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FRACTAL FLUIDS OF CONDUCTIVE TYPE BEHAVIOR THROUGH SCALE RELATIVITY THEORY

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Abstract. Using a generalization of the Nottale's scale relativity theory in second order terms of an equation of motion for a complex speed field, it is shown that the "synchronization" of the movements at different scales (fractal scale, differential scale etc.) confers conductive type properties to the fractal fluid. The behavior of a conductive fractal fluid is illustrated through the numerical simulation of plasma diffusion generated by laser ablation in the presence and respectively in the absence of a wall.

Key words: fractal space-time, conductive type fractal fluid, laser ablation plasma.

1. INTRODUCTION

The idea that the quantum space-time of microphysics is fractal, rather than flat and Minkowskian as assumed up to now, was suggested in [1, 2]. This proposal was itself based on earlier results [3-7], obtained at first by Feynman, concerning the geometrical structure of quantum paths. These have shown that the typical trajectories of quantum mechanical particles are continuous but non-differentiable, and can be characterized by a fractal dimension which jumps from $D_F = 1$ at large length-scales to $D_F = 2$ at small length-scales, the transition occurring around the de Broglie scale ([8, 9]).

Now, the fractal dimension $D_F = 2$ plays a particular role in physics. It is well-known that this is the fractal dimension of Brownian motion [10], *i.e.* from the mathematical view-point, of a Markov-Wiener process. This remark leads us to recall a related attempt at understanding the quantum behavior, namely, Nelson's stochastic quantum mechanics [11, 12]. In this approach, it is assumed that any particle is subjected to an underlying Brownian motion of unknown origin, which is described by two (forward and backward) Wiener processes: when combined together they yield the complex nature of the wave function and they transform Newton's equation of dynamics into the Schrödinger equation.

This is precisely one of the aims of the fractal space-time theory, and particularly of the Scale Relativity Theory (SRT), to relate the fractal and stochastic approaches [1, 9, 13-22]: the hypothesis that the space-time is non-differentiable and fractal implies that there are an infinity of geodesics between any couple of points [9] and provides us with a fundamental and universal origin for the double Wiener process of Nelson [11, 12]. SRT is a new approach to understand quantum mechanics, and moreover physical domains involving scale laws, such as chaotic systems. "It is based on a generalization of Einstein's principle of relativity to scale transformations. Namely, one redefines space-time resolutions as characterizing the state of scale of reference systems, in the same way as speed characterizes their state of motion. Then one requires that the laws of physics apply whatever the state of the reference system, of motion (principle of motion-relativity) and of scale (principle of SRT). The principle of SRT is mathematically achieved by the principle of scale-covariance, requiring that the equations of physics keep their simplest form under transformations of resolution. In such conjecture, it was demonstrated that, in the fractal dimension $D_F = 2$, the geodesics of the space-time are given by a Schrödinger's type equation" [13-22].

Three scales of interaction of SR were developed: (i) Galilean version corresponding to the standard fractals with constant fractal dimensions (for details see refs.[23, 34-36]. In such version, a quantum mechanics model was developed [13-22]; (ii) Special scale-relativistic version. This implied the high energy physics [8, 9, 15-20, 25-33]; (iii) General scale-relativistic version. This involved the cosmology [8, 9, 25-29, 37-40].

However, we note that all the treatments of Nottale are limited to the motion on fractal curves of fractal dimension $D_F = 2$ and second order terms in the equation of motion of a complex speed field.

In the present paper, assuming that the microparticle movement takes place on fractal curves (continuous but non-differentiable) of arbitrary fractal dimension D_F, the fractal fluids of conductive type behavior through scale relativity theory is studied in the second order approximation of the equation of motion, *i.e.* in an extended Nottale's model of SRT. The paper is structured as follows: in Section 2 the mathematical model is developed and correspondences with known results are given. In Section 3 numerical simulations concerning plasma expansion in the presence and respectively the absence of a wall were performed.

