Romanian Reports in Physics, Vol. 60, No. 4, P. 1065–1070, 2008

APPLICATION OF THE MATRIX FORMALISM IN A MUELLER MATRIX IMAGING POLARIMETRY★

O. TOMA, E. DINESCU

Faculty of Physics, University of Bucharest, P.O.Box MG-11,
0771253 Bucharest – Magurele, Romania
E-mail: thtoma72@yahoo.com

(Received September 1, 2008)

Abstract. We present the practical realization and the theoretical analysis in the frame of the Mueller matrix formalism of a Mueller imaging polarimeter, functioning in transmission. The Mueller matrices of both the polarization state generator and the polarization state analyzer of the Mueller matrix imaging polarimeter will be theoretically calculated.

Key words: light polarization, anisotropy, Mueller matrix polarimetry.

1. INTRODUCTION

Ellipsometry is a modern technique with lot of applications in various fields of physics and science, in general [1]. Some of the up to date directions of the modern ellipsometry are: the dynamic ellipsometry [2], the spectroscopic ellipsometry [3] and the imaging polarimetry [4].

The description of various ellipsometers can be handled in various matrix (Jones and Mueller) [5–8], or pure operatorial [9–12] formalisms.

This work is focused on the theoretical calculation, based on a Mueller matrix approach of the Mueller matrices for PSG (polarization state generator) and PSA (polarization state analyzer) systems.

As it is known, Mueller matrix polarimetry imaging [13] is a powerful imaging technique used to provide high precision measurements for the Mueller matrices at every pixel of an image captured with a CCD detector.

We remind two classical configurations Fig. 1 for a Mueller matrix imaging polarimeter.

The first configuration uses a focused beam: the sample (which must posses a polarization inhomogeneous structure depending on its internal structure and on the different interfaces) that might be a biological tissue, or a thin film, is illuminated

★ Paper presented at the Annual Scientific Conference, June 6, 2008, Faculty of Physics, Bucharest University, Romania.

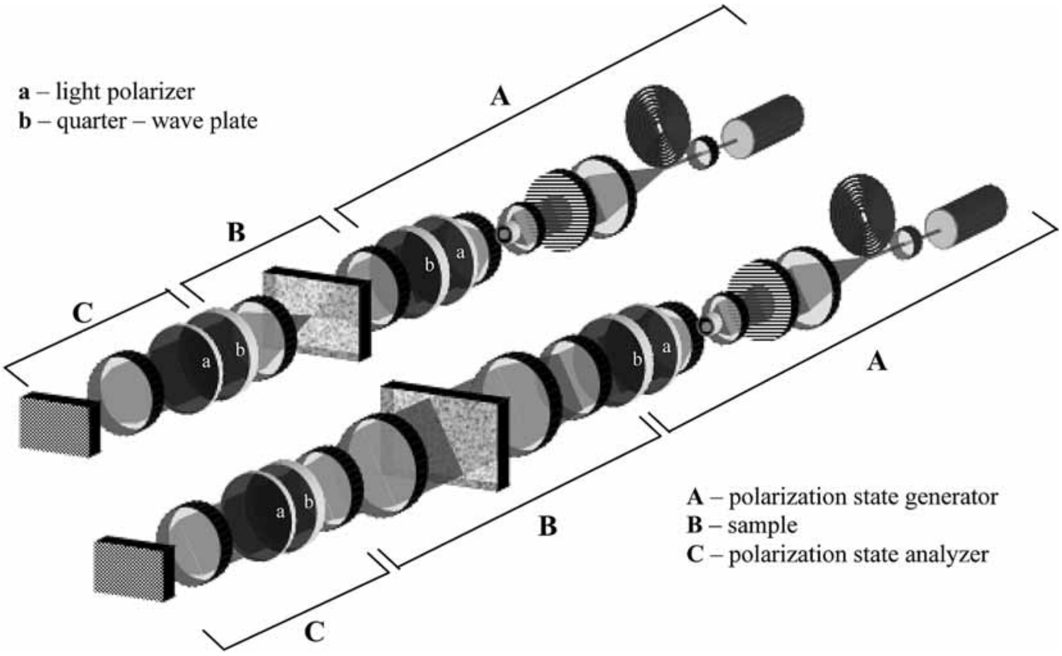

Fig. 1 – Classical configurations for a Mueller matrix imaging polarimeter, in the focused beam (above) or expanded beam (below) setup: a) light polarizer; b) quarter – wave plate.

with a focused beam and a second lens (or microscope objective) collects the transmitted light. This second lens is imaged onto the CCD, so that each pixel will correspond to a different incidence angle on the sample. Thus, this configuration is used for determining how the polarization properties of a plan sample vary with the incident direction of the light.

