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SPINOR MODEL OF A PERFECT FLUID: EXAMPLES

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Abstract. Different characteristic of matter influencing the evolution of the Universe has been simulated by means of a nonlinear spinor field. Using the nonlinear spinor field corresponding to different types of perfect fluid or dark energy as a source of gravitational field, evolution of a Bianchi type-I universe has been illustrated.

Key words: spinor field, perfect fluid, dark energy.

1. INTRODUCTION

One of the principal goal of cosmological models is the description of the different phases of the Universe. In doing so researchers use different types of sources from perfect fluid to dark energy. In recent time many authors have studied the evolution of the Universe where the source is given by a nonlinear spinor field [1–8]. In those papers it was shown that a suitable choice of nonlinearity (i) accelerates the isotropization process, (ii) gives rise to a singularity-free Universe and (iii) generates late time acceleration. In a recent paper [9] the authors have simulated perfect fluid using spinor field with different nonlinearity. That paper was followed by one [10], where perfect fluid and dark energy were modeled by nonlinear spinor field. In doing so we used two types of nonlinearity, one occurs as a result of self-action and the other resulted fromthe interaction between the spinor and scalar field. It was shown that the case with induced nonlinearity is the partial one and can be derived from the case with self-action. So, in this paper we stick to the case with self-action. Here we repeat some of the previous results, give the description of generalized Chaplygin gas and modified quintessence [11] in terms of spinor field and study the evolution of the Universe filled with nonlinear spinor field within the scope of a Bianchi type-I cosmological model.

2. SIMULATION OF PERFECT FLUID WITH NONLINEAR SPINOR FIELD

First of all let us note that one of the simplest and popular model of the Universe is a homogeneous and isotropic one filled with a perfect fluid with the energy density $\varepsilon = T_0^0$ and pressure $p = -T_1^1 = -T_2^2 = -T_3^3$ obeying the barotropic equation of state

$$p = W\varepsilon \tag{2.1}$$

where W is a constant. Depending on the value of W (2.1) describes perfect fluid from phantom to ekpyrotic matter, namely

$W = 0$	(dust),	(2.2a)
$W = 1/3$	(radiation),	(2.2b)
$W \in (1/3, 1)$	(hard Universe),	(2.2c)
$W = 1$	(stiff matter),	(2.2d)
$W \in (-1/3, -1)$	(quintessence),	(2.2e)
$W = -1$	(cosmological constant),	(2.2f)
$W < -1$	(phantom matter),	(2.2g)
$W > 1$	(ekpyrotic matter).	(2.2h)

In order to describe the matter given by (2.2) with a spinor field let us now write the corresponding Lagrangian [2]:

$$L_{sp} = \frac{i}{2}\left[\bar{\psi}\gamma^{\mu}\nabla_{\mu}\psi - \nabla_{\mu}\bar{\psi}\gamma^{\mu}\psi\right] - m\bar{\psi}\psi + F, \tag{2.3}$$

where the nonlinear term F describes the self-action of a spinor field and can be presented as some arbitrary functions of invariant generated from the real bilinear forms of a spinor field. For simplicity we consider the case when $F = F(S)$ with $S = \bar{\psi}\psi$. We consider the case when the spinor field depends on t only. In this case for the components of energy-momentum tensor we find

$$T_0^0 = mS - F, \tag{2.4a}$$

$$T_1^1 = T_2^2 = T_3^3 = S\frac{\mathrm{d}F}{\mathrm{d}S} - F. \tag{2.4b}$$

Inserting $\varepsilon = T_0^0$ and $p = -T_1^1$ into (2.1) we find

$$S\frac{\mathrm{d}F}{\mathrm{d}S}-\left(1+W\right)F+mWS=0, \tag{2.5}$$

with the solution

$$F=\lambda S^{1+W}+mS, \tag{2.6}$$

with λ being an integration constant. Inserting (2.6) into (2.4a) we find that

$$T_0^0=-\lambda S^{1+W}. \tag{2.7}$$

Since energy density should be non-negative, we conclude that λ is a negative constant, i.e., $\lambda=-\nu$, with ν being a positive constant. So finally we can write the components of the energy momentum tensor

$$T_0^0=\nu S^{1+W}, \tag{2.8a}$$

$$T_1^1=T_2^2=T_3^3=-\nu WS^{1+W}. \tag{2.8b}$$

As one sees, the energy density $\varepsilon=T_0^0$ is always positive, while the pressure $p=-T_1^1=\nu WS^{1+W}$ is positive for $W>0$, i.e., for usual fluid and negative for $W<0$, i.e. for dark energy.

In account of it the spinor field Lagrangian now reads

$$L_{sp}=\frac{i}{2}\left[\bar{\psi}\gamma^\mu\nabla_\mu\psi-\nabla_\mu\bar{\psi}\gamma^\mu\psi\right]-\nu S^{1+W}, \tag{2.9}$$

Thus a massless spinor field with the Lagrangian (2.9) describes perfect fluid from phantom to ekpyrotic matter. Here the constant of integration ν can be viewed as constant of self-coupling. A detailed analysis of this study was given in [9].

