WIND-SHEAR EFFECT ON THE INSTABILITIES IN THE MID-LATITUDE E REGION PLASMA

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Abstract. In this paper we consider the effect of a vertical wind shear on the dispersion relationship for the gradient drift instability. For small-scale waves the effect is negligible, but for larger scale waves the shear can increase the growth rate. However, the effect of the wind shear is better seen in the decrease of the marginal relative drift velocity, which means that the presence of a wind shear favors the generation of GD waves.

Key words: ionospheric plasma instabilities, mid-latitude E region.

1. INTRODUCTION

In the mid-latitude E region most of the coherent backscatter radar echoes are the result of the gradient-drift (GD) instability. The collision frequencies \( v_e \) and \( v_i \) of the plasma components with the neutrals are different and vary strongly with the altitude. For altitudes between 80 and 120 km the electrons are magnetized and their movement normally to the magnetic field is limited. The ions, on the other hand, are unmagnetized and, at these latitudes, are subject to the neutral drag. Usually, the dispersion relationship for the E region waves takes into consideration only the electric field and its effect on the electrons, while the ions are considered at rest (since at low altitudes they are unaffected by the electric field) [1]. It has been shown that in the lower E region, at 100–120 km, the velocity of the neutral wind reaches values of 120–150 m/s. These values are comparable or even greater than the electron speeds induced by the electric field, thus the ion movement cannot be neglected [2]. Following this observation, the dynamics of the neutral component of the plasma has been included in [2], who have shown that the neutral wind itself could generate an instability whose growth rate is at least comparable with a similar instability generated by the electric field. In this altitude range, the neutral wind is not constant with the altitude. On the contrary, vertical shears in the winds (abrupt change of the wind velocity on a relatively small scale length) can have sometimes values of 0.2 s\(^{-1}\). In the present paper we will investigate, theoretically, the effect of a wind shear on the phase velocity and on the growth
rate for the GD instability occurring in the E region for different background conditions. The GD instability occurs in the ionospheric E region plasma (see for instance [1, 3]).

2. MODEL AND RESULTS

The calculation of the gradient drift dispersion relationship starts from the bi-fluid model of the plasma, where the gravity and the electron production and loss are neglected and where fluctuations of the geomagnetic field, B, are too small at this altitude to be considered.

\[
\frac{\partial n}{\partial t} + \text{div}(n \cdot \mathbf{V}) = 0
\]  

(1)

\[
\frac{\partial n}{\partial t} + \text{div}(n \cdot \mathbf{V}) = 0
\]  

(2)

\[-\nabla p - ne(E + \mathbf{V}_e \times \mathbf{B}) - mn\mathbf{v}_e(V_e - \mathbf{U}) = 0\]

(3)

\[
\left(\frac{\partial}{\partial t} + \mathbf{V}_i \cdot \nabla\right)\mathbf{V}_i = -\nabla p + ne(E + \mathbf{V}_i \times \mathbf{B}) - mn\mathbf{v}_i(V_i - \mathbf{U}) = 0
\]

(4)

where \(\mathbf{V}_{e,i}\) are the electron, respectively ion, velocities, \(\mathbf{U}\) the neutral wind (neutral component velocity) and \(\mathbf{v}_{i,e}\) – the collision frequencies of the charged plasma particles with the neutrals. The wind shear is described by

\[w = \frac{\partial U_z}{\partial z},\]

with \(z\) the altitude and \(V_{ixy}\) the horizontal component of the ion velocity (\(V_i = \mathbf{U}\)).

The perturbations in this system are:

\[f = f_0 \exp\left(i(\omega t - k \cdot \mathbf{r})\right),\]  

where \(\omega = \omega_r - i\gamma\).

The following notations are introduced:

\[\Omega_e = -\frac{eB}{m_e}, \quad \Omega_i = \frac{eB}{m_i}\]

(5)

where \(m_{e,i}\) are the electron, respectively ion, mass and \(B\) the terrestrial magnetic field

\[\Psi = \frac{\mathbf{v}_i \cdot \mathbf{V}_i}{\Omega_e \Omega_i},\]

(6)

\[\mathbf{V}_d = \mathbf{V}_e - \mathbf{V}_i,\]

(7)
and the density gradient scale length, which measures the plasma density, \( n \), variations:

\[
L = \frac{1}{n} \left( \frac{\partial n}{\partial z} \right)^{-1}.
\]  

(8)

The detailed calculus of the dispersion relationship is given in [3] and here, for the sake of brevity, we will outline only the main steps of that calculation. Each perturbed quantity can be written:

\[
F_1 = F + f,
\]

where \( F \) is the unperturbed quantity, \((N, V_e, V_i, \phi)\) and \( f \) is the linear perturbation given above.

The calculation is done under several assumptions:

- the electron inertial effects are negligible;
- the wind has a zonal component \((U_x)\);
- the electron velocity can be perturbed in any directions, while for the ion the propagation is restricted only in the \( x \) direction. This assumption is justified also by the fact that in the lower \( E \) region the ions are practically unmagnetized:

The linearly perturbed equations are projected on the three axes of the coordinate system so that a system of equations is obtained (see [3]). The condition for non-trivial solution leads to:

\[
\begin{align*}
\omega - k \cdot V_e &= \left( -\frac{i G_{eL}}{k_x L \Omega_i} + \frac{\psi}{V_i} \right) \\
&\times \left[ (\omega - k \cdot V_i) (i k \cdot V_i - i \omega - V_i) + i k_i^2 C_s^2 \right],
\end{align*}
\]

where the fact that \( \beta_e^2 \gg 1 \) in the \( E \) region was taken into account. The real and imaginary parts of \( \omega \) are, as shown above, the angular frequency, \( \omega_r \), and the growth rate, \( \gamma \), \( \omega = \omega_r - i \gamma \). In the linear approximation, \( \gamma \ll \omega_r \) and \( \gamma \ll \nu_i \).

