THEORETICAL STUDY
OF HIGH POWER LASERS CALORIMETRY

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Abstract. In the present paper, we discuss the differences between classical laser calorimetry
and high power laser calorimetry. We want to answer the fundamental question: Is it realistic, for
example, to speak up about free electron laser (FEL) calorimetry? The first problem has a simple
answer: the difference (between the two kind of calorimetries) is made by the huge intensity provided
by free electron lasers or CO₂ (KW) in comparison with other “atomic” lasers. This observation leads
us to the second question. The problem is how to avoid the thermal runaway. We have found three ways.

Key words: high power lasers, heat transfer.

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1. INTRODUCTION

For the last twenty years the advances in laser processing [1] have encouraged
the development of model calculations for spatial and temporal temperature fields in
laser heated solids. Recently the integral transform method [2] has been successfully
applied to the study of thermal fields in laser-solid interaction [3–5]. Elaborated
mathematical models, both analytical and numerical, have been developed for
describing the heat flow under a large number of simplifying approximations and
assumptions regarding the laser beam and the sample, like for example in the case of
thermal photoemission from a dispenser cathode of a free electron laser.

In order to understand the physical absorption process, it is better and
necessary to take into account of the multi-photon processes.

Three-dimensional model calculations in which full account is been taken
not only of the two-, three- and four-photon absorption but also of the time and
space characteristics of the laser beam as a heating source. The influence of the
two-, three- and four-photon absorption coefficient in establishing the thermal
profiles is also discussed. Although the influence of the heat transfer coefficient is
of great importance. Nonlinear spectroscopy has proved to be invaluable in
determining the optical and electronic properties of crystalline solids; e.g., when
one-photon absorption is forbidden by the selection rules, a higher-order multi-
photon absorption may be allowed. That is valid even when one-photon absorption
is allowed, because most of the experimentally studied properties are characteristic
to the surface rather than to the volume as a consequence of the drastic attenuation
of the radiation as it is propagating into the sample. Multi-photon experiments, on
the other hand, can enable the study on the properties of the crystalline volume,
because of the significantly smaller values of the multi-photon coefficients.

2. THE GENERAL ANALYTICAL MODEL FOR HEAT EQUATION
IN LASER-SOLID INTERACTION

In the present paper, the macroscopic heat equation is employed to
investigate the temperature field in a semiconductor [6–8] exposed to a free
electron laser with a Gaussian spatial profile and a rectangular nanosecond pulse.
The sample is supposed to be homogeneous and therefore there is no angular
dependence on the temperature variation. The equation describing the heat
diffusion inside a cylindrical solid sample irradiated by a laser beam centered to the
probe, is fully described by the partial differential equation:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} - \frac{1}{\gamma} \frac{\partial T}{\partial t} = \frac{-A(r, \varphi, z, t)}{k},$$

(1)

where: $k$ – the sample thermal conductivity; $\gamma$ – the sample thermal diffusivity
($\gamma = \frac{k}{c \cdot \rho}$); $c$ – the sample heat capacity; $\rho$ – the sample mass density. The tempera-
ture, $T$, is a function of $(r, z, t)$ and it is defined here as a temperature variation
rather than an absolute temperature: $T(r, z, t) = T_{f} - T_{in}$, where $T_{f}$ and $T_{in}$ are the
final and the initial absolute temperatures of the sample. If we consider a linear
heat transfer at the sample surface (the “radiation” boundary condition), than:

$$k \frac{\partial T(r, z, t)}{\partial r} \bigg|_{rz=b} + hT(b, z, t) = 0,$$

$$k \frac{\partial T(r, z, t)}{\partial r} \bigg|_{z=0} - hT(r, 0, t) = 0,$$

$$k \frac{\partial T(r, z, t)}{\partial r} \bigg|_{z=a} + hT(r, a, t) = 0,$$

(2)
where: $h$ – the heat transfer coefficient of the sample surface; $a$, $b$ – the thickness and the radius of the sample respectively. In the presence of one-, two-, three- and four-absorption, described by coefficients $\alpha_i$, the change in the intensity of the light as it passes through the sample is given by [9]:

$$\frac{dI}{dx} = -\alpha_1 I - \alpha_2 I^2 - \alpha_3 I^3 - \alpha_4 I^4,$$

when free carrier absorption is negligible.

In order to consider the multi-photon absorption, the heat rate per unit volume and time will be determined by the laser intensity according to Beer’s law:

$$A(r, z, t) = (\alpha_1 \cdot I_{00}(r, z) + \alpha_2 I_{00}^2(r, z) + \alpha_3 I_{00}^3 + \alpha_4 I_{00}^4 + r_S \delta(z)) \cdot (h(t) - h(t - t_0)), \quad (3)$$

where $t_0$ is the pulse duration and $h(t)$ is the step function and $r_S$ is the surface absorption coefficient.