2. MATHEMATICAL MODEL

Let us suppose that the motion of the particles takes place on continuous but non-differentiable curves, *i.e.* on fractals [15-23]. The "non-differentiability" in the topological dimension D_F implies the replacement of the standard time derivative $\mathrm{d}/\mathrm{d}t$ by a new complex operator $\hat{\partial}/\partial t$ (for details see ref. [37]),

$$\frac{\hat{\partial}}{\partial t}=\frac{\partial}{\partial t}+\boldsymbol{V}\cdot\nabla-\mathrm{i}\frac{\lambda^2}{2\tau}\left(\frac{\mathrm{d}t}{\tau}\right)^{\left(\frac{2}{D_F}\right)-1}\Delta \tag{1}$$

where V is the complex speed field, $\mathrm{d}t$ is a time resolution, τ is a fractal – non-fractal transition time and λ a characteristic length scale. We are now able to write the conservation law of a fractal function ε (for details see Refs. [41–48]) in a fractal space-time under its covariant form:

$$\frac{\hat{d}\varepsilon}{dt}=\frac{\partial\varepsilon}{\partial t}+\boldsymbol{V}\cdot\nabla\varepsilon-\mathrm{i}\frac{\lambda^2}{2\tau}\left(\frac{\mathrm{d}t}{\tau}\right)^{\left(\frac{2}{D_F}\right)-1}\Delta\varepsilon=0 \tag{2}$$

or more, by separating the real and imaginary parts,

$$\frac{\partial\varepsilon}{\partial t}+\boldsymbol{V}\cdot\nabla\varepsilon=0\,,\quad -\boldsymbol{U}\cdot\nabla\varepsilon=\frac{\lambda^2}{\tau}\left(\frac{\mathrm{d}t}{\tau}\right)^{\left(\frac{2}{D_F}\right)-1}\Delta\varepsilon\,. \tag{3 a, b}$$

Consequently, at the differentiable scale the local temporal variation, $\partial\varepsilon/\partial t$, and the term, $\boldsymbol{V}\cdot\nabla\varepsilon$, are equal, while at the non-differentiable scale, the term, $\boldsymbol{U}\cdot\nabla\varepsilon$, and $\Delta\varepsilon$, compensate each other.

Particularly, for $\boldsymbol{V}=\boldsymbol{U}$ (*i.e.* "synchronal" movements at differentiable and fractal scales), from (3a,b) we get the diffusion type equation,

$$\frac{\partial\varepsilon}{\partial t}=\frac{\lambda^2}{\tau}\left(\frac{\mathrm{d}t}{\tau}\right)^{\left(\frac{2}{D_F}\right)-1}\Delta\varepsilon\,. \tag{4}$$

Such an equation is implied by the Fourier type law

$$\mathrm{j}(\varepsilon)=\frac{\lambda^2}{\tau}\left(\frac{\mathrm{d}t}{\tau}\right)^{\left(\frac{2}{D_F}\right)-1}\nabla\varepsilon \tag{5}$$

with a current density $\boldsymbol{j}(\varepsilon)$. Therefore, Eqs. (4) and (5) describe the fractal fluid of conductive type behavior.

Particularly, for movements on fractal curves of the Peano's type, *i.e* in the fractal dimension, $D_F \equiv 2$ (for details see refs. [9, 37]), the equations (4) and (5) take the standard forms

$$\frac{\partial \varepsilon}{\partial t} = \frac{\lambda^2}{\tau}\Delta\varepsilon \tag{6}$$

and respectively

$$\boldsymbol{j}(\varepsilon) = \frac{\lambda^2}{\tau}\nabla\varepsilon\,. \tag{7}$$

3. NUMERICAL SIMULATIONS

Let us now apply the previous considerations in the numerical simulations of laser produced plasma. The plasma expansion is solved in the planar coordinate system in the region above the target surface (Fig. 1). The *y*-axis coincides with the laser beam axis and is directed along the outer normal to the target surface. The plasma evolution is described with the following assumptions: i) local thermo-dynamical equilibrium plasma satisfies the quasi-neutrality condition; ii) the expansion is described in the approximation of a diffusion type equation; iii) the source term is introduced through the boundary conditions.

