{A}\right)_1 + \frac{C_{44}}{C} - \frac{C_{11}}{C} = -\left[\frac{F_{12}^2}{2\bar{\mu}A^2B^2}\right]A^2, \tag{15}$$

$$\left(\frac{A_4}{A}\right)_4 - \left(\frac{A_1}{A}\right)_1 + \frac{B_{44}}{B} - \frac{B_{11}}{B} = \left[\frac{F_{12}^2}{2\bar{\mu}A^2B^2}\right]A^2, \tag{16}$$

$$\begin{aligned} -\frac{B_{11}}{B} - \frac{C_{11}}{C} + \frac{A_1}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right) + \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - \frac{B_1C_1}{BC} + \frac{B_4C_4}{BC} = \\ = \left[\rho + \frac{F_{12}^2}{2\bar{\mu}A^2B^2}\right]A^2, \end{aligned} \tag{17}$$

$$\frac{B_{14}}{B} + \frac{C_{14}}{C} - \frac{A_4}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right) - \frac{A_1}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) = 0, \tag{18}$$

where the sub indices 1 and 4 in A, B, C and elsewhere denote ordinary differentiation with respect to x and t respectively.

The velocity field u^i is irrotational. The scalar expansion θ, shear scalar σ^2, acceleration vector $\dot{u}_i$ and proper volume V^3 are respectively found to have the following expressions:

$$\theta = u^i_{;i} = \frac{1}{A}\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right), \tag{19}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\theta^2 - \frac{1}{A^2}\left(\frac{A_4B_4}{AB} + \frac{B_4C_4}{BC} + \frac{C_4A_4}{CA}\right), \tag{20}$$

$$\dot{u}_i = u_{i;j}u^j = \left(\frac{A_1}{A}, 0, 0, 0\right) \tag{21}$$

$$V^3 = \sqrt{-g} = A^2BC, \tag{22}$$

where g is the determinant of the metric (1). Using the field equations and the relations (19) and (20) one obtains the Raychaudhuri's equation as

$$\dot{\theta} = \dot{u}^i_{;i} - \frac{1}{3}\theta^2 - 2\sigma^2 - \frac{1}{2}\rho_p, \tag{23}$$

where dot denotes differentiation with respect to t and

$$R_{ij}u^iu^j = \frac{1}{2}\rho_p. \tag{24}$$

With the help of Eqs. (1)–(4), (9) and (10), the Bianchi identity $\left(T^{ij}_{;j}\right)$ reduced to two equations:

$$\rho_4 - \frac{A_4}{A}\lambda + \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)\rho = 0 \tag{25}$$

and

$$\lambda_1 - \frac{A_1}{A}\rho + \left(\frac{A_1}{A} + \frac{B_1}{B} + \frac{C_1}{C}\right)\lambda = 0. \tag{26}$$

Thus, due to all the three (strong, weak and dominant) energy conditions, one finds $\rho \geq 0$ and $\rho_p \geq 0$, together with the fact that the sign of λ is unrestricted, it may take values positive, negative or zero as well.

3. SOLUTION OF THE FIELD EQUATIONS

As in the case of general-relativistic cosmologies, the introduction of inhomogeneities into the string cosmological equations produces a considerable increase in mathematical difficulty: non-linear partial differential equations must now be solved. In practice, this means that we must proceed either by means of approximations which render the non- linearities tractable, or we must introduce particular symmetries into the metric of the space-time in order to reduce the number of degrees of freedom which the inhomogeneities can exploit.

To get a determinate solution, let us assume that expansion (θ) in the model is proportional to the eigen value $\sigma^1{}_1$ of the shear tensor $\sigma^i{}_j$. This condition leads to

$$A = (BC)^n, \tag{27}$$

where n is a constant. Equations (15) and (16) lead to

$$\frac{F_{12}^2}{\bar{\mu}B^2} = \frac{B_{44}}{B} - \frac{B_{11}}{B} - \frac{C_{44}}{C} + \frac{C_{11}}{C}. \tag{28}$$

and

$$2\left(\frac{A_4}{A}\right)_4 - 2\left(\frac{A_1}{A}\right)_1 + \frac{B_{44}}{B} - \frac{B_{11}}{B} + \frac{C_{44}}{C} - \frac{C_{11}}{C} = 0. \tag{29}$$

Using (27) in (18) reduces to

$$\frac{B_{41}}{B} + \frac{C_{41}}{C} - 2n\left(\frac{B_4}{B} + \frac{C_4}{C}\right)\left(\frac{B_1}{B} + \frac{C_1}{C}\right) = 0. \tag{30}$$

To get the deterministic solution, we assume

$$B = f(x)g(t) \text{ and } C = h(x)k(t) \tag{31}$$

and discuss its consequences below in this paper.

