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ILLUSTRATION OF THE COHERENCE OF NON-STATIONARY OPTICAL FIELD STHROUGH A TWO BEAM INTERFERENCE EXPERIMENT*

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Abstract. The interference phenomenon in non-stationary optical fields is investigated. We present a method of describing the interference of partially coherent optical beams by using only observable quantities, expressed entirely as temporal averages of the instantaneous signals, which better relates the experimental observations to the coherence properties of the field. We show that this method of description allows, even through a simple experimental method, to obtain a quantitative measure of the degree of optical coherence. In order to do so, we simulate the results of an interference experiment in which the optical field is non-stationary, recorded by means of the revolving disk method. In our analysis, we take into consideration all the aspects of the detection process, including the sensitivity of the photographic film upon which the intensity of the optical field is recorded. Graphical representations of the simulated experimental results are presented for all relevant cases.

Key words: optical coherence, non-stationary optical fields, interference.

1. INTRODUCTION

The theory of partial coherence is the modern scientific language which supports the description of optical phenomena related to the coherence of light. The modern formulation of the theory has been given in a series of papers by Wolf [1-3], Blanc-Lapierre and Dumontet [4] and has been continuously developed until the present [5].

The most important case with which the theory is concerned is the case of stationary, ergodic light [5]. The two special statistical properties of the optical processes allow a simplification of the mathematical formalism, the most important being that the correlation functions, the fundamental observable optical quantities, depend only on the temporal delay between the optical signals.

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In the last years, an increased interest has been shown for the description of the non-stationary optical fields, because of the large number of optical applications in which the light ceases to be stationary and ergodic and cannot be described using the standard formulation of the theory of partial coherence. Such applications include heterodyne interferometry [6], modulated optical fields, as the ones in optical telecommunication systems [7], and even experimental applications of pulsed lasers [8].

Some studies of non-stationary light have been performed even in the early stage of development of the theory of partial coherence. A first approach was initiated by Fischer [9] and extensively developed in a series of papers by Brătescu [10] and Tudor [10–13], based on a mixed temporal and spectral description. Other studies [15, 16] focused mainly on spectral considerations. Most modern approaches to the description of partially coherent non-stationary optical signals are based on or at least refer to the recently introduced two-frequency cross-spectral density [16, 17], which represents the correlation function of the generalized Fourier components of the optical signals. In the case of a stationary optical signal, this correlation function is null for Fourier components of different frequencies [5], while in the case of a non-stationary optical signal at least some of the Fourier components of different frequencies must be correlated.

Some models of non-stationary light have been proposed and analyzed, such as intrinsically stationary light (obtained through the modulation of stationary light) [18] and cyclostationary light, for which the correlation function of the optical signals is periodical [19]. Also, some representations of the correlation functions (in the temporal or in the spectral domain) have been recently introduced [20, 21].

In the analysis of non-stationary optical fields, the correlation functions defined as ensemble averages do not necessarily equal the results of time-integrated detector measurements. Another problem is raised by the definition of the coherence functions, which, if identified with the correlation function itself, mixes the coherence related information with other aspects of the optical phenomena (such as the dynamics of the intensity pattern). This because the non-stationarity of an optical signal is often determined by a non-stochastic component, not related to the coherence properties, the latter being of stochastic manifestation.

The last observation is also the reason for which, in many practical cases, the optical signals are separable [22], in the sense that their non-fluctuating components can be factorized as completely separate functions. These components, which are responsible for the non-stationarity of the optical fields, multiply in each of the optical signals stochastic functions which contain *all* the coherence related information and are stationary and ergodic.

In the present paper, we will use the advantages brought by such a separability property to offer a description of the interference phenomena in non-stationary optical fields based entirely on observable quantities. We will show that the separation allows the obtainment of a general interference law in an advantageous form, which can be applied, even through a classic experimental method (the revolving disk method [9–13]), for determining the degree of optical coherence of two interfering optical signals. We will perform numerical simulations of the observable results of such an interference experiment, in which the non-stationary optical field is obtained by the superposition of two quasi-monochromatic light beams of different frequencies. In obtaining the expressions of the relevant quantities, as well as in the graphical representations, we will take into consideration multiple aspects of the detection process.