Let us now generate a Chaplygin gas by means of a spinor field. A Chaplygin gas is usually described by a equation of state

$$p=-A/\varepsilon^\gamma. \tag{2.10}$$

Then in case of a massless spinor field for F one finds

$$\frac{\left(-F\right)^\gamma d\left(-F\right)}{\left(-F\right)^{1+\gamma}-A}=\frac{\mathrm{d}S}{S}, \tag{2.11}$$

with the solution

$$-F=\left(A+\lambda S^{1+\gamma}\right)^{1/(1+\gamma)}. \tag{2.12}$$

On account of this for the components of energy momentum tensor we find

$$T_0^0 = \left(A + \lambda S^{1+\gamma}\right)^{1/(1+\gamma)}, \tag{2.13a}$$

$$T_1^1 = T_2^2 = T_3^3 = A / \left(A + \lambda S^{1+\gamma}\right)^{\gamma/(1+\gamma)}. \tag{2.13b}$$

As was expected, we again get positive energy density and negative pressure. Thus the spinor field Lagrangian corresponding to a Chaplygin gas reads

$$L_{sp} = \frac{i}{2}\left[\bar{\psi}\gamma^{\mu}\nabla_{\mu}\psi - \nabla_{\mu}\bar{\psi}\gamma^{\mu}\psi\right] - \left(A + \lambda S^{1+\gamma}\right)^{1/(1+\gamma)}. \tag{2.14}$$

Setting $\gamma = 1$ we find the result obtained in [10].

Finally, we simulate modified quintessence with a nonlinear spinor field. It should be noted that one of the problems that face models with dark energy is that of eternal acceleration. In order to get rid of that problem quintessence with a modified equation of state was proposed which is given by [11]

$$p = -W\left(\varepsilon - \varepsilon_{cr}\right), \qquad W \in (0,1). \tag{2.15}$$

Here ε_{cr} some critical energy density. Setting $\varepsilon_{cr} = 0$ one obtains ordinary quintessence. It is well known that as the Universe expands the (dark) energy density decreases. As a result, being a linear negative function of energy density, the corresponding pressure begins to increase. In case of an ordinary quintessence the pressure is always negative, but for a modified quintessence as soon as ε_q becomes less than the critical one, the pressure becomes positive.

Inserting $\varepsilon = T_0^0$ and $p = -T_1^1$ into (2.15) we find

$$F = -\eta S^{1-W} + mS + \frac{W}{1-W}\varepsilon_{cr}, \tag{2.16}$$

with η being a positive constant. On account of this for the components of energy momentum tensor we find

$$T_0^0 = \eta S^{1-W} - \frac{W}{1-W}\varepsilon_{cr}, \tag{2.17a}$$

$$T_1^1 = T_2^2 = T_3^3 = \eta W S^{1-W} - \frac{W}{1-W}\varepsilon_{cr}. \tag{2.17b}$$

We see that a nonlinear spinor field with specific type of nonlinearity can substitute perfect fluid and dark energy, thus give rise to a variety of evolution scenario of the Universe.

3. ANISOTROPIC COSMOLOGICAL MODELS WITH A SPINOR FIELD

In the previous two sections we showed that the perfect fluid and the dark energy can be simulated by a nonlinear spinor field. In the section 2 the nonlinearity was the subject to self-action, while in section 3 the nonlinearity was induced by a scalar field. It was also shown the in our context the results of section 3 is some special cases those of section 2. Taking it into mind we study the evolution an anisotropic Universe filled with a nonlinear spinor field given by the Lagrangian (2.3), with the nonlinear term F is given by (2.6) of (2.12).

We consider the anisotropic Universe given by the Bianchi type-I (BI) space-time

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a_1^2\mathrm{d}x^2 - a_2^2\mathrm{d}y^2 - a_3^2\mathrm{d}z^2, \tag{3.1}$$

with a_i being the functions of t only. From the spinor field equation, it can be shown that [2]

$$S = \frac{C_0}{\tau}, \tag{3.2}$$

where we define

$$\tau = \sqrt{-g} = a_1 a_2 a_3. \tag{3.3}$$

For the components of the spinor field we obtain

$$\psi_{1,2}(t) = \frac{C_{1,2}}{\sqrt{\tau}}\,\mathrm{e}^{i\int \mathscr{D}\mathrm{d}t}, \quad \psi_{3,4}(t) = \frac{C_{1,2}}{\sqrt{\tau}}\,\mathrm{e}^{-i\int \mathscr{D}\mathrm{d}t}, \tag{3.4}$$

where $\mathscr{D} = \mathrm{d}F/\mathrm{d}S$. Solving the Einstein equation for the metric functions one find [2]

$$a_i = D_i \tau^{1/3} \exp\left(X_i \int \frac{\mathrm{d}t}{\tau} \right) \tag{3.5}$$

with the constants D_i and X_i obeying

$$\prod_{i=1}^{3} D_i = 1, \quad \sum_{i=1}^{3} X_i = 0. \tag{3.6}$$

Thus the components of the spinor field and metric functions are expressed in terms of τ. From the Einstein equations one finds the equation for τ [2]

$$\frac{\ddot{\tau}}{\tau} = \frac{3}{2}\kappa\left(T_1^1 + T_0^0\right). \tag{3.7}$$

In case of (2.9) on account of (3.2) Eq. (3.7) takes the form

$$\ddot{\tau} = (3/2)\kappa\nu C_0^{1+W}(1-W)\tau^{-W} \tag{3.8}$$

with the solution in quadrature

$$\frac{d\tau}{\sqrt{3\kappa\nu C_0^{1+W}\tau^{1-W} + C_1}} = t + t_0. \tag{3.9}$$

Here C_1 and t_0 are the integration constants.

