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# A MATHEMATICAL MODEL FOR KINEMATIC LINKAGES MADE OF DEFORMABLE COMPOSITE BARS BUILT BY USING THE HAMILTON'S VARIATION PRINCIPLE

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*Abstract*. This paper consists in building up a new perspective concerning some analytical approach of modeling the behavior of kinematic linkages made of linear-elastic deformable bars. We have been focused on case of composite orthotropic bars having elastic symmetry.

*Key words*: spatial linkages, deformable elements, composite bars, elastic symmetry, new HSDT deformation hypothesis.

## 1. INTRODUCTION

Describing the elasto-dynamics of composite bars by using the Classic Theory of Elasticity is practically impossible due to the fact that the Bernoulli hypothesis simply does not work in case of composite bars. Composite materials are generally non-homogenous and non-isotropic [4, 5]. For such kind of materials new methods of study have been developed [11]. This paper proposes a new HSDT deformation theory [11, 12] which is to be used to kinematic linkages made of deformable composite bars.

## 2. THE MECHANICAL WORK OF THE EXTERNAL CHARGES

We shall consider that on the $j$ bar axis are continuously distributed the next linear specific force:

$$\vec{p}_j = \left\{p^j\right\}^t \left\{\vec{i}^j\right\}, \tag{1}$$

the linear specific couple:

$$\vec{m}_j = \left\{m^j\right\}^t \left\{\vec{i}^j\right\}. \tag{2}$$

Due to the fact that the Gay conditions are fully satisfied, we can write the expression of the mechanical work of the external charges:

$$L_e = \sum_{j=1}^{n} \int_{0_j}^{L_j} \left( \left\{ p^j \right\}^t \left\{ u^j \right\} + \left\{ m^j \right\}^t \left\{ \theta^j \right\} \right) dx_1^j . \tag{3}$$

We shall note "$0_j$" the boundary point in which is to be placed the origin $O_j$ of the own reference system of *j* bar, the second boundary point will be obviously $L_j$, where $L_j$ is the length of the *j* bar. In our holonomic case we have:

$$\delta \int_{t_0}^{t_1} L_e \, dt = \int_{t_0}^{t_1} \delta \, L_e \, dt = \int_{t_0}^{t_1} \left( \sum_{j=1}^{n} \int_{0_j}^{L_j} \left( \left\{ p^j \right\}^t \left\{ \delta u^j \right\} + \left\{ m^j \right\}^t \left\{ \delta\theta^j \right\} \right) dx_1^j \right) dt . \tag{4}$$

Concentrated forces and concentrated couples are not to be considered.

## 3. THE MATHEMATICAL MODEL FOR THE VIBRATIONS OF EACH BAR OF A LINKAGE MADE OF DEFORMABLE COMPOSITE BARS

For the kinetic energy which has been calculated in [13] we will further apply the δ operator in order to obtain an even more simple form for the so-called "differentiated" kinetic energy. Considering all kinds of symmetries described in [13] and [14] we finally obtain:

$$\delta \int_{t_0}^{t_1} T dt = \sum_{j=1}^{n} \int_{t_0}^{t_1} \int_{0_j}^{L_j} \iint_{S_j} \left\{ \left\{ -\dot{A}^j \right\}^t \left[ {}^0S^j \right]^t - 2 \left\{ \dot{u}^j \right\}^t \left[ {}^0\Omega^j \right] - \left\{ \left\{ x^j \right\}^t + \left\{ u^j \right\}^t \right\} \cdot$$

$$\cdot \left[ \left[ {}^0\dot{\Omega}^j \right] + \left[ {}^0\Omega^j \right]^2 \right] - \left\{ \ddot{u}^j \right\}^t \right\} \left\{ \delta u^j \right\} \rho_j dS_j dx_1^j dt + \sum_{j=1}^{n} \int_{t_0}^{t_1} \int_{0_j}^{L_j} \iint_{S_j} \left\{ \left\{ 2 \left\{ \dot{\theta}^j \right\}^t \left[ r_j \right] \cdot \right.\right.$$