2. EXPERIMENTAL ARRANGEMENT AND GENERAL INTERFERENCE LAW

Our analysis will be carried out for the interference of two optical fields produced in a Mach-Zehnder arrangement. The experimental setup, thoroughly described in [23], is schematically presented in Fig. 1. The light beam enters the Mach-Zehnder interferometer through the first beam-splitter (BS_1) and is half reflected, half transmitted. The two beams obtained this way travel along the two arms and, after passing through the second beam-splitter (BS_2), interfere on any of the two paths emerging from it. We will analyze the interference on the horizontal path, for which the default phase shift is null. The second beam-splitter is slightly tilted and a cylindrical lens (L) is used in order to produce two real image sources, equivalent to the slits of a Young arrangement. In order to produce a non-stationary interference field, one must insert at least in one of the two arms of the interferometer a dynamic device, as a frequency translator or modulator.

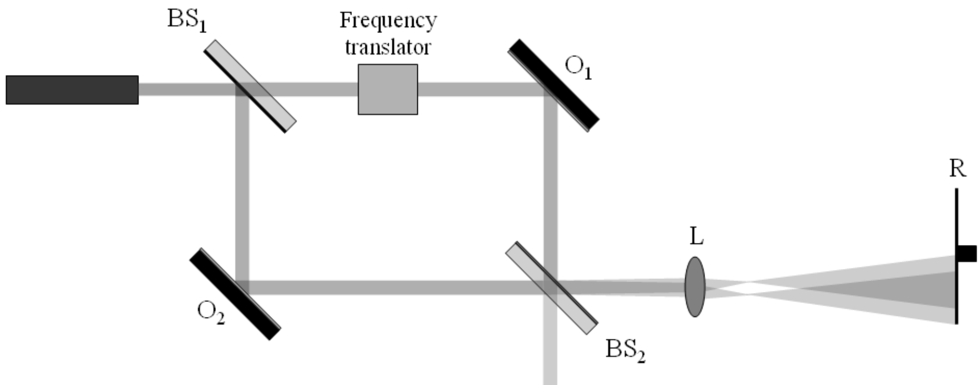

Fig. 1 – Experimental setup.

Let us consider that the optical signals emerging from the two image sources are $V_1(\mathbf{r}_1,t)$ and $V_2(\mathbf{r}_2,t)$. Since we consider the two optical signals separable, in the sense of the previous section, each of them can be written as:

$$V_1(\mathbf{r}_1,t)=U_1(\mathbf{r}_1,t)\,W_1(\mathbf{r}_1,t),$$
$$V_2(\mathbf{r}_2,t)=U_2(\mathbf{r}_2,t)\,W_2(\mathbf{r}_2,t), \tag{1}$$

in which the first functions (U) are non-fluctuating, while the second functions (W) are stationary and ergodic stochastic processes.

The superposition field at any point of space is given by the sum:

$$V(\mathbf{r},t)=K_1V_1(\mathbf{r}_1,t-t_1)+K_2V_2(\mathbf{r}_2,t-t_2), \tag{2}$$

in which K_1, K_2 are pure imaginary numbers and t_1, t_2 are the time intervals required for the light to travel from the two sources to the observation point [24].

If the optical field satisfies some ergodicity property, the detected intensity is given by:

$$I(\mathbf{r},t)=\left\langle V(\mathbf{r},t)V^*(\mathbf{r},t)\right\rangle_e . \tag{3}$$

The subscript e denotes the ensemble average and the subscript T will denote in the following the temporal average of the function between brackets.

