Romanian Reports in Physics, Vol. 62, No. 2, P. 249–262, 2010

INHOMOGENEOUS MESONIC VISCOUS FLUID MODELS IN MODIFIED THEORY OF GENERAL RELATIVITY

R.C. SAHU*, L.K MOHAPATRA**, G. MOHANTY***

**Department of Mathematics, K.S.U.B. College, Bhanjanagar - 761 126, Orissa, India*
E-mail: rcsahu2@rediffmail.com
*** Principal, UCP Engineering School, Berhampur-10,Ganjam,Orissa,India*
E-mail: laxmikanta_puri@rediffmail.com
**** Retired Professor, Department of Mathematics, Sambalpur University, Orissa, India*
E-mail: ganeshwarm@yahoomail.com

(Received January 20, 2009)

Abstract. Mesonic bulk viscous solutions are found in modified theory of general relativity (proposed by Barber, [1]) in the plane symmetric inhomogeneous space-time. Some interesting and important properties of the solutions with singularity involved in the models are discussed. The models found in this paper can be applied to all the stages of the evolution of the Universe.

Key words: cosmology, plane symmetric, bulk viscous model, modified theory.

1. INTRODUCTION

In an attempt to make Einstein's theory of general relativity more satisfactory, Barber [1] proposed his second self-creation theory of gravitation by modifying general relativity. In this modified theory of general relativity, the scalar field does not gravitate directly but simply divides the matter tensor acting as a reciprocal to gravitational constant *G*. This theory predicts local effects that are within the observational limits. Moreover, this theory is capable of verification or falsification as it can be done by observing the behavior of both bodies of degenerate matter and photons. An observation of anomalous precessions in the orbits of pulsars about central masses and an accurate determination of the deflection of light and radio waves passing close to sun would verify or falsify such a theory and determines λ. The theory predicts the same precession of the perihelia of the planets as general relativity and in that respect agrees with observation to within 1%. In the limit $\lambda\to 0$ this modified theory approaches the Einstein's theory in every respect. Many authors have studied the modified theory of general relativity in various angles. Pimentel [2] has solved the Friedmann-Barber field equations under the assumptions of power law dependence of the scalar field on the scale factor Soleng [3, 4] has generalized the work of Pimentel [2] and got solutions for the vacuum dominated, radiation dominated and dust filled universe of the flat FRW space-time. Reddy and Venkateswaralu [5] have got Bianchi type VI_O cosmological solutions both in vacuum and in presence of perfect fluid with pressure equal to energy density. Venkateswarlu and Reddy [6] have also got spatially homogeneous and anisotropic Bianchi type-I cosmological macro models when the source of gravitational field is a perfect fluid. Shanti and Rao [7] have also got spatial homogeneous and anisotropic Bianchi type-II and III cosmological models both in vacuum and in presence of stiff fluid. Carvalho [8] has obtained a homogeneous and isotropic model of the early universe in which parameter gamma of "gamma law" equation of state varies continuously with cosmological time. Also he has presented a unified descriptions of early universe for inflationary period and radiation dominated era. Shri Ram and Singh [9] have obtained spatially homogenous and isotropic R-W model of the universe in the presence of perfect fluid by using 'gamma law' equation of state. Mohanty *et. al.*, [10] have obtained vacuum and Zeldovich fluid models for plane symmetric anisotropic homogeneous space-time. Mohanty *et. al.* [11, 12] have obtained an anisotropic homogeneous Bianchi type-I cosmological micro model in Barber's second theory of gravitation wherein the scaler field describe the elementary particles and their interactions (Srivastav and Sinha [13]). Also they have obtained a micro and macro cosmological model in the presence of massless scalar field interacted with perfect fluid. Panigrahi and Sahu [14, 15, 16, 17] have obtained plane symmetric inhomogeneous macro models, plane symmetric mesonic stiff fluid models, plane symmetric cosmological micro models in Barber's second theory of gravitation. Sahu and Panigrahi [18] have obtained Bianchi type-I vacuum modes and recently Sahu and Mohanty [19] have investigated inhomogeneous mesonic perfect fluid models in modified theory of general relativity.