The solution of the heat equation is:

$$T_{\alpha_1, \alpha_2, \alpha_3, \alpha_4}(r, z, t) =$$

$$= \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \left[ \frac{1}{1 + \frac{\gamma}{2} + \lambda_j^2} \cdot f_{\alpha_1, \alpha_2, \alpha_3, \alpha_4}(\mu_i, \lambda_j) \cdot (1 - e^{-\theta_i^j}) - (1 - e^{-\theta_j^j(t-t_0)}) \cdot h(t - t_0) \right] \times$$

$$\times K_r(\mu_i, r) \cdot K_z(\lambda_j, z),$$

where: $\theta_j^j = \gamma(\mu_i^2 + \lambda_j^2)$ and

$$f_{\alpha_1, \alpha_2, \alpha_3, \alpha_4}(\mu_i, \lambda_j) = \frac{1}{k C_i C_j} \int_0^a \delta^\mu \cdot \delta^\lambda_{r0} \cdot \delta^\lambda_{\mu i} \cdot \delta^\lambda_{\lambda j} \cdot (\alpha_1 I_{00} + \alpha_2 I_{00}^2(r, z) + \alpha_3 I_{00}^3 + \alpha_4 I_{00}^4 + r_S \delta(z)) \cdot$$

$$K_r(\mu_i, r) \cdot K_z(\lambda_j, z) \cdot \frac{a}{b} dr dz.$$

The coefficients $C_i$ and $C_j$ are normalizing coefficients. The eigenvalues $\mu_i$ and $\lambda_j$ correspond to the eigenfunctions $K_r(\mu_i, r)$ and $K_z(\lambda_j, z)$. The integral operators corresponding to the eigenfunctions $K_r(\mu_i, r)$ and $K_z(\lambda_j, z)$ are normalized by the following coefficients:

$$C_i = \int_0^b K_r^2(\mu_i, r) dr = \frac{b^2}{2 \mu_i^2} \left[ \frac{h}{k} \lambda_j + \frac{h}{k} \lambda_j + \frac{h}{k} \lambda_j - 2 \frac{h}{k} \lambda_j \cos(2a \lambda_j) - \frac{h}{k} \lambda_j \sin(2a \lambda_j) \right].$$

$$C_j = \int_0^a K_z^2(\lambda_j, z) dz = 1 \left[ \frac{2 h}{k} \lambda_j + 2a \frac{h}{k} \lambda_j + 2a \lambda_j^2 - 2 \frac{h}{k} \lambda_j \cos(2a \lambda_j) - \frac{h}{k} \lambda_j \sin(2a \lambda_j) \right].$$
The eigenvalues $\mu_i$ and $\lambda_j$ are determined from the boundary conditions by the following equations [3]:

$$\frac{h}{k} J_0(\mu_i b) - \mu_i J_1(\mu_i b) = 0$$

and

$$2 \cdot \cot(\lambda_j a) = \frac{\lambda_j}{h} - \frac{h}{\lambda_j k}.$$  \hspace{1cm} (6)

### 3. RESULTS AND DISCUSSION

In the previous section the heat diffusion equation was analytically solved in order to determine the temperature field inside a semiconductor sample heated by a laser beam.

Let us discuss some properties of the thermal field, which has the analytical expression given by equation (4). We are interested to avoid the thermal runaway in order to can measure the temperature of the sample.

If we increase the heat transfer coefficient (which is compose from the radiation heat transfer coefficient and convection heat transfer coefficient), we have a decrease of the thermal field. Of course we can manipulate the heat transfer coefficient just via the convection heat transfer coefficient. From mathematical point of view if we increase the heat transfer coefficient, we increase the eigenvalues and in consequence we decrease the temperature. From physical point of view, the things are also very simple, the higher the heat transfer coefficient the higher the heat quantity which goes from the sample to the surrounding medium. For a full discussion about heat transfer coefficient, one can see the Appendix B from reference [4].

Another ideas is to take into account just few (one) ultra-short pulses. The shorter the exposure time, the smaller the thermal field is. This can be demonstrated from mathematical point of view if we take the limit of equation (3):

$$\lim_{t=0^-} T_{\alpha_1, \alpha_2, \alpha_3, \alpha_4}(r, z, t) = 0.$$  \hspace{1cm} (7)

The third ideas, which we consider that is the most interesting is to choose samples with null linear absorption coefficient [9] (because is forbidden by the quantum mechanics selection rules). The higher order absorptions are very small and in consequence the thermal field will be weak. From mathematical point of view we can rewrite the equation (3) in the form:

$$T_{\alpha_1, \alpha_2, \alpha_3, \alpha_4}(r, z, t) \equiv \alpha_2 I_{00}(r, z, t)T_{2\text{ photons}} + \alpha_3 I_{00}(r, z, t)T_{3\text{ photons}} +$$

$$+ \alpha_4 I_{00}(r, z, t)T_{4\text{ photons}},$$  \hspace{1cm} (8)
where we have defined the “reduced” temperatures produces by absorption of two, three and four photons.

This can be also a very interesting formula for solid state physics experimentalists, which are interested to find out the cross section for higher order photon absorption.

In Fig. 1 is present a practical example, when one considers only two-photon absorption.

Fig. 1 – The thermal field in a InSb sample \((r = 10 \text{ mm}, z = 4 \text{ mm})\) espouse 250 ns at a laser beam of 100 W TEM\(_{00}\) CO\(_2\), considering 
\( h = 3 \times 10^{-7} \text{ Wmm}^{-2}\text{K}, \alpha_2 = 15 \text{ cmMW}^{-1} \) and \( \alpha_1 = 0 \).

4. CONCLUSIONS

The research in developing an analytical model to study the temperature distribution in solids materials heated by powerful free electron laser beams resulted in getting a new form of heat equation: semi-classical heat equation; considering in the source term two-, three- and four photon absorption phenomena which are of pure quantum nature. We have find three ways to avoid the thermal runaway. One can also combine this ideas in order to make more complex experiments. The target of next papers is to combine the present theory with our latest theories regarding the beam-solid interaction [10-13]. For example, one can easily combine the theories from references [4] and [14], in order to find out the thermal field for multi-mode free electron lasers and a solid sample.

REFERENCES