By substituting Eq. (1) in Eq. (2) and further in Eq. (3) and by taking into consideration the properties of the U and W functions, one obtains the following general interference law [22]:

$$I(\mathbf{r},t)=I_1(\mathbf{r},t)+I_2(\mathbf{r},t)+2\,\mathrm{Re}\left[K_1K_2^*\ U_1(\mathbf{r}_1,t-t_1)\ U_2^*(\mathbf{r}_2,t-t_2)G_{12}(\mathbf{r}_1,\mathbf{r}_2,\tau)\right]=$$
$$=I_1(\mathbf{r},t)+I_2(\mathbf{r},t)+2\sqrt{I_1(\mathbf{r},t)}\sqrt{I_2(\mathbf{r},t)}\,\mathrm{Re}\left[\ \Phi(\mathbf{r}_1,\mathbf{r}_2,t-t_1,t-t_2)g_{12}(\mathbf{r}_1,\mathbf{r}_2,\tau)\right], \tag{4}$$

in which $G_{12}(\mathbf{r}_1,\mathbf{r}_2,\tau)$ and $g_{12}(\mathbf{r}_1,\mathbf{r}_2,\tau)$ are the refined forms of the mutual coherence function and of the complex degree of mutual coherence, respectively, defined as:

$$G_{12}(\mathbf{r}_1,\mathbf{r}_2,\tau)=\left\langle W_1(\mathbf{r}_1,t)\ \ W_2^*(\mathbf{r}_2,t+\tau)\right\rangle_T ,$$
$$g_{12}(\mathbf{r}_1,\mathbf{r}_2,\tau)=\frac{G_{12}(\mathbf{r}_1,\mathbf{r}_2,\tau)}{\sqrt{G_{11}(\mathbf{r}_1,\mathbf{r}_1,0)}\sqrt{G_{22}(\mathbf{r}_2,\mathbf{r}_2,0)}} . \tag{5}$$

As can be seen in Eqs. (5), the coherence functions are still defined using the temporal averages and are only functions of the relative temporal delay between the two signals, $\tau = t_1 - t_2$, because of the stationarity and ergodicity of the W

processes. Equation (4) could be obtained because statistically non-fluctuating functions emerge unchanged from the ensemble averaging operations.

The Φ function in Eq. (4) is the normed product of the two U functions and determines the dynamics of the interference pattern. Taking into consideration that all the coherence related information is contained in the W functions, it is justified to define the coherence functions as in Eqs. (5).

3. INTERFERENCE OF TWO QUASI-MONOCHROMATIC OPTICAL BEAMS OF DIFFERENT FREQUENCIES

If one inserts in one of the arms of the Mach-Zehnder interferometer in Fig. 1 a frequency translator, one obtains at the output the interference field of two quasi-monochromatic light beams of different frequencies:

$$\begin{aligned} I(\mathbf{r},t) &= I_1(\mathbf{r})(\ \ ) + I_2(\mathbf{r}) + 2\,\mathrm{Re}\left[K_1 K_2^* \mathrm{e}^{\mathrm{i}\left[(\omega_1-\omega_2)t + k_2 s_2 - k_1 s_1\right]} G_{12}(\mathbf{r}_1,\mathbf{r}_2,\tau)\right] = \\ &= I_1(\mathbf{r}) + I_2(\mathbf{r}) + 2\sqrt{I_1(\mathbf{r})}\sqrt{I_2(\mathbf{r})}\left|g_{12}(\mathbf{r}_1,\mathbf{r}_2,\tau)\right| \times \\ &\times \cos\left[(\omega_1-\omega_2)t + k_2 s_2 - k_1 s_1 + \beta_{12}(\mathbf{r}_1,\mathbf{r}_2,\tau)\right] \end{aligned}, \quad (6)$$

or, if one considers that the detection is performed in the far field (plane wave approximation) and also that the intensities emerging from the two paths of the Mach-Zehnder interferometer are equal:

$$I(\mathbf{r},t) = 2I_0(\mathbf{r})\left\{1 + \left|g_{12}(\tau)\right|\cos\left[\Delta\omega t - \Delta\mathbf{k}\cdot\mathbf{r} + \beta_{12}(\tau)\right]\right\}, \quad (7)$$

where we have renounced to explicitly mention the two vectors $\mathbf{r}_1$ and $\mathbf{r}_2$. The intensity field is dynamic, taking the form of an intensity wave [11, 12], which propagates in the direction given by the vector $\Delta\mathbf{k} = \mathbf{k}_1 - \mathbf{k}_2$. The velocity of this intensity wave is given by [11]:

$$\mathrm{u} = \frac{\Delta\omega}{\left|\Delta\mathrm{k}\right|^2}\Delta\mathrm{k}\,. \quad (8)$$