Usually the investigation of relativistic models has the energy momentum tensor of matter and generated by a perfect fluid. But to obtain more realistic models, one must consider the viscosity mechanism. The viscosity mechanism in cosmology has attracted the attention of many researchers as it can account for high entropy of the present universe (Weinberg [20, 21]). High entropy per baryon and the remarkable degree of isotropy of the cosmic microwave background radiation suggests that one should analyze dissipative effects in cosmology. Moreover, there are several processes which are expected to give rise viscous effects. These are the decoupling of neutrinos during the radiation era and the decomposition of matter and radiation during the recombination era (Kolb and Turner [22]), decay of massive superstring models into massless models (Myung and Cho [23]), gravitational string production (Turok [24] and Barrow [25]) and particle creation effect in the grand unification era. Murphy [26] shown that introduction of bulk viscosity can avoid the big bang singularity. Hence one should consider the presence of material distribution other than the perfect fluid to get realistic cosmological models (see Gron [27] for a review on cosmological models with bulk viscosity).

To our knowledge none of the authors has studied the modified theory of general relativity for plane symmetric inhomogeneous space time in presence of mesonic viscous fluid. Hence in the present paper, we have studied the consistency of this theory to the case of a viscous fluid in presence of massless scalar field. The field equations are derived in section 2 and solved completely in section 3. In section 4 we have analysed the consequences of the results through different physical quantities involved in the solutions. The most important physical consequences are discussed in the concluding remarks in section 5.

2. FIELD EQUATIONS

Here we consider the space-time described by inhomogeneous and anisotropic metric of the form

$$\mathrm{d}s^2 = D^2\mathrm{d}t^2 - A^2\mathrm{d}x^2 - B^2\left(\mathrm{d}y^2 + \mathrm{d}z^2\right), \tag{1}$$

where A, B and D are functions of x and t only.

The Einstein-Barber's field equations are given by

$$G_{ij} \equiv R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi\phi^{-1}\left(T_{ij}^{v} + T_{ij}^{m}\right), \tag{2}$$

and

$$\Box\,\phi = \frac{8\pi\lambda}{3}T, \tag{3}$$

where

$$T_{ij}^{v} = \left(\rho + \overline{p}\right)u_i u_j - \overline{p}g_{ij}, \tag{4}$$

with

$$\overline{p} = p - \eta u^{i}_{;i} = p - 3\eta H, \tag{5}$$

and

$$T_{ij}^{m} = v_i v_j - \frac{1}{2}g_{ij}v_k v^k \tag{6}$$

are respectively the energy-momentum tensors corresponding to viscous fluid and massless scalar field satisfying the Klein-Gordan wave equation

$$g_{ij}v_{;ij} = 0. \tag{7}$$

Here $\rho, p, \overline{p}, \eta, u_i, \phi$, $\Box\phi$, ν, H and T are respectively the energy density, isotropic pressure, effective pressure, bulk viscous coefficient, four velocity vector of the fluid, Barber's scalar, the invariant d'Alembertian, scalar meson field, Hubble parameter and trace of the energy momentum tensors of the distribution. In general, η is a function of time and λ is the coupling constant to be determined from experiment where $|\lambda| < 10^{-1}$. Hereafter, the semicolon(;) denotes covariant differentiation. Since the bulk viscous pressure represents only a small correction to the thermodynamical pressure, the inclusion of viscous term in the energy momentum tensor is a reasonable assumption which doesn't change fundamentally the dynamics of the cosmic evolution. Using comoving coordinate system the set of field equations (2) and (3) for the metric (1) reduces to the following forms:

$$\frac{2}{BD^2}\left[B_{44}-\frac{DB_1D_1}{A^2}-\frac{B_4D_4}{D}\right]-\frac{1}{B^2}\left[\frac{B_1^2}{A^2}-\frac{B_4^2}{D^2}\right]= -8\pi\phi^{-1}.\left[\overline{p}+\tfrac{1}{2}\left(\frac{v_1^2}{A^2}+\frac{v_4^2}{D^2}\right)\right], \tag{8}$$

$$\frac{2}{B}\left[B_{14}-\frac{B_1A_4}{A}-\frac{D_1B_4}{D}\right]=-8\pi\phi^{-1}\nu_1\nu_4, \tag{9}$$