The second configuration is in the expanded beam and it is used to confirm the level of polarization aberrations in the beam expanding optics (this kind of polarimeter have increased costs because the polarization elements costs increase exponentially with the clear aperture).

Both these systems permit the acquisition of 16 Mueller matrix images and therefore the possibility of extracting a relatively reduced quantity of information regarding the polarization properties of the investigated sample.

2. EXPERIMENTAL SET-UP

In Fig. 2 it is presented the optical scheme [14] used in the acquisition of the Mueller matrix images. We have build up this polarimeter using a blue laser diode (with the wavelength of 457 nm) and with all the optical polarization components adapted to this wavelength.

Fig. 2 – Optical scheme of the Mueller polarization imaging system.

A collimated laser beam (1), enters into the PSG system formed by two quarter-wave plates (3) and (5) and a polarizer (4).

Thus, this system provides, for different rotation angles of its mobile parts, the corresponding Stokes vectors of the incident beam of light on the sample (6).

The Mueller matrix polarization images are projected onto the plane of a light sensitive CCD camera (10) with the minimum resolution 800 × 600 pixels, using a microscope objective (7) with approx. focal length 15 cm. This experimental set-up permits the recording of 24 polarization images, thanks to the supplementary quarter-wave plate (5) and therefore the possibility of extracting more information regarding the investigated object.

The PSA system is formed by a quarter-wave plate (8) and another polarizer (9), so this system generates the ensemble of the Stokes vectors for the light arriving at the CCD.

3. THEORETICAL CONSIDERATIONS AND RESULTS

We will obtain some analytical expressions for the Mueller matrix elements for both systems involved in our experimental set-up: the polarization state generator system (PSG) and the polarization state analyzer system (PSA).

The first quarter-wave plate (3) (an ideal, homogeneous, linear retarder with ρ – the azimuth of its fast axis, equal to 0), has the following associated matrix:

$$M_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}. \tag{1}$$

The rotating polarizer (4) (an ideal, homogeneous, linear polarizer, with θ – the azimuth of its transmission axis) has this corresponding matrix:

$$M_2 = \frac{1}{2}\begin{pmatrix} 1 & \cos 2\theta & \sin 2\theta & 0 \\ \cos 2\theta & \cos^2 2\theta & \cos 2\theta \sin 2\theta & 0 \\ \sin 2\theta & \cos 2\theta \sin 2\theta & \sin^2 2\theta & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{2}$$

The second quarter-wave plate (5) (an ideal, homogeneous, linear retarder with ρ # 0) has its corresponding matrix as it follows:

$$M_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 2\rho & \cos 2\rho \sin 2\rho & -\sin 2\rho \\ 0 & \cos 2\rho \sin 2\rho & \sin^2 2\rho & \cos 2\rho \\ 0 & \sin 2\rho & -\cos 2\rho & 0 \end{pmatrix}. \tag{3}$$

Thus, the polarization state generator matrix will be:

$$M_G = M_3 M_2 M_1, \tag{4}$$

and the Mueller matrix elements are calculated below:

$m_{11} = 1$

$m_{12} = \cos 2\theta$

$m_{13} = 0$

$m_{14} = \sin 2\theta$

$m_{21} = \cos 2\rho \cos 2(\delta - \theta)$

$m_{22} = \cos 2\rho \cos 2\theta \cos 2(\rho - \theta)$

$m_{23} = 0$

$m_{24} = \cos 2\rho \sin 2\theta \cos 2(\rho - \theta)$

$m_{31} = \sin 2\rho \cos 2(\rho - \theta)$

$m_{32} = \sin 2\rho \cos 2\theta \cos 2(\rho - \theta)$

$m_{33} = 0$

$m_{34} = \sin 2\rho \sin 2\theta \cos 2(\rho - \theta)$

$m_{41} = \sin 2(\rho - \theta)$

$m_{42} = \cos 2\theta \sin 2(\rho - \theta)$

$m_{43} = 0$

$m_{44} = \sin 2\theta \sin 2(\rho - \theta)$.

In the same manner we will calculate the elements corresponding to the PSA system.