If one considers that the frequency translation is not too great, the optical field satisfies the local stationarity assumption [25], and thus the ensemble statistics predict well the results of a slow detector intensity measurement. The visibility of the interference pattern is given, as in the stationary case, by the complex degree of optical coherence, but in its refined definition of Eq. (5):

$$\mathcal{V} = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \left|g_{12}(\tau)\right|, \quad (9)$$

thus making the visibility of the interference pattern still the experimentally relevant measure of the degree of optical coherence.

The experimental method employed for recording the dynamic intensity fringes is the revolving disk method. In the far field, the experimental arrangement of Fig. 1 contains a disk (R), revolving at a constant angular velocity, Ω, upon which is placed a photographic film (Fig. 2).

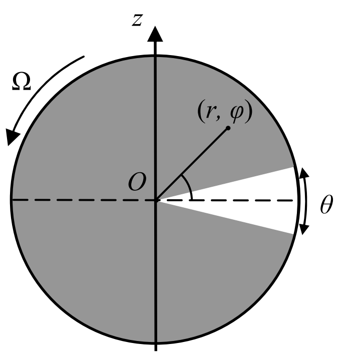

Fig. 2 – Revolving disk.

The revolving disk is placed so that the interference fringes move in planes parallel to its surface and in the direction of the *Oz* axis. A sectorial slit of angle θ is placed in front of the film, allowing the exposure of only the white region in Fig. 2, for which the velocities of the disk and of the fringes are equal.

The elementary areas of film have in the exposed region a different velocity than the interference fringes, except for a certain circumference, on which the interference pattern is recorded without distortion. It is the reason for which a detailed description of the detection process is necessary and will be given in the following.

In order to do so, one must calculate the exposure distribution on the film surface, which determines the observable response of the film. The finite time exposure to the intensity $I(t)$ can be defined, up to some constant factors, as [24]:

$$E = \int_0^{t_{ex}} I(t)\,\mathrm{d}t\,, \tag{10}$$

in which t_{ex} is the exposure time, given by:

$$t_{ex} = \frac{\theta}{\Omega}\,. \tag{11}$$

An easy way to calculate the exposure in a point of the photographic film, characterized by the radius r and the polar angle φ, during a pass through the exposed area, is to consider the intensity pattern static and the elementary portion of film moving with the relative velocity, $v_r(r) = \Omega\, r - \Delta\omega/|\Delta\mathbf{k}|$. As a result, using Eq. (7) in Eq. (10), one obtains the following expression of the exposure:

$$E(r,\varphi) = \int_0^{t_{ex}} 2I_0 \left\{1 + |g_{12}(\tau)| \cos\left[\frac{\Delta\omega}{\Omega}\varphi - \Delta k \cdot z(t) + \beta_{12}(\tau)\right]\right\} \mathrm{d}t = \\ = 2I_0 \int_0^{t_{ex}} \left\{1 + |g_{12}(\tau)| \cos\left[\frac{\Delta\omega}{\Omega}\varphi - \Delta k \cdot v_r(r) \cdot t + \beta_{12}(\tau)\right]\right\} \mathrm{d}t. \tag{12}$$

where $\Delta k = |\Delta\mathbf{k}|$. In the above expression we have taken into account that the delay with which the elementary film portion arrives at the sectorial slit, φ/Ω, determines the initial phase of the intensity distribution.

After some lines of calculus, one obtains for the exposure distribution:

$$E(r,\varphi) = 2I_0 t_{ex} \left[1 + |g_{12}(\tau)| \operatorname{sinc}\left(\frac{\Delta k \cdot v_r(r) \cdot t_{ex}}{2}\right) \times \right. \\ \left. \times \cos\left(\frac{\Delta\omega}{\Omega}\varphi - \frac{\Delta k \cdot v_r(r) \cdot t_{ex}}{2} + \beta_{12}(\tau)\right)\right]. \tag{13}$$

The exposure of the film after N complete rotation periods is obtained by multiplying Eq. (13) with N (for synchronism, $\Delta\omega$ must be a multiple of the angular velocity Ω). One must note in Eq. (13) that the complex degree of coherence is not integrated, thus implying that its variation is negligible in the region of the sectorial slit. This is an approximation, but it is well satisfied, taking into consideration the possibility of using a very narrow sectorial slit.