$$\cdot \left[ {}^0\Omega^j \right] + \left\{ \left\{ r_j \right\}^t + \left\{ \dot{\theta}^j \right\}^t \left[ r_j \right] \right\} \cdot \left[ \left[ {}^0\dot{\Omega}^j \right] + \left[ {}^0\Omega^j \right]^2 \right] + \left\{ \ddot{\theta}^j \right\}^t \left[ r_j \right] \right\} \left[ r_j \right] +$$

$$+ \left\{ -2 \left\{ \dot{\theta}^j \right\}^t \left[ Q_j \right]^t \left[ {}^0\Omega^j \right] - \left\{ \theta^j \right\}^t \left[ Q_j \right]^t \left[ \left[ {}^0\dot{\Omega}^j \right] + \left[ {}^0\Omega^j \right]^2 \right] - \left\{ \ddot{\theta}^j \right\}^t \left[ Q_j \right]^t \right\} \cdot$$

$$\cdot \left[ Q_j \right] \left\{ \delta\theta^j \right\} \cdot \rho_j dS_j dx_1^j dt . \tag{5}$$

The Hamilton's variation principle is about the extreme of the next functional:

$$\mathrm{S} = \int_{t_0}^{t_1} \left( T - L_{\mathrm{d}} + L_{\mathrm{e}} \right) dt \,, \tag{6}$$

considered with its own synchronic conditions: $\delta \overrightarrow{w^j}(t_0) = \delta \overrightarrow{w^j}(t_1) = \vec{0}$.

Let's put (36) of [13], (44) of [14] and (4) in (6) and taking into account that the synchronic conditions are to be satisfied which means that $\left\{\delta u^j\right\}$ and $\left\{\delta\theta^j\right\}$ are to have arbitrary values. In such a case the expression being in a position to multiply them are to have zero value. So it means that, for the expression related to $\left\{\delta u^j\right\}$:

$$\iint_{S_j} \left\{ \left\{ -\dot{A}^j \right\}^t \left[ {}^0S^j \right]^t - 2\left\{ \dot{u}^j \right\}^t \left[ {}^0\Omega^j \right] - \left\{ \left\{ x^j \right\}^t + \left\{ u^j \right\}^t \right\} \left[ \left[ {}^0\dot{\Omega}^j \right] + \left[ {}^0\Omega^j \right]^2 \right] - \right.$$

$$\left. - \left\{ \ddot{u}^j \right\}^t \right\} \rho_j dS_j + \iint_{S_j} \left\{ \left\{ u^j \right\}^t_{,11} \left[ E^1_j \right]^t + \left\{ \theta^j \right\}^t_{,1} \left[ \left[ I^j_2 \right] \left[ E^2_j \right]^t - \left[ Q_j \right]^t_{,2} \left[ E^2_j \right]^t + \right. \right.$$

$$\left. \left. + \left[ I^j_3 \right] \left[ E^3_j \right]^t - \left[ Q_j \right]^t_{,3} \left[ E^3_j \right]^t \right] \right\} dS_j + \left\{ p^j \right\}^t = \left\{ 0 \right\}^t . \tag{7}$$

And, for the expression related to $\left\{\delta\theta^j\right\}$:

$$\iint_{S_j} \left\{ \left\{ 2\left\{ \dot{\theta}^j \right\}^t \left[ r_j \right] \left[ {}^0\Omega^j \right] + \left\{ \left\{ r_j \right\}^t + \left\{ \theta^j \right\}^t \left[ r_j \right] \right\} \cdot \left[ \left[ {}^0\dot{\Omega}^j \right] + \left[ {}^0\Omega^j \right]^2 \right] + \right. \right.$$

$$\left. + \left\{ \ddot{\theta}^j \right\}^t \left[ r_j \right] \right\} \left[ r_j \right] + \left\{ -2\left\{ \dot{\theta}^j \right\}^t \left[ Q_j \right]^t \left[ {}^0\Omega^j \right] - \left\{ \theta^j \right\}^t \left[ Q_j \right]^t \left[ \left[ {}^0\dot{\Omega}^j \right] + \left[ {}^0\Omega^j \right]^2 \right] - \right.$$