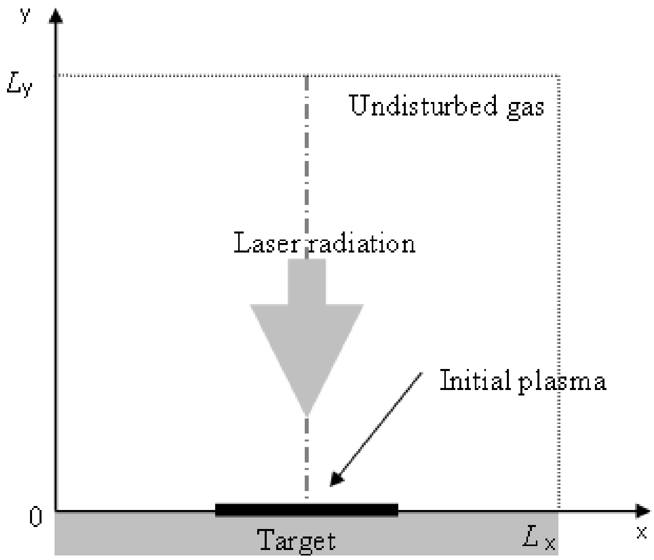

Fig. 1 – Integration domain used for the numerical simulation of the laser-produced plasma expansion.

In such circumstances, the two-dimensional gas dynamics is described by the equation of concentration,

$$\frac{\partial n}{\partial t} = D\Delta n\,, \quad D = \frac{\lambda^2}{\tau}\,. \tag{8 a, b}$$

This equation is obtained from equation (6) for Peano's type movements.

For the numerical integration, the following initial and boundary conditions are taken:

i) The box integration domain is initially filled with undisturbed gas,

$$n = n_0 \quad \text{for} \quad t = 0, \qquad 0 \le (x \times y) \le (L_x \times L_y)\,, \tag{9}$$

that is preserved on the boundaries,

$$n(t, x, L_y) = n(t, L_x / 2, y) = n(t, -L_x / 2, y) = n_{\max} / 1000\,. \tag{10}$$

ii) The interaction of the laser beam with the target produces a plasma source located on the target surface, which is assumed to have a Gaussian space-time profile,

$$n = n_{\max} \exp\left[-\frac{(t-\tau)^2}{(\tau_L / 2)^2} \right] \exp\left[-\frac{x^2}{(d_L / 2)^2} \right], \tag{11}$$

with d_L, τ_L similarly with the laser beam space-time full widths. We underline that the ablation takes place only in a region with a characteristic diameter of about 100 μm. The maximum atoms density n_{max} is taken according to the critical electron density ($n_{e\,c}$= $3.9\cdot10^{21}$ cm^{-3} [49]) at the laser wavelength (λ = 532 nm).

The diffusion equation of concentration with the initial and boundary conditions is numerically solved using finite differences [50] and the following parameters: L_x = 800 μm, L_y = 400 μm, τ_L=10 ns, d_L=100 μm, n_{max}=$1.95\cdot10^{21}$ cm^{-3}, n_0=n_{max}/1000. Moreover, if the diffusion takes place in the presence of a wall, the previous condition is replaced by $\partial n / \partial y(t, x, L_y) = 0$.

In Figs. 2 and 3 the two dimensional contour curves of the total atom density at the time moments t = 2 ns (a), t = 6 ns (b), and t = 10 ns (c) are given as obtained from the numerical simulations (Fig. 2 in the absence of wall, Fig. 3 in the presence of wall). The following considerations results: i) the plasma plume "disappears" by diffusion; ii) near the wall the plasma plume is 'regenerating'.

