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Dedicated to Professor Ioan Gottlieb's
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OCCURRENCE OF AN ITERATIVE EXPONENTIAL FUNCTION IN COSMOLOGY WITHOUT THE BIG BANG SINGULARITY

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Abstract. Application of the 5-dimensional Projective Unified Field Theory of the author to a homogeneous isotropic and spherical-symmetric cosmological model leads to a regular solution of the field equations. On the way to this non-big-bang model iterative exponential functions occur, having never been met in this field of research.

Key words: iterative exponential function, projective unified field theory, cosmology.

1. INTRODUCTION

Following ideas to generalize Einstein's 4-dimensional General Relativity Theory (1915) to five-dimensionality (Th. Kaluza 1921, P. Jordan 1945, etc.), half a century ago the author started his research by elaborating his Projective Unified Field Theory (PUFT) on a different geometrical basis. According to his physical experience he chose a particular 5-dimensional field equation [1], using projectors as excellently suitable mathematical basic quantities. By an appropriate vectorial projection calculus, elaborated by him, he projected this mentioned 5-dimensional field equation into the 4-dimensional space-time [2]. Thus he received a set of 4-dimensional tensorial field equations being presented in the following.

2. FIELD EQUATIONS IN THE SPACE-TIME

As in astrophysics mostly applied, I use the Gauss system of units. The projection described leads to following results.

2.1. Generalized gravitational field equation

$$R^{mn} - \frac{1}{2} g^{mn} R - \frac{\lambda_s}{S_0^2} \mathrm{e}^{-2\sigma} g^{mn} = \chi_0 \left(E^{mn} + S^{mn} + \Theta^{mn} \right). \tag{1}$$

Here $R^{mn} = R^i_{mni}$ and $R = R^m_m$ are the usual 4-dimensional curvature quantities. Further the identifications are valid:

a) Θ^{mn} (energy-momentum tensor of the substrate),

b) $E^{mn} = \frac{1}{4\pi}\left(B^{mk} H^n_k + \frac{1}{4} g^{mn} B_{kl} H^{kl} \right)$ (electromagnetic energy-momentum tensor), (2)

c) $S^{mn} = \frac{2}{\chi_0}\left(\sigma'^m \sigma'^n - \frac{1}{2} g^{mn} \sigma_{,k}\, \sigma^{,k} \right)$ (scalaric energy-momentum tensor).

The electromagnetic field strength tensor B_{ij} and electromagnetic induction tensor H_{ij} will be explained immediately in context with the electromagnetic field equations.

The scalaric field σ is a new quantity in space-time and occurred from the projection described above. I should mention that according to my hypothesis the scalaric field (with its origin in the five-dimensionality) represents a new hypothetical phenomenon of Nature, parallel to the phenomena of gravitation and electromagnetism, which I called scalarism/scalarity whose existence, of course, has to be tested empirically in future.

Furthermore, in the field equation (1) occur three fundamental constants:

a) $\chi_0 = \frac{8\pi\gamma_N}{c^4}$ (Einstein's gravitational constant, where γ_N is Newton's gravitational constant),

b) $S_0 = e_0 \sqrt{\frac{\chi_0}{2\pi}}$ (scalaric length constant of the same order of magnitude as Planck's length constant, e_0 electrical elementary charge), (3)

c) λ_S (scalaric cosmological constant).

The cosmological model to be treated within the framework of PUFT is, in spite of the use of numerical computer programs for solving the set of three coupled nonlinear differential equations, for mathematical reasons rather simple:

restriction to a 2-component gas mixture consisting of a substrate (matter) gas (firstly dark matter particles with the property of later clumping) and of an (electromagnetic) photon gas. For the description of the substrate gas I used the perfect gas energy-momentum tensor

$$\theta^{mn} = -\left(\mu + \frac{p}{c^2}\right)u^m u^n - p g^{mn} \quad (u^m \text{ four-velocity, } \mu \text{ mass density, } p \text{ pressure}). \tag{4}$$

2.2. Generalized electromagnetic field equations

$$\begin{aligned}
&\text{a)} \quad H^{mn}_{;n} = \frac{4\pi}{c} j^m \qquad \text{(inhomogeneous electromagnetic equation)},\\
&\text{b)} \quad B_{<ij,k>} = 0 \qquad \text{(cyclic electromagnetic equation)},\\
&\text{c)} \quad H^{mn} = \varepsilon B^{mn} \quad \text{with}\\
&\text{d)} \quad \varepsilon = \mathrm{e}^{2\sigma} \qquad \text{(vacuum dielectricity/polarisation)}.
\end{aligned} \tag{5}$$

The quantity j^m means the electric current density, e.g. in the convective case: $j^m = \rho_0 u^m$, where ρ_0 is the rest charge density.