$$\frac{1}{BD^2}\left[B_{44}-\frac{DB_1D_1}{A^2}-\frac{B_4D_4}{D}\right]-\frac{1}{A^2B}\left[B_{11}-\frac{A_1B_1}{A}-\frac{AA_4B_4}{D^2}\right]+ +\frac{1}{A^2D^2}\left(AA_{44}-\frac{AA_4D_4}{D}-DD_{11}+\frac{DA_1D_1}{A}\right)= =-8\pi\phi^{-1}\left[\overline{p}+\tfrac{1}{2}\left(\frac{v_1^2}{A^2}+\frac{v_4^2}{D^2}\right)\right], \tag{10}$$

$$\frac{2}{A^2B}\left[B_{11}-\frac{A_1B_1}{A}-\frac{AA_4B_4}{D^2}\right]+\frac{1}{B^2}\left[\frac{B_1^2}{A^2}-\frac{B_4^2}{D^2}\right]= =-8\pi\phi^{-1}\left[\rho+\tfrac{1}{2}\left(\frac{v_1^2}{A^2}+\frac{v_4^2}{D^2}\right)\right], \tag{11}$$

$$\frac{\varphi_{44}}{D^2}+\left(\frac{A_4}{A}+\frac{2B_4}{B}-\frac{D_4}{D}\right)\frac{\varphi_4}{D^2}+\left(\frac{A_1}{A}-\frac{2B_1}{B}-\frac{D_1}{D}\right)\frac{\varphi_1}{A^2}-\frac{\varphi_{11}}{A^2}= =\frac{8\pi\lambda}{3}\left(\rho-3\overline{p}+\frac{v_1^2}{A^2}-\frac{v_4^2}{D^2}\right) \tag{12}$$

The Klein-Gorden equation (7) for metric (1) yields

$$\frac{v_{44}}{D^2}+\left(\frac{A_4}{A}+\frac{2B_4}{B}-\frac{D_4}{D}\right)\frac{v_4}{D^2}+\left(\frac{A_1}{A}-\frac{2B_1}{B}-\frac{D_1}{D}\right)\frac{v_1}{A^2}-\frac{v_{11}}{A^2}=0. \tag{13}$$

Here and afterwards, the suffixes 1 and 4 after a field variable indicate partial differentiation with respect to x and t respectively. Now we find an underdetermined field equation system having six equations in seven unknowns viz. ρ, $\overline{p}$, ϕ, ν, A, B and D. It is observed from above field equation system that the bulk viscosity coefficient η has not appeared in explicit form. For the specification of η, we assume the equation of state

$$p=\gamma\,\rho,\quad 0\le\gamma\le 1. \tag{14}$$

Here γ is called the adiabatic parameter. Assigning different values for γ, we can solve the field equations for different epochs.

3. SOLUTIONS OF THE FILED EQUATIONS

In order to avoid the mathematical complexity due to inhomogeneity of the space-time and highly non-linearity of the field equation system, we consider some explicit solutions of the field equations which are physically important. Thus the metric functions A, B and D are of the following form (Davidson [28]):

$$A=t^{\alpha}(1+x^2)^a, B=t^{\beta}(1+x^2)^b \quad \text{and} \quad D=(1+x^2)^d, \tag{15}$$

where α, β, a, b and d are real constants.

Since Barber's scalar field ϕ is function of time, we consider the scalar field ν to be the function of t only. Using the values of A, B and D from eq. (15) in eq. (13), we get

$$v_{44}+\frac{kv_4}{t}=0, \tag{16}$$

where $k=\alpha+2\beta$.

On integration, eq. (16) reduces to

$$v\;=\;\frac{k_1 t^{-k+1}}{-\mathrm{k}+1}+k_2, \tag{17}$$

where $k_{i,}$ $i=1, 2$ are constants of integration and $k_1\neq 0$. In view of eq. (15) and (17), equations (8) – (12) yield:

$$\left[\frac{\left(3\beta^2-2\beta\right)}{t^2(1+x^2)^{2d}}-\frac{4b\left(2d+b\right)x^2}{t^{2\alpha}(1+x^2)^{2a+2}}\right]\phi+\frac{4\pi k_1^2}{t^{2k}(1+k^2)^{2d}}=-8\pi\overline{p}, \tag{18}$$

