

## ROLE OF PHASE AND GROUP VELOCITIES IN CERENKOV RADIATION AND APPLICATIONS IN HEP EXPERIMENTS\*

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*Abstract.* Emission of Cerenkov radiation represents a phenomenon used in detector systems, in a very wide type of applications: high energy physics, nuclear physics, astroparticles, high energy neutrino and gamma-ray astronomy. This phenomenon is supposed to exist in strong interactions and gravity as well. Some peculiarities of the electromagnetic Cerenkov radiation, very important in HEP experiments, are not always completely considered and could conduce to mistaken interpretations. In this paper, the role of phase and group velocities is revised and discussed for media with different dispersive and symmetry properties. The energy distribution of the electric field of the emitted radiation is analysed and the essential role of group velocity is evidenced. For possible practical applications, these characteristics are calculated and essential aspects are analysed for some media where Cerenkov radiation could be produced: water, ice, noble gases (liquids and solids) and alkali.

*Key words:* Cerenkov radiation, group and phase velocities, optical refractive index.

### 1. INTRODUCTION

The detection of particles with very high energy from accelerators and from astrophysical sources based on optical Cerenkov effect is a difficult and expensive task. The actual directions of investigation are represented by the search for new materials with optical properties adequate for this scope, or studies of Cerenkov phenomena in other wavelength regions than visible, where this phenomenon could be produced. This second direction of investigation was open by Askarian's idea. It was outlined already in 1962 that electron-photon cascades in matter could emit coherent radiation in the radio and microwave regions.

Cerenkov radiation is electromagnetic radiation emitted when a charged particle passes through a dielectric medium which is optically transparent for light,

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with a velocity greater than the phase speed of light in that medium. The threshold condition is that the velocity of the particle should be equal to the phase velocity of the radiation.

The discovery of Cerenkov radiation has a long history which started with the first correct prediction by O. Heaviside in 1888, and with the partially correct Sommerfeld's idea of the radiation emitted by an electron moving with constant velocity greater than that of light, in vacuum (in 1904). P. A. Cerenkov discovered the phenomenon and the explanation was given by I. E. Tamm and I. M. Frank [1, 2, 3, 4] a short time after this, and a very rich field of applications was developed. This phenomenon was predicted to exist also in strong interactions – see for example references [5, 6] and for the gravitational field [7, 8].

Because some peculiarities of the Cerenkov effect are not always completely considered in applications in HEP and could lead to mistaken interpretations, in this paper we review the role of phase and group velocities and we discuss these aspects in concrete cases of media of interest in applications. The essential role of group velocity, an unmeasurable quantity, is put in evidence. Media with different dispersive and symmetry properties and in different states of aggregation are considered and their essential characteristics for Cerenkov radiation are determined. The energy distribution of the electric field of the emitted radiation is analysed.

The regions of wavelength where the ratio between group and phase velocities is close to unity defines the domains where Cerenkov radiation could be used in practical applications and the experimental errors in measurements are minimized.

## 2. CERENKOV RADIATION AND THEIR CHARACTERISTICS

### 2.1 GENERAL CONSIDERATIONS

In the Cerenkov radiation the photons are emitted perpendicular to the wave front and fill the rings in the plane transverse to the direction of motion of the charge. Let  $u$  be the phase velocity of the photons. Rings of light in Cerenkov detectors are used to measure the velocity of the incident particle if the refractive index is known.

This radiation is a very directional one – waves of a given frequency  $\omega$  are emitted only under a definite angle with respect to the direction of motion of the system, this angle being determined by the relation

$$\cos \theta = \frac{c}{n v}. \quad (1)$$

Here  $c$  is the speed of light in vacuum,  $v$  is the velocity of particle in the considered medium and  $n=n(\omega)$  is the refractive index.

In the direction which satisfies the above condition the waves reach the observer with an optical path-length difference equal to zero, and accordingly the radiation will only result in this direction.