In this case Eq. (30) reduces to

$$\frac{f_1/f}{h_1/h}=-\frac{(2n-1)(k_4/k)+2n(g_4/g)}{(2n-1)(g_4/g)+2n(k_4/k)}=K(\text{constant}). \tag{32}$$

which leads to

$$\frac{f_1}{f}=K\frac{h_1}{h}, \tag{33}$$

and

$$\frac{k_4/k}{g_4/g}=\frac{K-2nK-2n}{2nK+2n-1}=a(\text{constant}). \tag{34}$$

From Eqs. (33) and (34), we obtain

$$f=\alpha h^K, \tag{35}$$

and

$$k=\delta g^a, \tag{36}$$

where α and δ are integrating constants.

From Eqs. (29) and (27), we obtain

$$\begin{aligned}(2n+1)\frac{B_{44}}{B}-2n\frac{B_4^2}{B^2}+(2n+1)\frac{C_{44}}{C}-2n\frac{C_4^2}{C^2}=\\ =(2n+1)\frac{B_{11}}{B}+(2n+1)\frac{C_{11}}{C}-2n\frac{B_1^2}{B^2}-2n\frac{C_1^2}{C^2}=\mathrm{N}\,(\text{constant}).\end{aligned} \tag{37}$$

Eqs. (31) and (37) lead to

$$gg_{44}+rg_4^2=sg^2, \tag{38}$$

where

$$r=\frac{a(a-1)-2n(a+1)}{(2n+1)(a+1)},\qquad s=\frac{N}{(2n+1)(a+1)}.$$

Integrating Eq. (38), we obtain

$$g=\left(c_2e^{bt}+c_3e^{-bt}\right)^{\frac{1}{(r+1)}}, \tag{39}$$

where $b=\sqrt{s(1+r)}$ and c_2, c_3 are constants of integration. Thus from (36), we obtain

$$k=\delta\left(c_2e^{bt}+c_3e^{-bt}\right)^{\frac{a}{(r+1)}}. \tag{40}$$

From Eqs. (33) and (37), we have

$$hh_{11}+\ell h_1^2=mh^2, \tag{41}$$

where

$$\ell = \frac{K(K-1)-2n(K+1)}{(2n+1)(K+1)},$$

$$m = \frac{N}{(2n+1)(K+1)}.$$

Integrating (41), we get

$$h = \left(r_2 e^{\beta x} + r_3 e^{-\beta x}\right)^{\frac{1}{(\ell+1)}}, \tag{42}$$

where $\beta = \sqrt{m(\ell+1)}$ and r_2, r_3 are constants of integration. Eqs. (35) and (42) lead to

$$f = \alpha\left(r_2 e^{\beta x} + r_3 e^{-\beta x}\right)^{\frac{K}{(\ell+1)}}. \tag{43}$$

Hence we obtain

$$B = fg = \alpha\left(r_2 e^{\beta x} + r_3 e^{-\beta x}\right)^{\frac{K}{(\ell+1)}}\left(c_2 e^{bt} + c_3 e^{-bt}\right)^{\frac{1}{(r+1)}}, \tag{44}$$

$$C = \delta\left(r_2 e^{\beta x} + r_3 e^{-\beta x}\right)^{\frac{1}{(\ell+1)}}\left(c_2 e^{bt} + c_3 e^{-bt}\right)^{\frac{a}{(r+1)}}, \tag{45}$$

$$A = (BC)^n = (\alpha\delta)^n\left(r_2 e^{\beta x} + r_3 e^{-\beta x}\right)^{\frac{n(K+1)}{(\ell+1)}}\left(c_2 e^{bt} + c_3 e^{-bt}\right)^{\frac{n(a+1)}{(r+1)}}. \tag{46}$$