$$\beta(d-b)+\alpha b=0, \tag{19}$$

$$\left[\frac{4b\left\{\left(3b-2a-1\right)x^2+1\right\}}{t^{2\alpha}(1+x^2)^{2a+2}}-\frac{\beta\left(2\alpha+\beta\right)}{t^2(1+x^2)^{2d}}\right]\phi+\frac{4\pi k_1^2}{t^{2k}(1+k^2)^{2d}}=-8\pi\rho, \tag{20}$$

and

$$\left[\phi_{44}+\frac{\left(\alpha+2\beta\right)}{t}\phi_4\right].\frac{1}{\left(1+x^2\right)^{2d}}=\frac{8\pi\lambda}{3}\left[\rho-3\overline{p}-\frac{k_1^2}{t^{2k}(1+x^2)^{2d}}\right]. \tag{21}$$

Now corresponding to the eq. (19), we have two sets of solutions *i.e.*

Case 1: When $b = 0$, $d = 0$,

$$\alpha=\beta=r(\text{say}). \tag{22}$$

Using the values from eq. (22), eq. (18) and (20) yield

$$\frac{1}{8\pi}\left[\frac{\left(2r-3r^2\right)\phi}{t^2}-\frac{4\pi k_1^2}{t^{6r}}\right]=\overline{p}, \tag{23}$$

$$\frac{1}{8\pi}\left[\frac{3r^2}{t^2}\phi-\frac{4\pi k_1^2}{t^{6r}}\right]=\rho. \tag{24}$$

Again, using equations (22), (23) and (24) in eq. (21),we obtain

$$t^2\phi_{44}+3rt\phi_4-m^2\phi=0, \tag{25}$$

where $m^2=2\lambda r(2r-1)$.

Solving eq. (25), we get two basic solutions for ϕ *i.e.*

$$\varphi_1 = c_1 t^{m_1} \text{ and } \varphi_2=c_2 t^{m_2} \tag{26a,b}$$

where c_1, c_2 are integration constants and m_1, m_2 are given by

$$m_1 = \frac{\left(1-3r\right)+\sqrt{1+r^2\left(9+16\lambda\right)-2r\left(3+4\lambda\right)}}{2},$$

$$m_2 = \frac{(1-3r) - \sqrt{1 + r^2(9+16\lambda) - 2r(3+4\lambda)}}{2}.$$

The first solution given in eq. (26a) is not acceptable as the theory does not lead to Einstein's theory when $\lambda \to 0$. Thus the second solution given in (26b) is only acceptable. Using (26b) in eq. (23) and (24), we get the effective pressure and density as:

$$\bar{p} = \frac{c_2(2r - 3r^2)}{8\pi t^{2-m_2}} - \frac{k_1^2}{2t^{6r}}, \tag{27}$$

$$\rho = \frac{3c_2 r^2}{8\pi t^{2-m_2}} - \frac{k_1^2}{2t^{6r}}. \tag{28}$$

Using eqs. (28) in eq. (14), we obtain

$$p = \gamma\left[\frac{3c_2 r^2}{8\pi t^{2-m_2}} - \frac{k_1^2}{2t^{6r}}\right]. \tag{29}$$

In view of eqs. (15), (27) and (29), eq. (5) yields

$$\eta = \frac{(p - \bar{p})t}{3r} = \frac{(\gamma\rho - \bar{p})t}{3r} = \left[\frac{3c_2 r^2(\gamma+1) - 2c_2 r}{8\pi t^{2-m_2}} + \frac{k_1^2(1-\gamma)}{2t^{6r}}\right]. \tag{30}$$

Thus the geometry of the space time (1) corresponding to solutions (26b) and (27) – (30) can be written as

$$\mathrm{d}s^2 = \mathrm{d}t^2 - t^{2r}\left(1 + x^2\right)^{2a}\mathrm{d}x^2 - t^{2r}\left(\mathrm{d}y^2 + \mathrm{d}z^2\right). \tag{31}$$

The model obtained in (31) is the plane symmetric inhomogeneous mesonic viscous fluid model in second self creation theory. It is interesting to see that when $a = 0$ then the model [31] reduces to Einstein – de-Sitter universe.