The equation obtained for  $\cos\theta=1$  expresses the energetic threshold for the emission of the radiation. The minimum energy of the particle (at the threshold),  $E_0$ , is determined by the refraction index  $n$ . Since it is not directly the energy of the particle that is required for the determination of this threshold, but its velocity, it is therefore obvious that  $E_0$  is also dependent on the mass of the particle. The threshold condition for the effect and maximum angle at which the light is emitted are:  $(\beta_{\text{threshold}} = 1/n, \theta = 0)$ , and  $(\beta_{\text{max}} \rightarrow 1, \theta_{\text{max}} = \cos^{-1}(1/n))$  respectively.

Cerenkov radiation result as a different reasoning from the balance of energy and momentum. For combined conditions of energy and momentum conservation, the following equation is obtained:

$$\Delta U = -E \left( 1 - \frac{v \cdot \cos \vartheta}{u} \right). \quad (2)$$

In this equation, a particle with velocity  $v$  radiates an energy  $E$ , in a direction characterised by a unit vector  $\vec{n}$ , modifying its translational velocity and its energy  $U$  that represent its internal degrees of freedom. If the particle does not have internal degrees of freedom, the usual condition for the effect is directly obtained. For systems with internal structure, the interpretation is more complicated. If the velocity of the particle is small,  $v \ll u$ , then, usually, the internal energy of the particle decreases with the energy radiated; but, if the condition  $v > u$  is produced, the system is self-excited,  $\Delta U > 0$ , and this process is simultaneously accompanied by a slowing down of its motion as a whole.

## 2.2. PHASE AND GROUP VELOCITIES OF THE CERENKOV RADIATION

Cerenkov radiation is emitted because the charged particle polarises atoms along its track so that they become electric dipoles. The time variation of the dipole field leads to the emission of electromagnetic radiation. For velocities below the threshold, the dipoles are symmetrically arranged along the particle path, so the integrated dipoles field is zero and no radiation occurs. The situation is very different for the case of particles with velocities above the threshold for Cerenkov radiation, because the dipoles are oriented along the path, the symmetry is broken, the dipoles moment is nonzero and thus a radiation is produced. Thus, this is a collective effect determined by the presence of the medium. The coherent nature of Cerenkov radiation could be explained intuitively using Huygens' principle.

Because a photon in a medium is not a free particle, the wave propagation is obtained as a result of coherent addition of waves of individual atoms. Thus, of importance for the onset of the wave is the collective motion that occurs in the atoms of the medium. From this point of view, let's ascribe to the photon in a medium a velocity  $w = \frac{d\omega}{dk} \left( p = \hbar k = \frac{h}{\lambda} = \frac{n}{c} \hbar \omega \right)$  which is equal to the group velocity of light. In a general form, the group velocity is given [1] by equation:

$$w = v_{group} = \frac{c}{n(\omega) + \omega \operatorname{Re} \left[ \frac{dn(\omega)}{d\omega} \right]}. \quad (3)$$

As it could be seen from the equation, it is evident that the behaviour of  $\frac{dn(\omega)}{d\omega}$  is an essential parameter: if the index of refraction is independent on the frequency, the group and phase velocities are equal; in all other cases the group velocity can be positive, infinite or negative. The threshold condition for the appearance of this effect is that the velocity of the particle coincides with the group velocity.

If the derivative decreases rapidly enough with frequency, the group velocity can be negative and this phenomenon is produced near an absorption region for light.

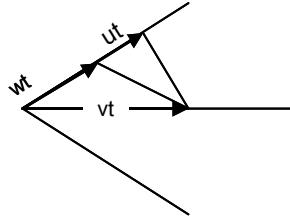


Fig. 1 – The wave fronts due to phase and group velocities.

The wave front of the radiation will be normal to the radiation cone only if  $\operatorname{Re} \left[ \frac{dn}{d\omega} \right]$  is zero, in the contrary case there will exist an angle between the wave front defined by phase velocity and the corresponding wave front for group velocity as is suggested in Fig. 1. Equation:

$$\cot \varphi = \left( \beta^2 n^2 - 1 \right)^{1/2} + n \omega \beta^2 \left( dn/d\omega \right) / \left( \beta^2 n^2 - 1 \right)^{1/2} \quad (4)$$

defines the angle between the wavefront and the direction of the particle.