So, the rotating quarter-wave plate (8) (an ideal, homogeneous, linear retarder with ρ # 0) and the rotating polarizer (9) (an ideal, homogeneous, linear polarizer, with θ – the azimuth of its transmission axis) have their associated matrices M_3 and, respectively, M_2, described by the relations (3) and (2).

Therefore, the polarization state analyzer matrix will be:

$$M_A = M_2 M_3 \tag{5}$$

and the Mueller matrix elements are also calculated below:

$$m_{11} = 1 \qquad m_{31} = \sin 2\theta$$
$$m_{12} = \cos 2\rho \cos 2(\rho - \theta) \qquad m_{32} = \sin 2\theta \cos 2\rho \cos 2(\rho - \theta)$$
$$m_{13} = \sin 2\rho \cos 2(\rho - \theta) \qquad m_{33} = \sin 2\rho \sin 2\theta \cos 2(\rho - \theta)$$
$$m_{14} = \sin 2(\theta - \rho) \qquad m_{34} = \sin 2\theta \sin 2(\theta - \rho)$$
$$m_{21} = \cos 2\theta \qquad m_{41} = 0$$
$$m_{22} = \cos 2\rho \cos 2\theta \cos 2(\rho - \theta) \qquad m_{42} = 0$$
$$m_{23} = \sin 2\rho \cos 2\theta \cos 2(\rho - \theta) \qquad m_{43} = 0$$
$$m_{24} = \cos 2\theta \sin 2(\theta - \rho) \qquad m_{44} = 0\,.$$

Each rotating polarization component of the experimental set-up permit us the obtaining of different values for the angles θ and ρ. Thus, different values for the Mueller matrix elements corresponding to the PSG and PSA systems Mueller matrices can be obtained, and from these matrices an important quantity of information regarding the polarization properties of the investigated sample can be extracted.

4. CONCLUSIONS

The calculus of the elements corresponding to the Mueller matrices for the polarization state generator and the polarization state analyzer systems involved in this Mueller matrix imaging polarimeter, was performed.

In comparison with the classical acquisition of 16 Mueller matrix images of the sample, the experimental set-up used permits the recording of 24 polarization images and therefore the possibility of extracting more information regarding the investigated object.

REFERENCES

1. R. M. Azzam, N. M. Bashara, *Ellipsometry and Polarized Light*, Elsevier, Amsterdam, 1996.
2. E. Bernabeu, J. J. Gil, *An experimental device for the dynamic determination of Mueller matrices*, J. Opt., **16**, *3*, 139–141 (1985).
3. H. Fujiwara, *Spectroscopic ellipsometry, Principles and applications*, John Wiley and Sons Ltd., 2007.
4. J. S.Tyo, D. L. Goldstein, D. B. Chenault, J. A. Shaw, *Review of passing image polarimetry for remote sensing applications*, Appl. Opt., **45**, 5452–5469 (2006).
5. D. H. Goldstein, E. Collett, *Polarized Light*, CRS Press, NewYork, 2003.
6. S. Huard, *Polarisation de la lumière*, Masson, Paris, 1994.
7. S. Y. Lu, R. A. Chipman, *Mueller matrices and the degree of polarization*, Opt. Commun., **146**, 11–14 (1998).
8. S. N. Savenkov, *Optimization and structuring of the instrument matrix for polarimetric measurements*, Opt. Eng., **41**, 965–972 (2002).

9. T. Tudor, *Spectral analysis of the device operators in polarization dynamics*, J. Mod. Opt., **48**, *11*, 1669–1689 (2001).
10. T. Tudor, *Dirac algebraic approach to the theory of device operators in polarization optics*, J. Opt. Soc. Am. A, **20**, *4*, 728–732 (2003).
11. T. Tudor, *Operatorial form of the theory of polarization optical devices: II. Spectral theory of the composite devices,* Optik, **115**, *5*, 173–180 (2004).
12. T. Tudor, *Operatorial form of the theory of polarization optical devices: I. Spectral theory of the basic devices*, Optik, **114**, *12*, 539–547 (2003).
13. J. L. Pezzaniti, R. A. Chipman, Opt. Eng., **34**, 1558 (1995).
14. A. Ushenko, S. Yermolenko, A. Prydij, S. Guminetsky, I. Gruia, O. Toma, K. Vladychenko, *Statistical and fractal approaches in laser polarimetry diagnostics of the cancer prostate tissues*, Proc. of SPIE, Volume 2008, Bellingham WA, 2008.