The points that have null relative velocity are situated at a distance from the center of the revolving disk equal to:

$$r_0 = \frac{\Delta\omega}{\Omega|\Delta\mathbf{k}|}, \tag{14}$$

obtained by imposing the condition $v_r = 0$. For these points, Eq. (13) takes a form similar to Eq. (7), the circumferential visibility defined in terms of exposure being equal to the intensity visibility of Eq. (9).

It is useful to represent Eq. (13) in the variable $\rho = r/r_0$, by using Eq. (14) and the expression of the relative velocity:

$$E(\rho,\varphi) = 2I_0 t_{ex} \left\{1 + |g_{12}(\tau)| \operatorname{sinc}\left[\frac{\Delta\omega}{\Omega}\frac{\theta}{2}(\rho - 1)\right] \times \right. \\ \left. \times \cos\left[\frac{\Delta\omega}{\Omega}\varphi - \frac{\Delta\omega}{\Omega}\frac{\theta}{2}(\rho - 1) + \beta_{12}(\tau)\right]\right\}. \tag{15}$$

Equation (15) shows that the exposure, written in the variable ρ, is completely defined by the parameters $\Delta\omega/\Omega$, θ and $g_{12}(\tau)$. In the following, we will analyze how these parameters influence the exposure distribution on the photographic film, making use of graphical representations. We will make the representations in the normed coordinate ρ, which allows some quite general results, requiring specific values only for the three parameters mentioned above. The fraction $\Delta\omega/\Omega$ gives the number of interference fringes recorded on a film circumference. In all our representations, we will set it equal to 30. Also, we will always scale the disk to have a radius of 3ρ.

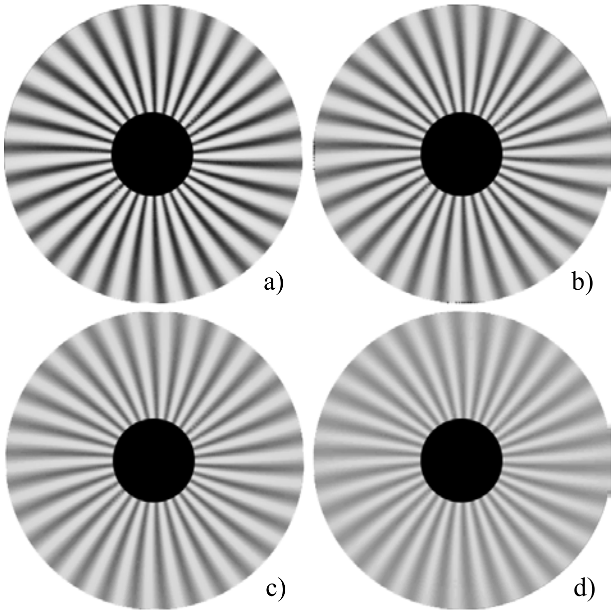

Fig. 3 – Simulation of the exposure distribution for decreasing values of $|g_{12}(\tau)|$: a) 1; b) 0.8; c) 0.6; d) 0.4.

With these settings, in Fig. 3 one can see graphical representations of the exposure distribution for decreasing absolute values of the degree of optical coherence. By analyzing Eqs. (13) and (15), one can see that the information regarding the interference field is distorted by the "sinc" factor, the circumferential exposure visibility being modulated as:

$$\mathcal{V}_e(r)=\left|g_{12}(\tau)\right|\left|\operatorname{sinc}\left[\frac{\Delta k\cdot v_r(r)\cdot t_{ex}}{2}\right]\right|;\qquad \mathcal{V}_e(\rho)=\left|g_{12}(\tau)\right|\left|\operatorname{sinc}\left[\frac{\Delta\omega}{\Omega}\frac{\theta}{2}(\rho-1)\right]\right|. \quad (16)$$

Taken as a whole, the exposure visibility is not a direct measure of the degree of optical coherence. The central maximum of the sinc factor has a width:

$$\Delta r = \frac{4\pi}{\Delta k \cdot \theta}, \tag{17}$$

which is inversely proportional to the angle of the sectorial slit.