2.3. Scalaric field equation

$$\sigma'^{k}_{;k} - \frac{\lambda_S}{S_0^2}\mathrm{e}^{-2\sigma} = -\frac{\chi_0}{2}\left(\frac{1}{8\pi} B_{ij} H^{ij} + \vartheta\right). \tag{6}$$

In this equation occurs the scalaric substrate energy density (short: scalerg density) ϑ, which is a basically new quantity within the framework of the traditional 4-dimensional physics.

3. BALANCE EQUATIONS IN THE SPACE-TIME

Einstein's General Relativity Theory has in contrast to most other field theories the basic advantage that by mathematical operations the fundamental balance equations (particularly the related conservation laws of physics) result from the fundamental field equations, i. e. that the balance equations mentioned are not independent axioms of his theory. Thus his theory exhibits a maximum of self-containment. Fortunately this advantage is also inherent in PUFT.

Here by the mathematical operations mentioned above the following local balance equations result:

$$\text{a)}\quad \Theta^{mk}_{;k} = -\frac{1}{c} B^m_k j^k + \vartheta\sigma'^{\mathrm{m}} \quad \text{(for the momentum-energy density)},$$
$$\text{b)}\quad j^m_{;m} = 0 \qquad\qquad \text{(for the electric current density)}. \tag{7}$$

The second balance equation reflects the conservation of the electric charge.

4. TREATMENT OF THE COSMOLOGICAL MODEL AND THE OCCURRENCE OF THE ITERATIVE EXPONENTIAL FUNCTION

As usual, the metric of such a cosmological model with the symmetry properties postulated above reads:

$$\mathrm{d}s^2 = K(\xi)^2 \left[\mathrm{d}\vartheta^2 + \sin^2\vartheta \left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2 \right) \right] - \mathrm{d}\xi^2 \tag{8}$$

(K world radius, ξ = ct). I would like to mention that for further calculations it is useful to introduce another dimensionless time parameter for rescaling the three differential equations resulting from the above field equations (1) and (6). For the integration of this system, mostly done numerically, I had to choose physically acceptable initial conditions.

Investigating an analogous cosmological model with the metric (8) within the Einstein theory, H. Hönl [3] derived the Hönl relation

$$K\overline{p} = \text{const.} \tag{9}$$

for a moving body in an expanding cosmos (m mass of the body, $\overline{p}$ impulse of the body). A similar formula to (9) was derived by M. von Laue (1931) for the frequency of a moving photon in an expanding cosmos. Let me emphasize that the well-tried relation (9) is in astrophysics and cosmology very important for the empirical test of new cosmological theories being proposed.

Following this treatment within PUFT, where the theoretical calculations are much more complicated as in the Einstein theory, I succeeded in arriving at an analogous relation as (9), however using a different scheme of definitions for the motion of a body. Since such a relationship as (9) was my intended goal [4], I had to postulate the following differential equation for the mass $m(\sigma)$ as a condition in order to arrive at the structure of (9):

$$\frac{\mathrm{d}m}{\mathrm{d}\sigma} + \alpha m \mathrm{e}^{-2\sigma} = 0 \qquad (\alpha \text{ free parameter}). \tag{10}$$

The solution of this equation is

$$m = m_0 \mathrm{e}^{\alpha/2\left(\mathrm{e}^{-2\sigma}-1\right)}, \tag{11}$$

where $m_0 = m(\sigma = 0)$ is a constant of integration with the physical meaning: mass at $\sigma = 0$, i. e. at the time $t = 0$. This time point designs the temporal beginning of the existence of the cosmos investigated. I called this regular begin "urstart", instead of the singular big bang ("urknall") in the Einstein theory.

Let me finish this paper with the annotation that in my theoretical PUFT concept applied to cosmology it seems to be unavoidable to accept that a body considered during the cosmological expansion changes his mass, induced through the influence of the temporally changing world function. This is a new relevant outcome from 5-dimensional physics with consequences for other fields of physics, now permanently discussed in physical research literature: cosmological time dependence of the gravitational constant, of Sommerfeld's fine structure constant, etc.

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