Case II: When $b = 0$, $d = 0$,

$$\alpha = -2\beta + 1. \tag{32}$$

Substitution of the value of "α "from (32) in equation (16), we get

$$v_{44} + \frac{v_4}{t} = 0. \tag{33}$$

On integration, equation (33) yields

$$v = k_3 \ln t + k_4 , \tag{34}$$

where k_i, $i = 3, 4$ are constants of integration and $k_3 \neq 0$.

Using rel. (32) in eq. (18), (20) and (21), we obtain

$$\bar{p} = \rho = \frac{\beta\left(2 - 3\beta\right)\varphi}{8\pi t^2} - \frac{k_1^2}{2t^2} , \tag{35}$$

and

$$\phi_{44} + \frac{\varphi_4}{t} = \frac{8\pi\lambda}{3}\left(\rho - 3\bar{p} - \frac{k_1^2}{t^2}\right). \tag{36}$$

In view of eq. (35), eq. (36) gives

$$t^2\varphi_{44} + t\varphi_4 + \frac{2\lambda}{3}\left(2\beta - 3\beta^2\right)\varphi = 0. \tag{37}$$

On integration, eq. (37) yields

$$\varphi = c_3 t^{m_3} + c_4 t^{m_4} , \tag{38}$$

where c_3, c_4 are constants of integration and

$$m_3 = \sqrt{\frac{2\lambda\beta}{3}\left(3\beta - 2\right)},$$

$$m_4 = -\sqrt{\frac{2\lambda\beta}{3}\left(3\beta - 2\right)}.$$

With the use of equation (38), equation (35) reduces to

$$\bar{p} = \rho = \frac{\left(2 - 3\beta\right)}{8\pi t^2}\left(c_3 t^{m_3} + c_4 t^{m_4}\right) - \frac{k_1^2}{2t^2} . \tag{39}$$

Using eq. (39) in eq. (14), we get

$$p = \gamma\left[\frac{\left(2 - 3\beta\right)}{8\pi t^2}\left(c_3 t^{m_3} + c_4 t^{m_4}\right) - \frac{k_1^2}{2t^2}\right]. \tag{40}$$

Now using equations (15), (39) and (40) in equation (5), we obtain

$$\eta = (\gamma - 1)\rho t = (\gamma - 1)\left[\frac{(2\beta - 3\beta^2)}{8\pi t}\left(c_3 t^{m_3} + c_4 t^{m_4}\right) - \frac{k_3^2}{2t}\right]. \tag{41}$$

Therefore the model of the universe is described by the space time (1) as

$$\mathrm{d}s^2 = \mathrm{d}t^2 - t^{-4\beta+2}\left(1 + x^2\right)^{2a}\mathrm{d}x^2 - t^{2\beta}\left(\mathrm{d}y^2 + \mathrm{d}z^2\right). \tag{42}$$

As in Case 1, here also it is interesting to note that when $a = 0$ and $\beta = 1/3$, the model (42) reduces to Einstein – de-Sitter Universe.

4. SOME PHYSICAL AND GEOMETRIC PROPERTIES OF THE MODELS

The physical parameters $\overline{p}, \rho, p$ and η involved in the models (31) and (42) are given by equations (27) to (30) and (39) to (41) respectively. The Barbers scalar ϕ for the models (31) and (42) are given by (26b) and (38) respectively.

If $\eta = 0$ then $\overline{p} = p$. Thus the results in this case reduce to that of found earlier by Sahu and Mohanty [19]. Again if ν = a constant and $\phi = 1$ then the results reduce to that of already obtained by Pradhan et al. [29].

Moreover, if ν = a constant, $\eta = 0$ and $\phi = 1$ then the result to be obtained are same as with results found earlier by Pradhan et al. [30].

It is interesting to see that when $\lambda \to 0$, $\phi \to$ some constant (in both the cases). Hence Barber's second theory leads to Einstein's theory.