In an optically isotropic medium, the group velocity  $w$  of the radiated waves, as well as the phase velocity  $u$ , is directed at a characteristic acute angle of  $\theta = \cos^{-1}\left(\frac{c}{vn}\right)$  to the particle velocity. The radiated energy and the momentum of the photon propagate in the same direction. The energy of the photon is transmitted at the group velocity, not at the phase velocity [10]. The momentum carried away slows down the particle, and this result is similar with the previous one, but is obtained from different considerations (Fig. 2a). The situation is very different in a medium with negative group velocity. In this case the phase and group velocities are directed oppositely, and if  $|w| > |u|$ , the radiated cone would make an angle  $(\pi - \theta)$  with the direction of particle. So, the energy and momentum would be in opposite directions (Fig. 2.b) and momentum as a component would be oriented in the direction of particle.

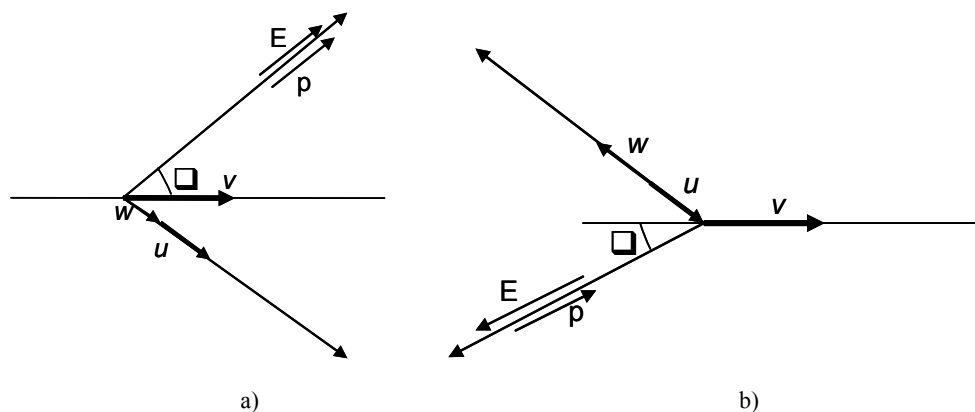


Fig. 2 – Cerenkov radiation in a medium with positive (a) and negative (b) group velocities.

Inside a dielectric medium the polarization due to the elementary dipoles is time dependent. In the case of isotropic and non-dispersive media, the group and phase velocities coincide, and the surface on which the intensity of electric peaks coincides with the well known wave cone orthogonal to the wavevector of the emitted light and the aperture is determined by the coherence condition for Cerenkov radiation. The situation is very different in other media [11, 12, 13]; in high dispersive media the intensity is instead peaked on the surface of a group cone that is not orthogonal to the frontwave and much narrower than the wave cone. So, the electric field intensity is peaked on the group cone described by the equation:

$$\frac{x^\perp (v - w^\parallel)}{w^\perp} + z = vt, \quad (5)$$

where we used the notation:  $\vec{x} \equiv (x^\perp, z)$  and  $w \equiv (w^\perp, w^\parallel)$  as the two perpendicular components of the position vector and group velocity respectively.

Other characteristics of the Cerenkov radiation are associated with inhomogeneous periodic and crystalline medium. Different behaviours are associated with single crystals or lattice crystal structures. In a single crystal, Cerenkov radiation is expected to be emitted at angles and frequencies satisfying the conditions for Bragg scattering. Details could be found for example in Refs. [14, 15] but these will not be discussed in this paper.

### 3. RESULTS AND DISCUSSIONS

#### 3.1. VELOCITIES IN CERENKOV RADIATION FOR DISPERSIVE AND ANISOTROPIC MEDIA

The behaviour of the radiation in different media and in different wavelength regions is investigated in relation with the group and phase velocities. The wavelength dependence of the refractive index for different materials: water [16], ice [17], noble gases (argon, krypton, xenon) in liquid and solid phase [18, 19, 20], and of NaCl [21] are used to evidence the behaviour of phase and group velocities in materials.

The choice of  $n$  rather than  $n_g$  or of  $v$  rather than  $v_g$  leads to errors in track reconstruction and in distance measurements. Thus, a good option to use Cerenkov radiation in detection is for these wavelength regions where ratio between two velocities to be close to one, because this way the errors in identifications are minimized.