$$\left. \left. - \left\{ \ddot{\theta}^j \right\}^t \left[ Q_j \right]^t \right\} \left[ Q_j \right] \right\} \rho_j dS_j - \iint_{S_j} \left\{ \left\{ \theta^j \right\}^t_{,11} \left[ r_j \right] \left[ E^1_j \right]^t + \left\{ \theta^j \right\}^t_{,1} \left[ - \left[ Q_j \right]^t_{,2} \left[ E^2_j \right]^t - \right. \right.$$

$$\left. \left. - \left[ Q_j \right]^t_{,3} \left[ E^3_j \right]^t \right] \right\} \left[ r_j \right] dS_j + \iint_{S_j} \left\{ \theta^j \right\}^t_{,11} \left[ Q_j \right]^t \left[ E^1_j \right]^t \left[ Q_j \right] dS_j - \iint_{S_j} \left\{ \left\{ u^j \right\}^t_{,1} \left[ E^1_j \right]^t + \right.$$

$$+\left\{\theta^j\right\}^t\left[\left[I_2^j\right]\left[E_j^2\right]^t-\left[Q_j\right]_{,2}^t\left[E_j^2\right]^t+\left[I_3^j\right]\left[E_j^3\right]^t-\left[Q_j\right]_{,3}^t\left[E_j^3\right]^t\right]\Big\}\left[I_1^j\right]dS_j-$$

$$-\iint_{S_j}\left\{-\left\{u^j\right\}_{,1}^t\left[E_j^1\right]^t+\left\{\theta^j\right\}_{,1}\left[-\left[r_j\right]\left[E_j^1\right]^t+\left[Q_j\right]^t\left[E_j^1\right]^t\right]\right\}\left[Y_3^j\right]\left[Q_j\right]_{,2}dS_j-$$

$$-\iint_{S_j}\left\{\theta^j\right\}^t\left[-\left[I_2^j\right]\left[E_j^2\right]^t+\left[Q_j\right]_{,2}^t\left[E_j^2\right]^t-\left[I_3^j\right]\left[E_j^3\right]^t+\left[Q_j\right]_{,3}^t\left[E_j^3\right]^t\right]\cdot$$

$$\cdot\left[Y_3^j\right]\left[Q_j\right]_{,2}dS_j-\iint_{S_j}\left\{\left\{u^j\right\}_{,1}^t\left[E_j^1\right]^t+\left\{\theta^j\right\}_{,1}\left[\left[r_j\right]\left[E_j^1\right]^t-\left[Q_j\right]^t\left[E_j^1\right]^t\right]\right\}\cdot$$

$$\cdot\left[Y_2^j\right]\left[Q_j\right]_{,3}dS_j-\iint_{S_j}\left\{\theta^j\right\}^t\left[\left[I_2^j\right]\left[E_j^2\right]^t-\left[Q_j\right]_{,2}^t\left[E_j^2\right]^t+\left[I_3^j\right]\left[E_j^3\right]^t-\right.$$

$$\left.-\left[Q_j\right]_{,3}^t\left[E_j^3\right]^t\right]\left[Y_2^j\right]\left[Q_j\right]_{,3}dS_j+\left\{m^j\right\}^t=\left\{0\right\}^t. \tag{8}$$

Considering (19) of [13] and considering that:

$$\left[{}^0\Omega^j\right]^t=-\left[{}^0\Omega^j\right];\ \left[{}^0\dot{\Omega}^j\right]^t=-\left[{}^0\dot{\Omega}^j\right]\text{ and }\left[\left[{}^0\Omega^j\right]^2\right]^t=\left[{}^0\Omega^j\right]^2, \tag{9}$$

and for $\rho$ = ct., we have: $\iint_{S_j}\left[Q_j\right]^t\left[{}^0\Omega^j\right]\left[Q_j\right]\rho dS_j=\left[0\right]_{3\times3}$; (10)