Thus the geometry of the space-time (1) reduces to the form

$$\begin{aligned} ds^2 &= (\alpha\delta)^{2n}\left(r_2 e^{\beta x} + r_3 e^{-\beta x}\right)^{\frac{2n(K+1)}{(\ell+1)}}\left(c_2 e^{bt} + c_3 e^{-bt}\right)^{\frac{2n(a+1)}{(r+1)}}(dx^2 - dt^2) + \\ &+ \alpha^2\left(r_2 e^{\beta x} + r_3 e^{-\beta x}\right)^{\frac{2K}{(\ell+1)}}\left(c_2 e^{bt} + c_3 e^{-bt}\right)^{\frac{2}{(r+1)}} dy^2 + \\ &+ \delta^2\left(r_2 e^{\beta x} + r_3 e^{-\beta x}\right)^{\frac{2}{(\ell+1)}}\left(c_2 e^{bt} + c_3 e^{-bt}\right)^{\frac{2a}{(r+1)}} dz^2. \end{aligned} \tag{47}$$

4. SOME PHYSICAL AND GEOMETRIC PROPERTIES

The energy density (ρ), the string tension density (λ), the particle density (ρ_p) for the model (47) are given by

$$\rho = \frac{1}{(\alpha\delta)^{2n}\left(r_2 e^{\beta x} + r_3 e^{-\beta x}\right)^{\frac{2n(K+1)}{(\ell+1)}}\left(c_2 e^{bt} + c_3 e^{-bt}\right)^{\frac{2n(a+1)}{(r+1)}}} \times$$

$$\times\left[\frac{\beta^2\{n(K+1)^2 + K(\ell-K) + \ell\}}{(\ell+1)^2}\frac{\left(r_2 e^{\beta x} - r_3 e^{-\beta x}\right)^2}{\left(r_2 e^{\beta x} + r_3 e^{-\beta x}\right)^2} + \right.$$

$$+\frac{b^2\{n(a+1)^2+a\}}{(r+1)}\frac{\left(c_2e^{bt}-c_3e^{-bt}\right)^2}{\left(c_2e^{bt}+c_3e^{-bt}\right)^2}-\frac{\beta^2(K+1)}{(\ell+1)}-$$

$$\left.-\frac{F_{12}^2}{2\bar{\mu}\alpha^2\left(r_2e^{\beta x}+r_3e^{-\beta x}\right)^{\frac{2K}{(\ell+1)}}\left(c_2e^{bt}+c_3e^{-bt}\right)^{\frac{2}{(r+1)}}}\right], \tag{48}$$

$$\lambda=\frac{1}{(\alpha\delta)^{2n}\left(r_2e^{\beta x}+r_3e^{-\beta x}\right)^{\frac{2n(K+1)}{(\ell+1)}}\left(c_2e^{bt}+c_3e^{-bt}\right)^{\frac{2n(a+1)}{(r+1)}}}\times$$

$$\times\left[\frac{b^2\{a(a-r)+r-n(a+1)^2\}}{(r+1)^2}\frac{\left(c_2e^{bt}-c_3e^{-bt}\right)^2}{\left(c_2e^{bt}+c_3e^{-bt}\right)^2}+\right.$$

$$+\frac{b^2(a+1)}{(r+1)}-\frac{\beta^2\{K(n+1)+n\}^2}{(\ell+1)}\frac{\left(r_2e^{\beta x}-r_3e^{-\beta x}\right)^2}{\left(r_2e^{\beta x}+r_3e^{-\beta x}\right)^2}- \tag{49}$$

$$\left.-\frac{F_{12}^2}{2\bar{\mu}\alpha^2\left(r_2e^{\beta x}+r_3e^{-\beta x}\right)^{\frac{2K}{(\ell+1)}}\left(c_2e^{bt}+c_3e^{-bt}\right)^{\frac{2}{(r+1)}}}\right],$$

where

$$F_{12}^2=\bar{\mu}\alpha^2\left(r_2e^{\beta x}+r_3e^{-\beta x}\right)^{\frac{2K}{(\ell+1)}}\left(c_2e^{bt}+c_3e^{-bt}\right)^{\frac{2}{(r+1)}}\times$$

$$\times\left[\frac{b^2(1-a)}{(r+1)}\left\{1-\frac{(r-a)}{(r+1)}\frac{\left(c_2e^{bt}-c_3e^{-bt}\right)^2}{\left(c_2e^{bt}+c_3e^{-bt}\right)^2}\right\}+\right. \tag{50}$$