4.1. ENERGY CONDITIONS FOR VISCOUS FLUID

The strong, weak and dominant energy condition *i.e.* $\rho + 3\overline{p} \geq 0, \rho \geq 0$ and $\rho + \overline{p} \geq 0,$ are given by in model 1 as

$$\rho + 3\overline{p} = \frac{3c_2 r(1-r)}{4\pi t^{2-m_2}} - \frac{2k_1^2}{t^{6r}},$$

$$\rho = \frac{3c_2 r^2}{8\pi t^{2-m_2}} - \frac{k_1^2}{t^{6r}},$$

$$\rho + \overline{p} = \frac{c_2 r}{4\pi t^{2-m_2}} - \frac{k_1^2}{t^{6r}}.$$

In model 2, we have

$$\rho + 3\overline{p} = \frac{(2-3\beta)\beta}{2\pi t^2}\left(c_3 t^{m_3} + c_4 t^{m_4}\right) - \frac{k_1^2}{2t^2},$$

$$\rho = \frac{(2-3\beta)\beta}{8\pi t^2}\left(c_3 t^{m_3} + c_4 t^{m_4}\right) - \frac{k_1^2}{2t^2},$$
$$\rho + \overline{p} = \frac{(2-3\beta)\beta}{4\pi t^2}\left(c_3 t^{m_3} + c_4 t^{m_4}\right) - \frac{k_1^2}{2t^2}.$$

In case of model 1, the strong energy condition is satisfied when $r \neq 0, 1$. The weak energy condition is satisfied when $r \neq 0$ and $c_2 > 0$. The dominant energy condition is also satisfied when $r \neq 0$ or when c_2 and r are of opposite sign.

If $t \to 0$, then ρ and p both are indetermined subject to restriction $m_2 < 2$, but $\rho, p \to \infty$ each subject to the condition $m_2 > 2$.

Again as $t \to \infty$, ρ and $p \to 0$ each subject to the condition $m_2 < 2$ and $r > 0$.

In case of model 2, we have $\overline{p} = \rho \to \infty$ as $t \to 0$ and $\overline{p} = \rho \to 0$ as $t \to \infty$. Thus the result shows the presence of big-bang singularity at initial epoch.

The strong energy condition in case of model 2 is satisfied only when $\beta < 2/3$. The other energy condition *i.e.* weak and dominant are also satisfied when $\beta < 2/3$.

4.2. BULK VISCOUS COEFFICIENT η

In both the models the bulk viscous coefficient $\eta \to 0$ when $t \to 0$ provided that $r \neq 0$ in case 1 and $\eta \to \infty$ when $t \to \infty$ provided $\gamma \neq 1$ in case 2. From above result, we observe the presence of big-bang singularity.

4.3. EXPANSION SCALAR

The values of expansion scaler $\theta = u^i_{;i}$ in model 1 and 2 are given by $\theta = \frac{3r}{t}$ and $\theta = \frac{1}{t}$ respectively. Thus in both the cases we have $\theta \to 0$ as $t \to \infty$ and $\theta \to \infty$ as $t \to 0$. It is evident from the above result that the models start expanding with a big-bang at $t = 0$ and the expansion in the models decreases as time increases. However, the expansion stops at infinite future or for the case $r = 0$.

4.4. HUBBLE PARAMETER H

The Hubble's parameter H in both the models are found as $H = \frac{r}{t}$ and $H = \frac{1}{3t}$ respectively. Thus H is a function of t and hence we concluded that the models are not of steady-state.

4.5. SHEAR SCALAR σ

The anisotropy (Raychoudhuri [31]) defined by

$$\sigma^2 = \frac{1}{12}\left[\left(\frac{g_{11,4}}{g_{11}} - \frac{g_{22,4}}{g_{22}}\right)^2 + \left(\frac{g_{22,4}}{g_{22}} - \frac{g_{33,4}}{g_{33}}\right)^2 + \left(\frac{g_{33,4}}{g_{33}} - \frac{g_{11,4}}{g_{11}}\right)^2\right]$$

for models (31) and (42) yield $\sigma = 0$ and $\sigma = \sqrt{\frac{2}{3}}.\left(\frac{1-3\beta}{t}\right)$ respectively.