For water and ice the wavelength dependence of the ratio of group versus phase velocity is determined for a region between  $2 \times 10^{-2}$  and to  $10^5 \mu\text{m}$  for water and from  $3 \times 10^{-2}$  to  $10^2 \mu\text{m}$  for ice respectively. For these regions, the optical refractive index presents a typical behaviour with smooth regions, alternating with regions with resonant form. It must be mentioned that for smooth regions of refractive index, a polynomial fit (fourth order at least) or Boltzmann function are adequate to reproduce the experimental values. These dependencies are presented in Figs. 3 (for water) and 4 (ice).

First, the great modifications of optical properties after the phase transition between liquid and solid states must be observed. For both situations, there exist wavelength regions for which group velocity has positive, negative and zero

values. The regions with negative group velocities correspond to high absorptive character for the radiation. For both optical media, there exist large regions of wavelength where Cerenkov radiation could be used for detection, with minimal errors in the identification.

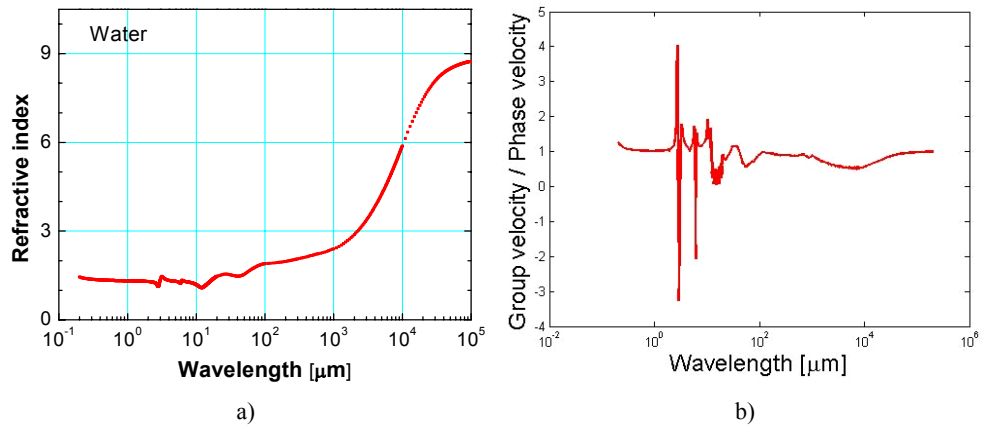


Fig. 3 – Refractive index (a) and ratio between group velocity and phase velocity (b) versus wavelength in water.

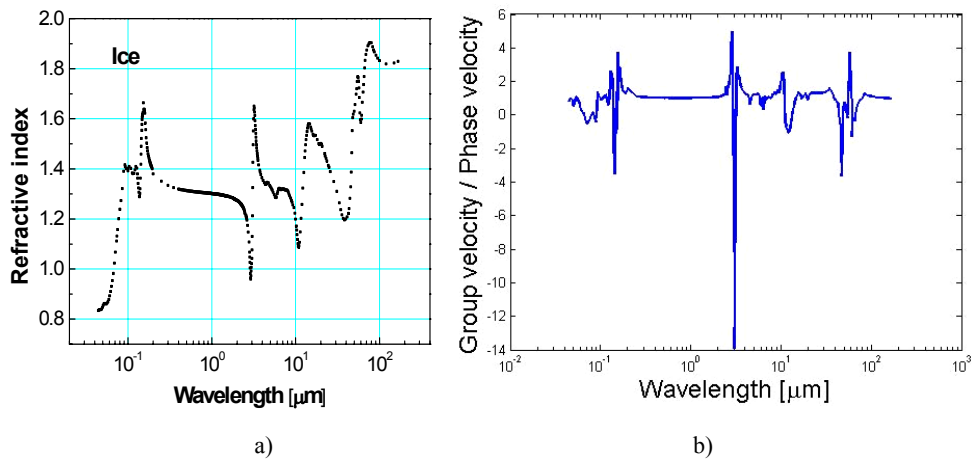


Fig. 4 – Refractive index (a) and ratio between group velocity and phase velocity (b) versus wavelength in ice.