$$\left.+\frac{\beta^2(1-K)}{(\ell+1)}\left\{1-\frac{(\ell-K)}{(\ell+1)}\frac{\left(r_2e^{\beta x}-r_3e^{-\beta x}\right)^2}{\left(r_2e^{\beta x}+r_3e^{-\beta x}\right)^2}\right\}\right].$$

$$\rho_p=\frac{1}{(\alpha\delta)^{2n}\left(r_2e^{\beta x}+r_3e^{-\beta x}\right)^{\frac{2n(K+1)}{(\ell+1)}}\left(c_2e^{bt}+c_3e^{-bt}\right)^{\frac{2n(a+1)}{(r+1)}}}\times$$

$$\times\left[\frac{\beta^2\{n(K+1)(K+2)+K(\ell-K)+(\ell+n)\}}{(\ell+1)^2}\frac{\left(r_2e^{\beta x}-r_3e^{-\beta x}\right)^2}{\left(r_2e^{\beta x}+r_3e^{-\beta x}\right)^2}+\right. \tag{51}$$

$$\left.+\frac{b^2(a-r)(1-a)}{(r+1)^2}\frac{\left(c_2e^{bt}-c_3e^{-bt}\right)^2}{\left(c_2e^{bt}+c_3e^{-bt}\right)^2}-\frac{\beta^2(K+1)}{(\ell+1)}+\frac{b^2(a+1)}{(r+1)}\right].$$

The scalar of expansion (θ), shear tensor (σ), acceleration vector $\dot{u}_i$ and the proper volume for (V^3) for the model (47) are given by

$$\theta = \frac{(n+1)(a+1)b\left(c_2e^{bt} - c_3e^{-bt}\right)}{(r+1)(\alpha\delta)^n\left(r_2e^{\beta x} + r_3e^{-\beta x}\right)^{\frac{n(K+1)}{(\ell+1)}}\left(c_2e^{bt} + c_3e^{-bt}\right)^{\frac{n(a+1)}{(r+1)}+1}}, \tag{52}$$

$$\sigma^2 = \frac{b^2\{(a+1)^2(n^2-n+1)-3a\}\left(c_2e^{bt} - c_3e^{-bt}\right)^2}{3(r+1)^2(\alpha\delta)^{2n}\left(r_2e^{\beta x} + r_3e^{-\beta x}\right)^{\frac{2n(K+1)}{(\ell+1)}}\left(c_2e^{bt} + c_3e^{-bt}\right)^{\frac{2n(a+1)}{(r+1)}+2}} \tag{53}$$

$$\dot{u}_i = \left(\frac{n(K+1)\beta}{(\ell+1)}\frac{\left(r_2e^{\beta x} - r_3e^{-\beta x}\right)}{\left(r_2e^{\beta x} + r_3e^{-\beta x}\right)}, 0, 0, 0\right), \tag{54}$$

$$V^3 = (\alpha\delta)^{2n+1}\left(r_2e^{\beta x} + r_3e^{-\beta x}\right)^{\frac{(2n+1)(K+1)}{(\ell+1)}}\left(c_2e^{bt} + c_3e^{-bt}\right)^{\frac{(2n+1)(a+1)}{(r+1)}}. \tag{55}$$

From Eqs. (52) and (53), we obtain

$$\frac{\sigma^2}{\theta^2} = \frac{(a+1)^2(n^2-n+1)-3a}{3(n+1)^2(a+1)^2} = \text{constant}. \tag{56}$$

The model (47) represents an expanding, shearing and non-rotating universe. The expansion in the model increases as time increases when $n < 0$ but the expansion in the model decreases as time increases when $n > 0$. The spatial volume V increases as time increases. If we set the suitable values of constants, we find that energy conditions $\rho \geq 0$, $\rho_p \geq 0$ are satisfied. We observe that $\frac{\sigma}{\theta}$ is constant throughout. Hence the model does not approach isotropy. The energy density (ρ) and string tension density (λ) decrease as electromagnetic field component (F_{12}) increases. The electromagnetic field tensor (F_{12}) becomes in uniform state when $x = 0$, $t = 0$ and it increases when x and t increase.

5. CONCLUDING REMARKS

In this paper, we have investigated the behaviour of a string in the cylindrically symmetric inhomogeneous universe in presence of electromagnetic field with perfect fluid distribution. Generally the model is expanding, shearing, non-rotating and accelerating. The solution obtained in this paper is new and different from the other author's solutions. In this solution all physical and kinematical quantities depend on at most one space coordinate and the time.

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