In first case, the model clearly approaches to isotropy as $\sigma = 0$ and $\lim_{t\to\infty}\frac{\sigma}{\theta} = 0$. But in second case, the model does not approaches to isotropy as $\sigma \neq 0$ and $\lim_{t\to\infty}\frac{\sigma}{\theta} \neq 0$. Thus in case of second model the anisotropy exists through out the evolution. Since $\lim_{t\to 0}\sigma^2 = \infty$ and $\lim_{t\to\infty}\sigma^2 = 0$ (subject to restriction $\beta \neq 1/3$), the shape of the universe changes uniformly in x and y direction only. But the rate of change of the shape of universe becomes slow with increase of time. Also it is clear that there is a real physical singularity in the model at $t = 0$. The present upper limit for $\frac{\sigma}{\theta} = 10^{-3}$, obtained from indirect arguments concerned to the isotropy of primordial black body radiation (Collins et al., [32]). In case 1, we see that $\frac{\sigma}{\theta} = 0$, which satisfies the inequality. But in case 2, we observe that $\frac{\sigma}{\theta} < 10^{-3}$, provided $\beta > \frac{1}{3} - \frac{1}{2}.10^{-6}$. Hence the models found here can be applied to all stages of the evolution of the universe.

4.6. EINSTEIN SPACE

A space-time is said to be an Einstein space if it holds the property $R_{ij} = \frac{R}{4}g_{ij}$. But here, the space-time does not satisfy the above property and hence not an Einstein space.

4.7. MASSLESS SCALAR FILED v

The massless scaler field v for model (31) and (42) are found respectively as $\nu = \frac{k_1}{1-3r}.\frac{1}{t^{3r-1}} + k_2$ and $\nu = k_3 \ln t + k_4$.

It is evident from above that ν is a function of cosmic time in both the cases. However in case 1, ν is not defined for $r = 1/3$. But when $r < 1/3$, ν is increasing function of time and when $r > 1/3$, ν is decreasing function of time. In case 1, we see that as $t \to 0$, $\nu \to \infty$ and as $t \to \infty$, $\nu \to a$ constant. But in case 2, we find that as $t \to 0$, $|v| \to \infty$ and as $t \to \infty$, $\nu \to \infty$. These results show that the presence of big-bang singularity.

The energy density associated with the massless scaler field ν is given as (Anderson [33]) as $\varepsilon = \frac{1}{2}\left(v_4^2 + m^2 v^2\right)$, where $m = 0$.

Thus from eqs. (17) and (34), we have $\varepsilon = \frac{k_1^2}{2t^{2k}}$ and $\varepsilon = \frac{k_3^2}{2t^2}$.

From above results, we observe that the energy density ε of the massless scaler field decreases with time (in both the cases) at faster rate than scalar field ν. For physically acceptable mesonic field, we have $\varepsilon > 0$. This situation leads to k_1 and k_3 which are both +ve or both –ve real constants.

4.8. THE PARAMETER γ

For realistic and physical situations, it is required that $\rho \geq p \geq 0$, which yields restrictions on the parameter γ *i.e.* $0 \leq \gamma \leq 1$.

Case 1

- If $\gamma = 0$ then eq. (29) reduces to $p = 0$. Thus corresponding to eq. (28) and (29) the model (31) leads to dust filled universe.
- If $\gamma = 1$ then from eq. (28) and (29) we have $p = \rho$ and in this case the model (31) reduces to stiff fluid filled universe.
- If $\gamma = 1/3$ then eq. (28) and (29) lead to $\rho = 3p$ and the model (31) reduces to radiating universe.

Case 2

- If $\gamma = 0$ then $p = 0$, $\overline{p} = \rho = -\eta$. In this case the model (42) reduces to dust dominated universe.
- If $\gamma = 1/3$ then $\rho = 3p = \overline{p}$ and hence the model (42) turns to radiating universe.
- If $\gamma = 1$ then we have $p = \overline{p} = \rho$ and $\eta = 0$. Thus in this case the results reduces to that of obtained by Sahu and Mohanty [19].

5. CONCLUSION

In this paper, we have described the plane symmetric space-time in modified theory of general relativity in the presence of mesonic viscous fluid as the source matter. The models found by using "gamma-law" equation of state are discussed.

The effect of bulk viscosity is to introduce a change in the perfect fluid models. Thus bulk viscosity exhibit essential influence on the character of the solution. We observe from the models found in section 3 that Murphy's conclusion (Murphy, [26]) about the absence of big-bang singularity in the infinite past or at initial epoch in models with bulk viscous fluid, is not true in general.