In the Figs. 1 and 2 we have plotted the evolution of the Universe defined by the nonlinear spinor field corresponding to perfect fluid and dark energy.

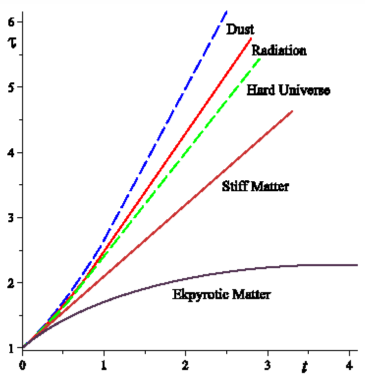

Fig. 1 – Evolution of the Universe filled with perfect fluid.

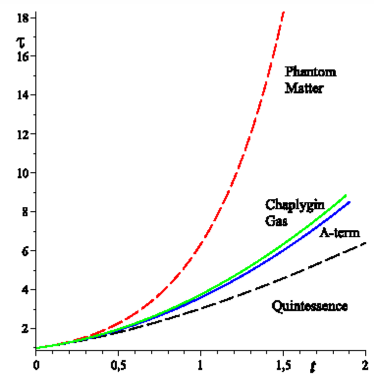

Fig. 2 – Evolution of the Universe filled with dark energy.

Let us consider the case when the spinor field is given by the Lagrangian (2.14). The equation for τ now reads

$$\ddot{\tau} = (3/2)\kappa\left[\left(A\tau^{1+\gamma} + \lambda C_0^{1+\gamma}\right)^{1/(1+\gamma)} + A\tau^{1+\gamma}/\left(A\tau^{1+\gamma} + \lambda C_0^{1+\gamma}\right)^{\gamma/(1+\gamma)}\right], \tag{3.10}$$

with the solution

$$\frac{d\tau}{\sqrt{C_1 + 3\kappa\tau\left(A\tau^{1+\gamma} + \lambda C_0^{1+\gamma}\right)^{1/(1+\gamma)}}} = t + t_0, \quad C_1 = \text{const.} \quad t_0 = \text{const.} \tag{3.11}$$

Inserting $\gamma = 1$ we come to the result obtained in [12].

Finally we consider the case with modified quintessence. In this case for τ we find

$$\ddot{\tau} = (3/2)\kappa\left[\eta C_0^{1-W}(1+W)\tau^{W} - 2W\varepsilon_{cr}\tau/(1-W)\right], \tag{3.12}$$

with the solution in quadrature

$$\frac{d\tau}{\sqrt{3\kappa\left[\eta C_0^{1-W}\tau^{1+W} - W\varepsilon_{cr}\tau^2/\left(1-W\right)\right]+C_1}} = t + t_0. \tag{3.13}$$

Here C_1 and t_0 are the integration constants.

In Fig. 3 we have illustrated the dynamics of energy density and pressure of a modified quintessence. In Fig. 4 the evolution of the Universe defined by the nonlinear spinor field corresponding to a modified quintessence has been presented. As one sees, in the case considered, acceleration alternates with declaration. In this case the Universe can be either singular (that ends in Big Crunch) or regular.

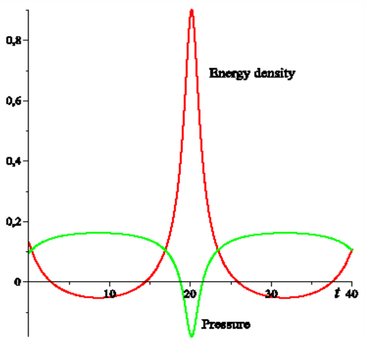

Fig. 3 – Dynamics of energy density and pressure for a modified quintessence.

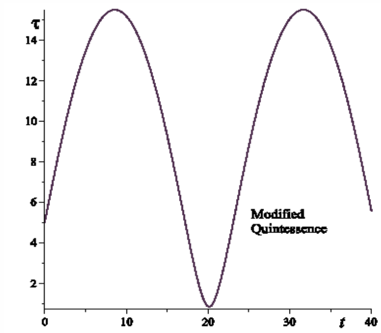

Fig. 4 – Evolution of the Universe filled with a modified quintessence.

4. CONCLUSION

Within the framework of cosmological gravitational field equivalence between the perfect fluid (and dark energy) and nonlinear spinor field has been established. It is shown that different types of dark energy can be simulated by means of a nonlinear spinor field. Using the new description of perfect fluid or dark energy one can study the evolution of the Universe.

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