The relations (7) and (8) are actually partial derivate equations having as unknowns $\left\{u^j\right\}$ and $\left\{\theta^j\right\}$. These unknowns depend only on the $x_1^j$ coordinate and on the time *t*. In many cases the kinematic linkages are made of composite orthotropic bars being mass homogenous (MEGS). Let's note $B_j$ and $H_j$ the dimensions of the transversal section of the *j* bar. So, (7) and (8) are to become:

$$\left[P_j^1\right]\left\{\ddot{u}^j\right\}+\left[P_j^2\right]\left\{\dot{u}^j\right\}+\left[P_j^3\right]\left\{u^j\right\}+\left[P_j^4\right]\left\{\theta^j\right\}_{,1}+\left[P_j^5\right]\left\{u^j\right\}_{,11}=\left\{f_j\right\}; \tag{11}$$

$$\left[V_j^1\right]\left\{\ddot{\theta}^j\right\}+\left[V_j^2\right]\left\{\dot{\theta}^j\right\}+\left[V_j^3\right]\left\{\theta^j\right\}+\left[V_j^4\right]\left\{u^j\right\}_{,1}+\left[V_j^5\right]\left\{\theta^j\right\}_{,11}=\left\{g_j\right\}; \tag{12}$$

where:

$$\left[P_j^1\right] = \begin{bmatrix} \rho_j B_j H_j & 0 & 0 \\ 0 & \rho_j B_j H_j & 0 \\ 0 & 0 & \rho_j B_j H_j \end{bmatrix}; \tag{13}$$

$$\left[P_j^2\right] = \begin{bmatrix} 0 & -2\rho_j B_j H_j\, {}^0\omega_3^j & 2\rho_j B_j H_j\, {}^0\omega_2^j \\ 2\rho_j B_j H_j\, {}^0\omega_3^j & 0 & -2\rho_j B_j H_j\, {}^0\omega_1^j \\ -2\rho_j B_j H_j\, {}^0\omega_2^j & 2\rho_j B_j H_j\, {}^0\omega_1^j & 0 \end{bmatrix}; \tag{14}$$

$$\left[{}^0\Omega^j\right] = \begin{bmatrix} 0 & {}^0\omega_3^j & -{}^0\omega_2^j \\ -{}^0\omega_3^j & 0 & {}^0\omega_1^j \\ {}^0\omega_2^j & -{}^0\omega_1^j & 0 \end{bmatrix}; \tag{15}$$

is the anti-symmetrical matrix attached to the angular speed of $T_j$ with respect to $T_0$:

$${}^0\vec{\omega}_{(j)}^{\,j} = {}^0\omega_1^j\, \vec{i}_1^{\,j} + {}^0\omega_2^j\, \vec{i}_2^{\,j} + {}^0\omega_3^j\, \vec{i}_3^{\,j} = \left\{{}^0\omega_1^j; {}^0\omega_2^j; {}^0\omega_3^j\right\} \left\{\vec{i}^{\,j}\right\}; \tag{16}$$

We have also the angular acceleration of $T_j$ with respect to $T_0$:

$${}^0\vec{\varepsilon}_{(j)}^{\,j} = {}^0\dot{\vec{\varepsilon}}_{(j)}^{\,j} = {}^0\dot{\omega}_1^j\, \vec{i}_1^{\,j} + {}^0\dot{\omega}_2^j\, \vec{i}_2^{\,j} + {}^0\dot{\omega}_3^j\, \vec{i}_3^{\,j} = \left\{{}^0\dot{\omega}_1^j; {}^0\dot{\omega}_2^j; {}^0\dot{\omega}_3^j\right\}\left\{\vec{i}^{\,j}\right\}; \tag{17}$$

where:

$$\left[{}^0\dot{\Omega}^j\right] = \begin{bmatrix} 0 & {}^0\dot{\omega}_3^j & -{}^0\dot{\omega}_2^j \\ -{}^0\dot{\omega}_3^j & 0 & {}^0\dot{\omega}_1^j \\ {}^0\dot{\omega}_2^j & -{}^0\dot{\omega}_1^j & 0 \end{bmatrix}; \tag{18}$$