In general, it is hard to precisely identify the circumference of radius r_0, for which the exposure visibility gives the correct absolute value of the complex degree of optical coherence. Still, Eq. (17) shows that, by sufficiently diminishing the angle of the sectorial slit, down to a value which does not produce any observable diffraction effects in the relevant region of the photographic film, one can obtain a wide sinc factor. In this way, the problem of precisely identifying the circumference of radius r_0 ceases to be critical, because the decrease of visibility is negligible on a wide enough area around this circumference. The effect of the sectorial slit angle θ on the width of the sinc factor is presented in Fig. 4.

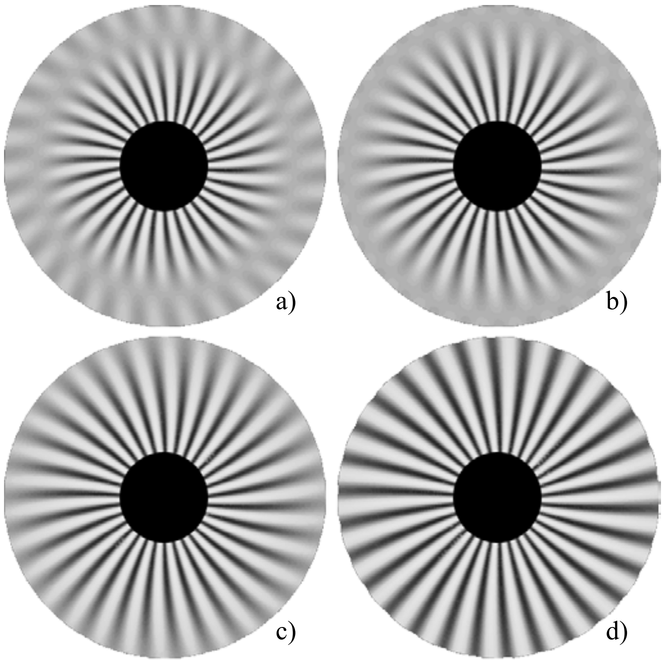

Fig. 4 – Simulation of the exposure distribution for decreasing values of θ: a) 9°; b) 7°; c) 5°; d) 3°.

4. DESCRIPTION OF THE EXPERIMENTALLY OBSERVABLE QUANTITIES

A significant problem in the real experimental situation is that the exposure distribution rarely determines a proportional observable effect on the photographic film. As a consequence, in the general case, the characteristic response of the photographic film to illumination leads to a distortion of the pattern described by Eqs. (13) or (15).

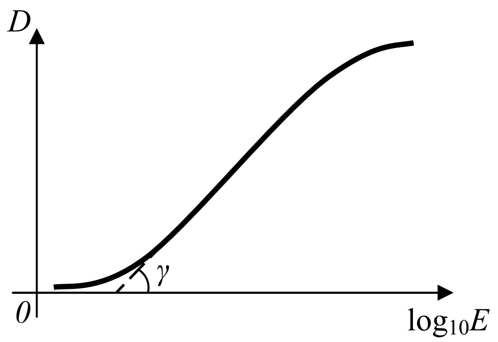

Fig. 5 – Characteristic (H-D) curve of the photographic film.

The degree to which a (black and white) film is darkened due to the interaction with the light is described by the film density, whose dependence on the light exposure is given by the Hurter-Driffield (H-D) curve of the film. The general form of the curve is presented in Fig. 5 [24]. The optical density is defined as:

$$D = -\log_{10} T = \log_{10} O, \tag{18}$$

where T denotes the transmittance and O the opacity of the photographic film, the latter being defined as the inverse of the transmittance, $1/T$.