For parallel propagation \( k_p \ll k_\perp \) and \( k \equiv k_\parallel \), \( \alpha \equiv 0 \), the wave frequency, \( \omega \), and the growth rate, \( \gamma \), are:

\[
\omega = \frac{k \cdot (V_e + \psi V_i)}{1 + \psi} = \omega^*,
\]  

(9)

\[
\gamma = \frac{\psi}{V_i (1 + \psi)} \left[ \left( \frac{k \cdot V_i}{1 + \psi} \right)^2 - k^2 C_s^2 \right] + \frac{V_i}{\Omega_i} \cos k \cdot \frac{k \cdot V_i}{L \cdot k} \left[ \frac{1}{1 + \psi} \right]^2 + \frac{\psi}{1 + \psi} \cdot \nu_i = \gamma^* + \frac{\psi}{1 + \psi} \cdot \nu_i.
\]  

(10)
where \( I \) is the inclination of the geomagnetic field line (the angle between the horizontal plane and the field vector, positive downwards) the stars stand for the “classic” frequency and growth rate, \( i.e. \) for the case when the ions are considered at rest and the dispersion relationship is considered in a frame moving with the neutrals.

Note that the wave frequency given by equation (9) is not modified ([3]). On the other hand equation (10) shows that the growth rate is different when the ion movement is considered compared to the case when ions are assumed to be at rest. When the shear values are below about \( 0.2 \text{ s}^{-1} \), the influence of the wind could be sometimes neglected. However for larger values of the shear the usual assumption that ions are at rest must be revised.

Figs. 1 and 2 depict the growth rate for different values for the wavelength, \( \lambda \), relative drift velocity, \( V \), sound velocity, \( C \), and gradient scale, \( L \), which are shown next to the figure.

Fig. 1 shows that for decameter scale waves, medium drift velocities and relatively abrupt plasma density variations, the two curves coincide, there is no difference between the growth rates with or without shear.

In Fig. 2 waves with a higher order of magnitude are considered. The growth rate is very small, thus the effect of the shear could be important, especially at smaller gradient scale lengths, \( i.e. \) for abrupt variations of the plasma density, as it happens at the walls of an \( \text{Es} \) layer, for instance.

The marginal drift is the smallest velocity of the electron drift relatively to the ions that generates the instability and it is given by [1]:

\[
k \cdot V_{dlimit} = kC_s(1 + \psi)
\]

\[\times \left[ F - \left( F^2 + 1 \right)^{\frac{1}{2}} \right], \tag{11}\]

where \( F = \frac{\nu_e \Omega_m \cos I}{2
\nu_e L k^2 C_s} \).

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Fig. 1 – The variation of \( \gamma \) (with shear) and of \( \gamma^* \) (without shear) with increasing altitude for decameter waves.
The marginal drift when the shear is included is:

\[ k \cdot V_{\text{dlim}} = kC_s(1 + \psi) \left[ F - \left( F^2 + G + 1 \right)^{1/2} \right], \tag{12} \]

where another term, \( G = \frac{wv_l}{k^2C_s^2} \), appears due to the shear. The variation of the marginal velocities is shown below, in Figs. 3 and 4.

For small wavelengths the difference between the two marginal drifts is negligible. The marginal drift decreases significantly below 105 km with the increase

Fig. 2 – The variation of \( \gamma \) (with shear) and of \( \gamma^* \) (without shear) with increasing altitude for larger scale waves.

Fig. 3 – The variation of the marginal drift velocities function of altitude with \( V_{\text{dlim}} \) and without \( V_{\text{dlim}}^* \) the shear for a 50 m wave.
increase of the wavelength. Also, the decrease of the gradient scale length contributes to the effect of the shear on the generation.

These plots show that the influence of the shear is important for wavelengths of the order of hundreds of meters and for high scale gradients, which means a relatively shallow density variation with height. For a wave with $\lambda = 100$ m the shear significantly reduces the marginal drift. The smallest electronic relative drift that can initiate the GD instability is reduced in the presence of the shear with 20–30 m/s.

3. CONCLUSIONS

The effect of the shear on the growth rate is negligible for small-scale waves. The contribution of the shear becomes important for large-scale waves, as long as
the local approximation ([4]) still holds. Also, the shear should be considered when large gradient scales are involved. The marginal drift is reduced in these cases.

Concluding, the vertical shear of the horizontal wind can be safely neglected for small-scale waves and/or abrupt density gradients but should be considered for large-scale waves and shallow variations of the plasma density in the low E region. The shear does not modify the frequency of the wave and has little effect on the wave development. In the presence of a appropriately oriented shear the GD instability is favored for waves of tens and hundreds of meters. These waves lead to secondary waves, via the cascade mechanism ([5]) so that finally the coherent backscatter echoes observed at VHF (corresponding to small-scale waves) should be seen more often when vertical shears of the wind are present. Coincident observations of echoes and wind measurements could be used to verify the theoretical results shown in this paper.

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REFERENCES