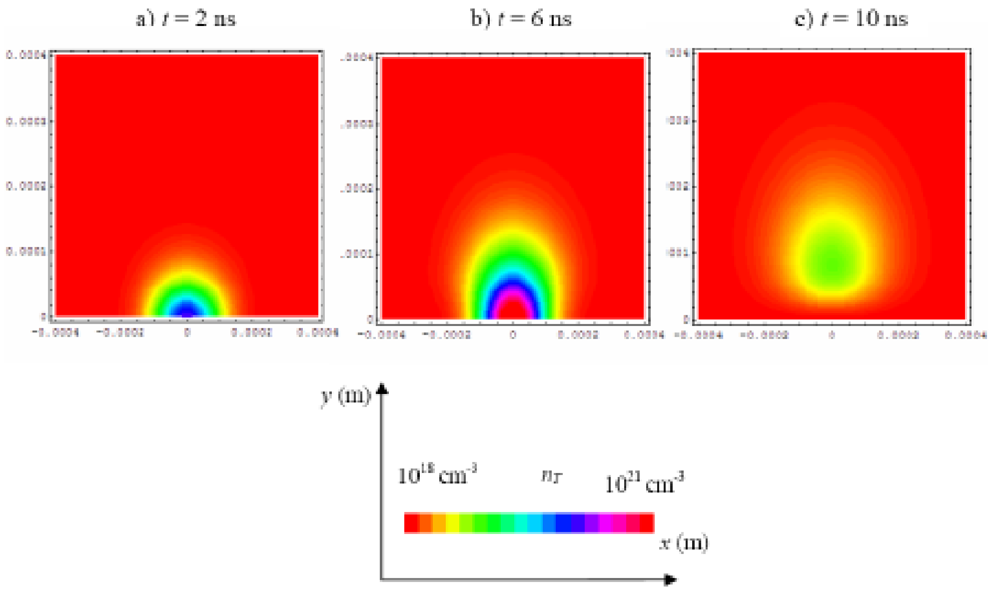

Fig. 2 – The contour curves of total atom density for various time moments as resulted from the numerical simulation of the diffusion equation.

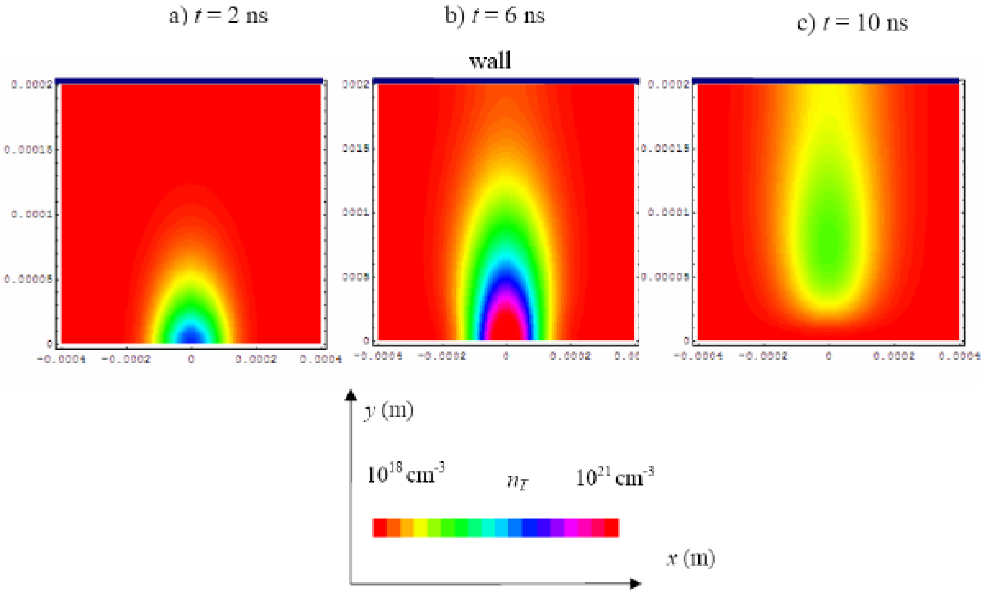

Fig. 3 – The contour curves of total atom density for various time moments as resulted from the numerical simulation of the diffusion equation in the presence of a wall.

4. CONCLUSIONS

The main conclusions of the present paper are the followings:

i) The synchronization of the movements at different scales gives conductive properties to the fractal fluid.

ii) By a numerical simulation of plasma expansion, it results that plasma plume either disappears by diffusion (in the absence of the wall), either is self-generating near a wall;

iii) Since the transport phenomena in nanostructures imply different space-time scales [51, 52], according with the previous result, new transport mechanisms result. Thus, for "synchronal" movement at different scales, a convection type transport mechanism is required.

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