For other media: NaCl crystals and noble elements (Kr, Ar, Xe), the analysis was realised only into a narrow wavelength interval in the visible region (Figs. 5 (a and b) and 6 (a, b and c)). Variation with temperature was also considered.

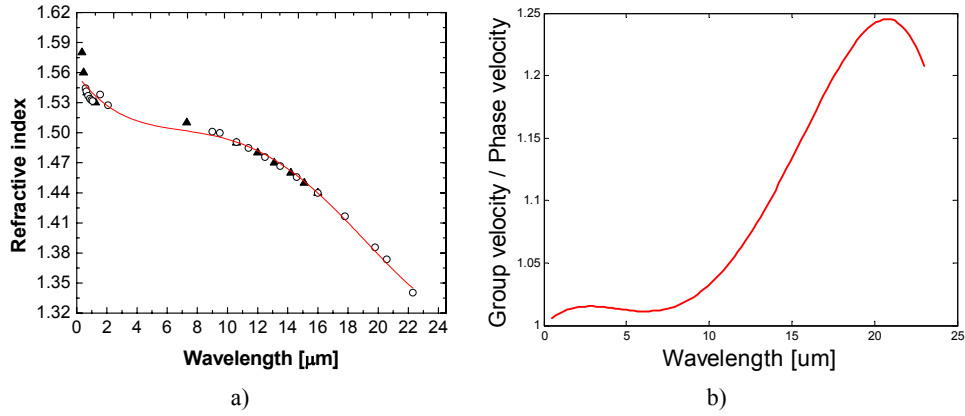


Fig. 5 – Refractive index (a) and ratio between group velocity and phase velocity (b) versus wavelength in NaCl.

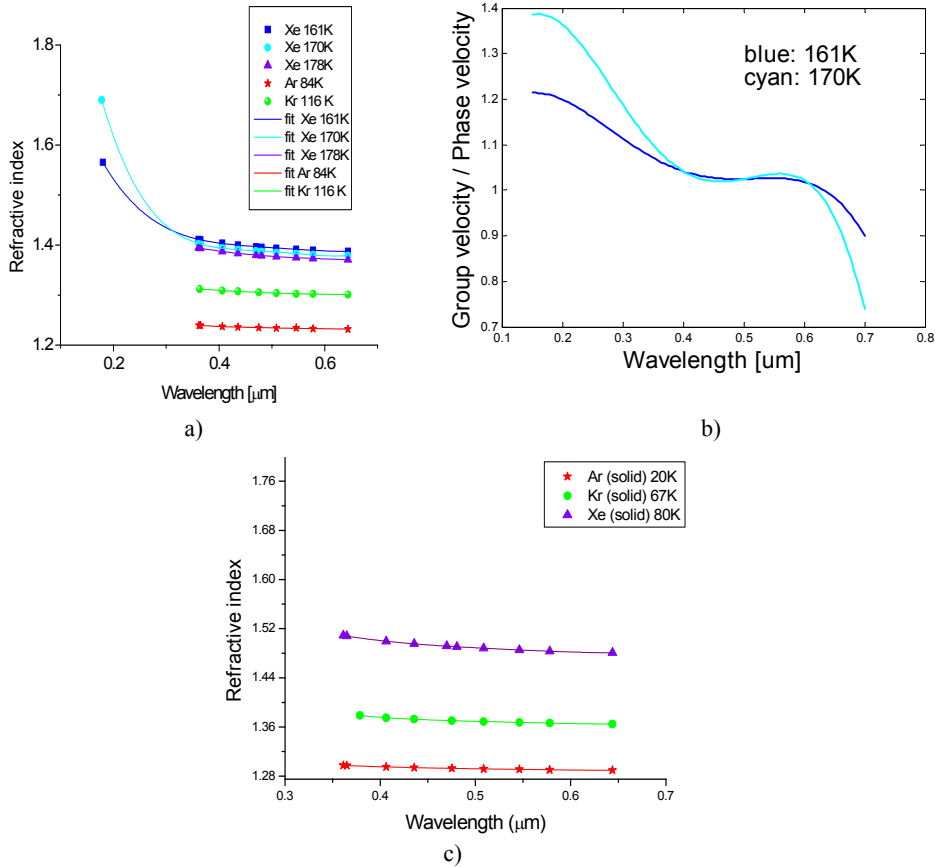


Fig. 6 – Refractive index (a) and ratio between group velocity and phase velocity (b) versus wavelength in noble gases in liquid states, and refractive index dependence versus wavelength in solid states (c).