In the following, we will consider that the exposure of the photographic film is situated in the linear region of the H-D characteristic. This means that the relation between the film density and the light exposure is [24]:

$$D = D_0 + \gamma \log_{10} \frac{E}{E_0}, \tag{19}$$

where D_0 and E_0 are constants.

The logarithmic dependency in Eq. (19) leads to very complicated expressions for the film density, which make it less advantageous for describing the observable effects of the intensity distribution. Much more useful from this point of view is the opacity of the photographic film, an easily measurable quantity, which, by means of Eqs. (18) and (19), can be expressed as:

$$O = O_0 \left(\frac{E}{E_0} \right)^{\gamma} . \tag{20}$$

Equation (20) shows that the opacity does not vary linearly with the exposure, except for the particular case γ = 1, which is achievable in practice. For this particular case, the opacity of the photographic film has an expression similar to Eqs. (13) or (15) and thus all the results of the previous section are applicable, only that they refer to an experimentally measurable quantity. In the general case, which takes into consideration all possible values of γ, Eq. (15) becomes:

$$\begin{aligned} O(\rho,\varphi) = \alpha \Bigg\{ 1 + \left| g_{12}(\tau) \right| \operatorname{sinc}\left[\frac{\Delta\omega}{\Omega} \frac{\theta}{2} (\rho - 1) \right] \times \\ \times \cos\left[\frac{\Delta\omega}{\Omega} \varphi - \frac{\Delta\omega}{\Omega} \frac{\theta}{2} (\rho - 1) + \beta_{12}(\tau) \right] \Bigg\}^{\gamma} , \end{aligned} \tag{21}$$

where we have grouped under the letter α all the factors that have the same value for all elementary areas of exposed photographic film.

In this case, the circumferential visibility of the recorded interference pattern, expressed in terms of the measurable opacity, is:

$$\begin{aligned} \mathcal{V}_o(\rho) &= \frac{O_{\max}(\rho) - O_{\min}(\rho)}{O_{\max}(\rho) + O_{\min}(\rho)} = \\ &= \frac{\left\{ 1 + \left| g_{12}(\tau) \right| \operatorname{sinc}\left[\frac{\Delta\omega}{\Omega} \frac{\theta}{2} (\rho - 1) \right] \right\}^{\gamma} - \left\{ 1 - \left| g_{12}(\tau) \right| \operatorname{sinc}\left[\frac{\Delta\omega}{\Omega} \frac{\theta}{2} (\rho - 1) \right] \right\}^{\gamma}}{\left\{ 1 + \left| g_{12}(\tau) \right| \operatorname{sinc}\left[\frac{\Delta\omega}{\Omega} \frac{\theta}{2} (\rho - 1) \right] \right\}^{\gamma} + \left\{ 1 - \left| g_{12}(\tau) \right| \operatorname{sinc}\left[\frac{\Delta\omega}{\Omega} \frac{\theta}{2} (\rho - 1) \right] \right\}^{\gamma}} , \end{aligned} \tag{22}$$

so it does not have the same radial variation as the exposure visibility.

The variation of the opacity distribution, for different values of the γ factor, is described in the two plots of Fig. 6. The left plot presents the radial distribution of the opacity visibility (as a function of ρ), while the right plot presents the distribution of the opacity normed by α, on the circle of radius r_0 (as a function of φ). The value of $|g_{12}(\tau)|$ is taken equal to 0.6.