The models found in section 3 are not of steady state but of expanding in nature. The models start expanding with a big-bang at initial epoch and the expansion stops at infinite future. Further, the first model is of isotropic in nature while second model is of anisotropic in nature through out the evolution. In case of second model the shape of the universe changes uniformly in X and Y direction but the rate of change becomes slow as time increases. Moreover, the space-time considered here is not an Einstein space. The models found in this paper can be applied to all the stages of the evolution of the universe.

REFERENCES

1. G.A. Barber, Gen. Rel. Grav., **14,** 117(1982).
2. L.O. Pimentel, Astrophys. Space Sci., **116**, 395 (1985)
3. H.H. Soleng, Astrophys. Space Sci., **139**, 373 (1987a). H.H.
4. H.H. Soleng, Astrophys. Space Sci., **102**, 67 (1987b).
5. D.R.K. Reddy, R. Venkateswarlu, Astrophysc. Space Sci., **155,** 135 (1989).
6. R. Venkateswarlu, D.R.K. Reddy, Astrophys. Space Sci., **168**, 193 (1990).
7. K. Shanti, V.U.M. Rao, Astrophys. Space Sci., **179,** 147 (1991).
8. J.C. Carvalho, Int. J. Theo. Phys., **35,** 2019 (1996).
9. Shri Ram, C. P. Singh, Astrophys. Space Sci., **257**, 123 (1998).
10. G. Mohanty, B. Mishra, R. Das, Bull. Inst. Math. Academia Sinica., **28**, 43 (2000).
11. G. Mohanty, U.K. Panigrahy, R.C. Sahu, Astrophysc. Space Sci., **281**, 633 (2002).
12. G. Mohanty, R.C. Sahu, U.K. Panigrahi, Astrophys. Space Sci., **284**, 1055 (2003).
13. S.K. Srivastav, K.P. Sinha, *Aspects of Gravitational Interactions*, Horizons in World Physics, Noya Science Publishers Inc., Commack, New York, 1998, p. 225, 111.
14. U.K. Panigrahi, R.C. Sahu, Bull. Cal. Math. Soc. India**, 95**, **3,** 183 (2003).
15. U.K.Panigrahi, R.C.Sahu, Science Letters, Allahabad, India, **25,** *11*, 12 (2002).
16. U.K.Panigrahi, R.C. Sahu, Theo. and Appl. Mech., **30**, 163 (2003).
17. U.K.Panigrahi, R.C. Sahu, Czech. J. Phys., **54**, 543 (2004).
18. R.C.Sahu, U.K.Panigrahi, Astrophys. Space Sci., **288**, 601 (2003).
19. R.C. Sahu, G. Mohanty, Astrophys Space Sci., **306**, 179 (2006).
20. S. Weinberg, Astronomical Physics Journal, **168**, 175 (1971).
21. S. Weinberg, *Gravitation and Cosmology*, Wiley and Sons, New York, 1972.
22. E.W. Kolb, M.S. Turner, *The Early Universe*, Additson – Wesley, U.S.A., 1990.
23. S. Myung, B.M. Cho., Mod. Phys. Lett., A **1**, 37 (1986).
24. N. Turok, Phys. Rev. Lett., **60**, 549 (1988).

25. J.D. Barrow, Nucl. Phys., B. **310**, 743 (1988).
26. G.L. Murphy, Phys. Rev., D **8**, 4231 (1973).
27. O. Gron, Astrophys. Space Sci., **173**, 191 (1990).
28. W. Davidson, J. Math. Phys., **32**, 1560 (1991).
29. A. Pradhan, R.V. Saraykar, A. Beesham, Astro Lett. Comm., **35**, 383 (1997).
30. A. Pradhan, K.L. Tiwari, A. Beesham, *The Plane Symmetric Inhomogeneous Cosmological models with perfect-fluid*, pre-print, University of Zululand.
31. A.K. Raychoudhuri, Physical Rev., **98** 1123 (1955).
32. C.B. Collins, E.N. Glass, D.A. Wilkinson, Gen. Rel. Grav., **12**, 805 (1980).
33. J.L. Anderson, *Principle of Relativity Physics*, Academic Press, New York, 1967.