$$\left[P_j^3\right]=\begin{bmatrix} -\rho_j B_j H_j\left[\left({}^0\omega_2^j\right)^2+\left({}^0\omega_3^j\right)^2\right] & \rho_j B_j H_j\left[{}^0\omega_1^j\cdot{}^0\omega_2^j-{}^0\dot{\omega}_3^j\right] & \rho_j B_j H_j\left[{}^0\omega_1^j\cdot{}^0\omega_3^j+{}^0\dot{\omega}_2^j\right] \\ \rho_j B_j H_j\left[{}^0\omega_1^j\cdot{}^0\omega_2^j+{}^0\dot{\omega}_3^j\right] & -\rho_j B_j H_j\left[\left({}^0\omega_1^j\right)^2+\left({}^0\omega_3^j\right)^2\right] & \rho_j B_j H_j\left[{}^0\omega_2^j\cdot{}^0\omega_3^j-{}^0\dot{\omega}_1^j\right] \\ \rho_j B_j H_j\left[{}^0\omega_1^j\cdot{}^0\omega_3^j-{}^0\dot{\omega}_2^j\right] & \rho_j B_j H_j\left[{}^0\omega_2^j\cdot{}^0\omega_3^j+{}^0\dot{\omega}_1^j\right] & -\rho_j B_j H_j\left[\left({}^0\omega_1^j\right)^2+\left({}^0\omega_2^j\right)^2\right] \end{bmatrix} \quad (19)$$

where:

$$\left[{}^0\,\Omega^j\right]=\begin{bmatrix} -\left({}^0\omega_2^j\right)^2-\left({}^0\omega_3^j\right)^2 & {}^0\omega_1^j\,{}^0\omega_2^j & {}^0\omega_1^j\,{}^0\omega_3^j \\ {}^0\omega_1^j\,{}^0\omega_2^j & -\left({}^0\omega_1^j\right)^2-\left({}^0\omega_3^j\right)^2 & {}^0\omega_2^j\,{}^0\omega_3^j \\ {}^0\omega_1^j\,{}^0\omega_3^j & {}^0\omega_2^j\,{}^0\omega_3^j & -\left({}^0\omega_1^j\right)^2-\left({}^0\omega_2^j\right)^2 \end{bmatrix}; \quad (20)$$

$$\left[P_j^4\right]=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{11}{9}E_{1212}^j B_j H_j \\ 0 & -\frac{11}{9}\,E_{1313}^j B_j H_j & 0 \end{bmatrix}; \quad (21)$$

$$\left[P_j^5\right]=\begin{bmatrix} -E_{1111}^j B_j H_j & 0 & 0 \\ 0 & -E_{1221}^j B_j H_j & 0 \\ 0 & 0 & -E_{1331}^j B_j H_j \end{bmatrix}; \quad (22)$$

$$\left\{f_j\right\}=\begin{Bmatrix} p_1^j-\rho_j B_j H_j\left[a_{o_j(j)1}^0-\left(\left({}^0\omega_2^j\right)^2+\left({}^0\omega_3^j\right)^2\right)x_1^j\right] \\ p_2^j-\rho_j B_j H_j\left[a_{o_j(j)2}^0+\left({}^0\omega_1^j\,{}^0\omega_2^j+{}^0\dot{\omega}_3^j\right)x_3^j\right] \\ p_3^j-\rho_j B_j H_j\left[a_{o_j(j)3}^0+\left({}^0\omega_1^j\,{}^0\omega_3^j-{}^0\dot{\omega}_2^j\right)x_1^j\right] \end{Bmatrix}; \quad (23)$$

$$\left[V_j^1\right]=\begin{bmatrix}\frac{\rho_j B_j H_j}{12}\left(B_j^2+H_j^2\right) & 0 & 0\\ 0 & \frac{1049}{11340}\rho_j B_j H_j^3 & 0\\ 0 & 0 & \frac{1049}{11340}\rho_j H_j B_j^3\end{bmatrix}; \quad (24)$$

$$\left[V_j^2\right]=\begin{bmatrix}0 & -\frac{\rho_j B_j H_j^3}{6}\,{}^0\omega_3^j & \frac{\rho_j H_j B_j^3}{6}\,{}^0\omega_2^j\\ \frac{\rho_j B_j H_j^3}{6}\,{}^0\omega_3^j & 0 & 0\\ -\frac{\rho_j H_j B_j^3}{6}\,{}^0\omega_2^j & 0 & 0\end{bmatrix}; \quad (25)$$

$$\left[V_j^3\right]=\begin{bmatrix}V_{j\,11}^3 & V_{j\,12}^3 & V_{j\,13}^3\\ V_{j\,21}^3 & V_{j\,22}^3 & V_{j\,23}^3\\ V_{j\,31}^3 & V_{j\,32}^3 & V_{j\,33}^3\end{bmatrix}; \quad (26)$$