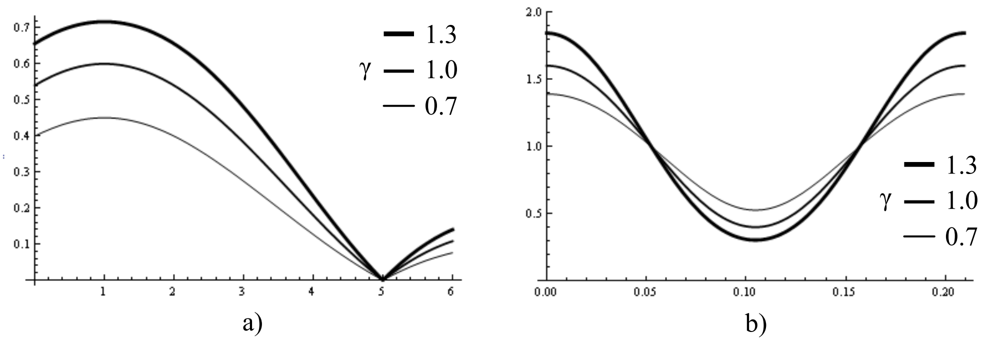

Fig. 6 – Effect of film sensitivity on experimentally observable quantities:
a) opacity visibility; b) circumferential opacity distribution.

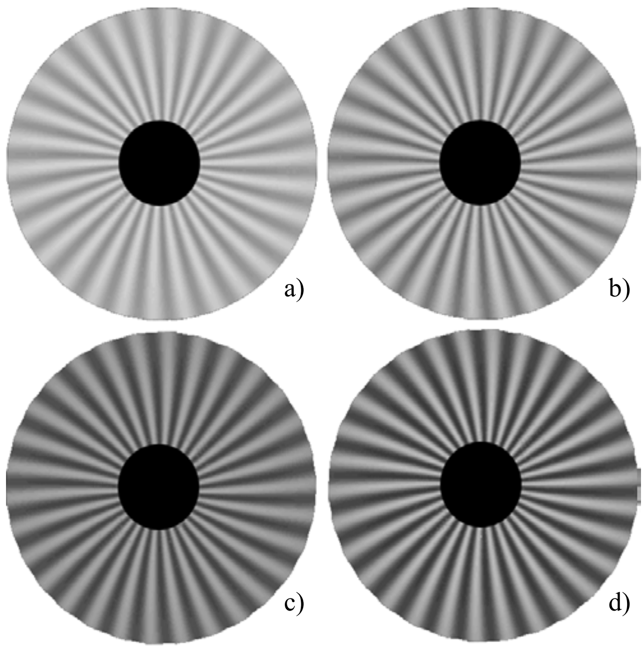

Fig. 7 – Simulation of the opacity distribution for increasing values of γ:
a) 0.7; b) 1.0; c) 1.5; d) 2.0.

One must note in Fig. 6 that the film sensitivity influences the absolute value of the opacity visibility and also the maximum and minimum values of the opacity on a certain circumference. More precisely, the opacity maxima are higher and the opacity minima are lower for films of higher γ. As a consequence, the opacity visibility increases on a certain circumference with the increase of γ. The influence of the sectorial slit angle on the opacity distribution does not depend on the film sensitivity, the behavior being the same as the one pointed out in the previous section for the exposure distribution. A disadvantage is the fact that on the circumference of radius r_0 the opacity visibility is not equal to $|g_{12}(\tau)|$:

$$\mathcal{V}_o(r_0) = \frac{\left[1+\left|g_{12}(\tau)\right|\right]^{\gamma} - \left[1-\left|g_{12}(\tau)\right|\right]^{\gamma}}{\left[1+\left|g_{12}(\tau)\right|\right]^{\gamma} + \left[1-\left|g_{12}(\tau)\right|\right]^{\gamma}}, \tag{23}$$

and the expression does not allow to obtain $|g_{12}(\tau)|$ analytically. There is however no problem to solve the equation through numerical methods.

In Fig. 7 is presented the influence of γ on the opacity distribution for an absolute value of the degree of optical coherence equal to 0.5.

5. CONCLUSIONS

The coherence phenomenon in non-stationary optical fields was illustrated through the simulation of an interference experiment with quasi-monochromatic light beams of different frequencies.

The experiment exemplified a widely encountered case, in which the components of the optical signals which determine the non-stationarity of the interference field can be completely factorized, carrying no coherence related information. Also, it showed that a refined definition of the coherence functions is justified in the case of such separable optical fields, because it is better connected to the experimentally observable quantities.

One could thus obtain, through a classical recording method, concrete information about the degree of optical coherence. We presented in detail, supported by graphical representations, the means by which this information can be extracted, including the influence of some experimental conditions on the observable results.

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