For both cases, only for a limited optical region the ratio between velocities is close to one and thus the errors are minimised.

### 3.2. DEPENDENCE ON PRESSURE

The effect of a uniaxial mechanical deformation on the dispersion of the refractive index of alkali-halide crystals, in particular of NaCl and KCl, in the visible and ultraviolet regions present interesting features [22]. Three regions can be distinguished: in the first region (from 0 to 4 kg/mm<sup>2</sup>), an increase almost monotonic of the refractive index, in the second region (from 5 to 6 kg/mm<sup>2</sup>), a sharp decrease to negative values, and in the third region, a clear decrease. This behaviour is observed in all analysed crystals. The data for NaCl and KCl crystals are presented in Fig. 7.

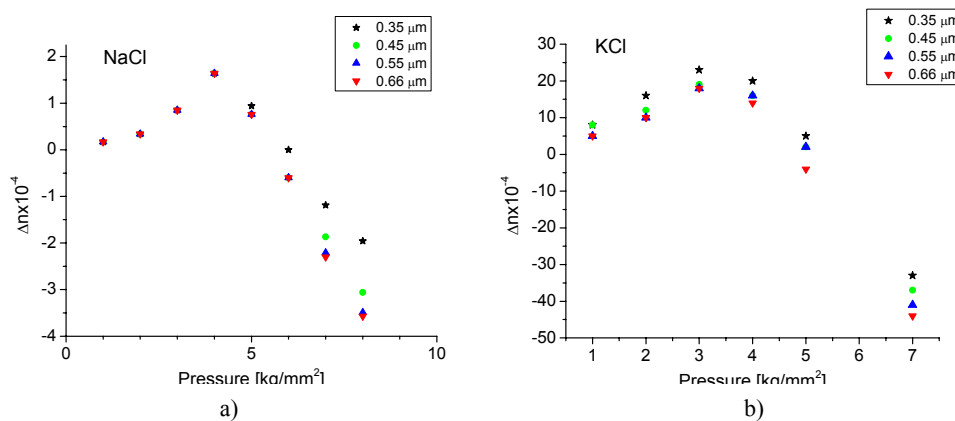


Fig. 8 – Dependence of the change of refractive index on pressure as kg/mm<sup>2</sup> for different wavelengths in NaCl and KCl crystals.

The change of refractive index with pressure can be explained by a change of both mechanical and optical properties of the crystal (density and absorption spectrum). In agreement with the paper cited, for low pressure the increase of the refractive index is due mainly to the increase of the crystal density. At higher pressure, the plastic compression of the crystal leads to a residual change of the density, which is the result of the effect of two concurrent processes: an increase of density and consequently of bulk compression, and a partial "crushing" of defects, and also a decrease of density, which is explained by the formation and development of new defects. Thus, the sharp decrease of the refractive index and negative values of  $\Delta n$  can be explained by the decrease in density of the crystals, which is due to the formation of microcracks, right up to the destruction of the

crystal. The effects are more pronounced for KCl crystals (multiplicative factor up to one order in magnitude) and observable for all wavelengths. Also, Chiba and coworkers [23, 24], observed important differences in the attenuation length of salt rock and synthesized samples in the region of radio frequencies.

#### 4. SUMMARY

In this paper, the role of phase and group velocities in Cerenkov radiation is revised. Because the group velocity is an essential characteristic of the phenomena, but it is not a measurable quantity, the regions of wavelength where the ratio between the group and the phase velocities is close to unity defines the regions where Cerenkov radiation could be used in practical applications and where the experimental errors in the measurements are minimized. Very interesting phenomena appear for the optical regions where the group velocity is negative. Concrete calculations were done for media with optical properties that permits the production of Cerenkov radiation and with different dispersive and symmetry properties.

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