where:

$$V_{j\,11}^3=-\frac{\rho_j B_j H_j^3}{12}\left({}^0\omega_3^j\right)^2-\frac{\rho_j H_j B_j^3}{12}\left({}^0\omega_2^j\right)^2-\frac{\rho_j B_j H_j}{12}\left(B_j^2+H_j^2\right)\left({}^0\omega_1^j\right)^2;$$

$$V_{j\,12}^3=-\frac{\rho_j B_j H_j^3}{12}\left({}^0\omega_1^j\,{}^0\omega_2^j+{}^0\dot{\omega}_3^j\right);$$

$$V_{j\,13}^3=-\frac{\rho_j H_j B_j^3}{12}\left({}^0\omega_1^j\,{}^0\omega_3^j-{}^0\dot{\omega}_2^j\right);$$

$$V_{j\,21}^3=-\frac{\rho_j B_j H_j^3}{12}\left({}^0\omega_1^j\,{}^0\omega_2^j-{}^0\dot{\omega}_3^j\right);$$

$$V_{j\,22}^{3} = -\frac{1049}{11340}\ \rho_j B_j H_j^3 \left[ \left( {}^0\omega_2^j \right)^2 + \left( {}^0\omega_3^j \right)^2 \right] + \frac{247}{135}\ E_{1313} B_j H_j + \frac{4}{9}\ E_{1212} \frac{H_j^3}{B_j}\,;$$

$$V_{j\,23}^{3} = 0\,;\ V_{j\,31}^{3} = -\frac{\rho_j H_j B_j^3}{12} \left( {}^0\omega_1^j\, {}^0\omega_3^j + {}^0\dot{\omega}_2^j \right);\ V_{j\,32}^{3} = 0\,;$$

$$V_{j\,33}^{3} = -\frac{1049}{11340}\ \rho_j H_j B_j^3 \left[ \left( {}^0\omega_2^j \right)^2 + \left( {}^0\omega_3^j \right)^2 \right] + \frac{247}{135}\ E_{1212} B_j H_j + \frac{4}{9}\ E_{1313} \frac{B_j^3}{H_j}\,; \quad (27)$$

$$\left[ V_j^4 \right] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{11}{9} E_{1331}^j B_j H_j \\ 0 & -\frac{11}{9} E_{1221}^j B_j H_j & 0 \end{bmatrix}; \quad (28)$$

$$\left[ V_j^5 \right] = \begin{bmatrix} -\frac{E_{1221} B_j H_j^3}{12} - \frac{E_{1331} H_j B_j^3}{12} & 0 & 0 \\ 0 & -\frac{1049}{11340} E_{1111} B_j H_j^3 & 0 \\ 0 & 0 & -\frac{1049}{11340} E_{1111} H_j B_j^3 \end{bmatrix}; \quad (29)$$

$$\left\{ g_j \right\} = \left\{ \begin{array}{c} m_1^j + \frac{\rho_j B_j H_j^3}{12} \left( {}^0\omega_2^j\, {}^0\omega_3^j - {}^0\dot{\omega}_1^j \right) - \frac{\rho_j H_j B_j^3}{12} \left( {}^0\omega_2^j\, {}^0\omega_3^j + {}^0\dot{\omega}_1^j \right) \\ m_2^j - \frac{\rho_j B_j H_j^3}{12} \left( {}^0\omega_1^j\, {}^0\omega_3^j + {}^0\dot{\omega}_2^j \right) \\ m_3^j + \frac{\rho_j H_j B_j^3}{12} \left( {}^0\omega_1^j\, {}^0\omega_2^j - {}^0\dot{\omega}_3^j \right) \end{array} \right\}. \quad (30)$$

We have to consider the situation which the boundary points of bars are the places of the only existing couplings. In such a case the first derivative with respect to time of (27) of [13] leads us to:

$$\left\{\overset{\bullet}{A}{}^{j}\right\} = \sum_{k=1}^{j}\left\{\left[{}^{0}S^{k-1}\right]^{t}\left\{\overset{\bullet\bullet}{r}{}_{k-1}^{k}\right\}+\left[{}^{0}\overset{\bullet}{S}{}^{k-1}\right]^{t}\left\{\overset{\bullet}{r}{}_{k}^{k-1}\right\}^{t}+\left[{}^{0}\overset{\bullet}{S}{}^{k-1}\right]^{t}\left\{\overset{\bullet}{r}{}_{k}^{k-1}\right\}+\right.$$

$$+\left[{}^{0}\overset{\bullet\bullet}{S}{}^{k-1}\right]^{t}\left\{r_{k}^{k-1}\right\}-\left[{}^{0}S^{k}\right]^{t}\left\{\overset{\bullet\bullet}{r}{}_{k-1}^{k}\right\}-\left[{}^{0}\overset{\bullet}{S}{}^{k}\right]^{t}\left\{\overset{\bullet}{r}{}_{k}^{k-1}\right\}-\left[{}^{0}\overset{\bullet}{S}{}^{k}\right]^{t}\left\{\overset{\bullet}{r}{}_{k}^{k-1}\right\}-$$

$$\left.-\left[{}^{0}\overset{\bullet}{S}{}^{k}\right]^{t}\left\{r_{k}^{k-1}\right\}\right. . \tag{31}$$

Remember that we are in the situation that the origin $0_j$ of $T_j$ is placed in the boundary point $L_{j-1}$ of the $j$–1 bar. In such a case we have:

$$\left\{r_{k}^{k-1}\right\}=\left\{0\right\}_{3\times 1}; \tag{32}$$

and $\left\{r_{k}^{k-1}\right\}$ are algebraic vectors having constant value with respect to $T_{k-1}$, so, whatever derivative of them has to have zero value. So, (31) becomes:

$$\left\{\overset{\bullet}{A}{}^{j}\right\}=\sum_{k=1}^{j}\left[{}^{0}\overset{\bullet\bullet}{S}{}^{k-1}\right]\left\{r_{k}^{k-1}\right\}=\frac{d^{2}}{dt^{2}}\left(\sum_{k=1}^{j}\left[{}^{0}S^{k-1}\right]^{t}\left\{r_{k}^{k-1}\right\}\right)=\frac{d^{2}}{dt^{2}}\left\{r_{0_j(0)}^{0}\right\}=$$

$$=\left\{\overset{\bullet\bullet}{r}{}_{0_j(0)}^{0}\right\}=\left\{a_{0_j(0)}^{0}\right\}. \tag{33}$$

Obviously we have:

$$\left\{a_{0_j(j)}^{0}\right\}=\left[{}^{0}S^{j}\right]\left\{a_{0_j(0)}^{0}\right\}=\left[{}^{0}S^{j}\right]\left\{\overset{\bullet}{A}{}^{j}\right\}. \tag{34}$$

## 4. CONCLUSIONS

An algorithm for establishing the mathematical model for the vibrations of each bar of the kinematic linkage. We propose such an algorithm which is consistent with all developments we have been working on. We shall attach to each element of the kinematic linkage its own reference system related to its non-deformed status. We made consideration on how is best to choose such a system. We shall determine the generalized coordinates of the linkage, considering its bars as being rigid ones.

We shall determine every matrix $\left[ {}^{k}S^{j} \right]$.

We shall determine every matrix: $\left[ {}^{0}\Omega^{j} \right] = \left[ {}^{0}\dot{S}^{j} \right] \cdot \left[ {}^{0}S^{j} \right]^{t}$.

We shall establish the position $A_{j,j-1}$, which means we shall determine the vectors $\vec{r}_{j-1}^{\,j}$ and $\vec{r}_{j}^{\,j-1}$.

We shall make a full-complete kinematic and dynamic analysis for the linkage, considering bars that being still rigid. We shall calculate all matrix of the mathematical model according to relations (13) ….. (30). We shall write the mathematical model in its classic form (7) and (8). We shall write all possible relations concerning the compatibility of displacements in all kinematic couplings. This kind of model can be improved in order to study more other sorts of bars like variable section composite bars and composite bars having some complex geometrical form. More, this sort of HSDT deformation hypothesis can well be put together with some sort of homogenization theory like this shown in [4, 5].

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