THE INTEGRATED SACHS-WOLFE EFFECT IN CROSS-CORRELATION WITH GALAXY SAMPLES – A RELIABLE INDEPENDENT PROBE FOR CONSTRAINING COSMOLOGY

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Abstract, The Λ-dominated Cold Dark Matter (ΛCDM) model predicts the existence of a secondary anisotropy of the Cosmic Microwave Background (CMB), called Integrated Sachs-Wolfe (ISW) effect, due to the passing of CMB photons, after the radiation-matter decoupling, through regions with time varying gravitational potential. Since the ISW effect is manifesting at large angular scales in the CMB anisotropies, where it is dominated by the primary CMB anisotropies and by the cosmic variance, has been proposed to measure the ISW effect of CMB from the cross-correlation with local tracers of the gravitational potential from Large Scale Structure (LSS) observations. We analyze the ability of this probe to constrain the cosmological model, especially the dark energy component. We found an agreement between the constraint on the dark-energy density parameter, Ω_Λ, from CMB anisotropy-galaxy overdensity cross-correlation data, and that from CMB anisotropy data alone. Though the ISW effect gives a weaker determination of Ω_Λ than CMB anisotropy data, it can be used as an independent test for the dark energy.

Key words: cosmic microwave background, cosmology, large-scale structure, cosmological parameters, data analysis.

1. INTRODUCTION

The ΛCDM model, which parameterize the “concordance Big-Bang cosmology”, is the most favored cosmological model, achieving confirmation from an increasing number of cosmological probes like CMB anisotropies observations, LSS observations of galaxy distribution and number density, as well as luminosity distance from Supernovae (SN) observations.

The Λ, or dark energy component, allowing for the current accelerating expansion of the Universe, is the dominant energy component, but its physical origin and properties are still insufficiently known. Even the existence of the dark energy, the mysterious component of negative pressure has been only inferred by
now. It has been first required by SN observations, which are consistent with a dark energy component of the Universe accounting for 71.3% of the critical density \cite{1}. The CMB observations, consistent with a flat Universe, with the total energy density equal to the critical density, and with a total matter energy density of only 27% \cite{2}, confirm the existence of dark energy, responsible for the remaining 73% of the total energy density of the Universe. Previous CMB observations \cite{3} give a matter contribution to the total energy density of the Universe today of 26% and a dark energy contribution of 74%. Also, the LSS observations agree with the CMB observations, being consistent with a total matter energy density of only 30% of the critical density.

A more direct and independent probe of the dark energy in a flat Universe is the Integrated Sachs-Wolfe effect of the CMB \cite{4}. This effect arises when CMB photons traverse time varying gravitational potentials of the large scale structures, because the energy gained when the photons enter in the potential well associated with a massive object is only partly lost when the photons escape from the potential well, due to the variation of the potential during the time the photons are traveling through it. The net effect is a contribution to the secondary anisotropies of the CMB, produced on the way of the CMB photons from the last scattering to the present. The existence of a gravitational potential which evolve in time at a given physical scale is an evidence for the existence of an energy component of the Universe, which is smooth at that scale and have not a contribution to the density perturbation, but have a contribution to the gravitational potential. This is the case of the dark energy, therefore the ISW effect of the CMB is a direct indicator of the dark energy. The ISW effect is expected to contribute to the CMB temperature anisotropies at the largest physical scales, corresponding to the lowest multipoles of the CMB temperature anisotropy power spectrum, where its detection is strongly limited by the cosmic variance. Therefore has been proposed, as an independent observational test of the ISW effect, the cross-correlation between CMB temperature fluctuations and local tracers of the gravitational potential like galaxy samples \cite{5}.

\textbf{2. THE ISW-GALAXY ANGULAR CROSS-CORRELATION FUNCTION}

\textbf{2.1. THEORETICAL FRAMEWORK}

The CMB temperature anisotropy corresponding to a given direction into the sky is defined as the relative difference between the CMB temperature for the considered direction and the sky averaged CMB temperature

\[
\Delta_T(\hat{n}) = \frac{T(\hat{n}) - T_0}{T_0},
\]  \hspace{1cm} (1)
where \( \hat{n} \) is a given direction into the sky and \( T_0 \) is the sky average of the CMB temperature. The total anisotropy is a superposition of different contributions, classified as primary and secondary anisotropies of CMB as they exist at the radiation-matter decoupling or are generated after the decoupling epoch. The CMB anisotropies can be understood as being generated by metric perturbations of the Friedmann – Robertson – Walker (FRW) homogeneous background associated to gravitationally unstable density fluctuations. At large angular scales, the CMB anisotropies consist in a contribution of the intrinsic anisotropy from the initial photon distribution and the contribution from the effect of the gravitational potential. The latter includes the ordinary Sachs-Wolfe effect, given by the existence of the gravitational potential from the last scattering until present epoch, and the differential gravitational redshift, or ISW effect, given by the time evolution of the gravitational potential along the line of sight. These effects have been first introduced in the original paper of Sachs and Wolfe [4]. The contribution of the total Sachs-Wolfe effect to the CMB temperature anisotropy can be understood as the effect of a scalar metric perturbation, introduced by the gravitational potential \( \Phi \), on the relative variation of the product between the photon energy and the scale factor \( a \) (which is related to the redshift \( z \) through \( a = 1/(1+z) \)), this product being constant in the unperturbed Universe. The gravitational potential completely describe a scalar perturbation of the FRW background metric and, choosing the particular longitudinal or Newtonian gauge, the line element for the perturbed metric can be expressed as

\[
dx^2 = a^2 (\eta)[-(1+2\Phi)d\eta^2 + (1-2\Phi)\delta_{ij}dx^i dx^j],
\]

where \( \eta \) is the conformal time, \( x^i \) are the space coordinates and \( \Phi \) is the scalar perturbation named Newtonian gravitational potential. Considering the components of the photon four-momentum linear in the perturbation and using the time component of the geodesic equation up to first order in the perturbation can be obtained the ISW contribution to the CMB temperature anisotropy:

\[
\Delta_T^{\text{ISW}}(\hat{n}) = -2 \int_0^{z_{\text{dec}}} dz \frac{d\Phi}{dz} (\hat{n}, z),
\]

where the subscript ‘dec’ refers to the decoupling time and ‘0’ refers to the present.

An intuitive derivation of the ISW anisotropy based on a potential approximation of Einstein’s field equation can be found in [6] and for an understanding of the generation of CMB anisotropies in connection with structure formation in the framework of general relativistic perturbations we recommend [7].

The above integral can be cut in two pieces at an intermediate redshift \( z_* \), chosen during full matter domination, when the gravitational potential is static, resulting into two contributions in the CMB anisotropies: the early ISW,
corresponding to the epoch immediately after decoupling, before full matter domination, when the gravitational potential vary in time, and the late ISW, generated during the dark energy domination, in which we are interested.

On the other side, the galaxy overdensity in a given direction into the sky, is defined as the relative difference between the galaxy number density in the direction \(\hat{n}\), \(N_G(\hat{n})\), and the map averaged galaxy number density, \(\bar{N}_G\),

\[
\delta_G(\hat{n}) = \frac{N_G(\hat{n}) - \bar{N}_G}{\bar{N}_G}.
\] (3)

The galaxy overdensity of a given galaxy sample is related to the total matter overdensity \(\delta_m\) through the galaxy bias \(b_G\) (\(\delta_G = b_G\delta_m\)) depending also on the normalized redshift distribution of the galaxy sample, \(\phi_G(z)\), and is given by

\[
\delta_G(\hat{n}) = \int dz b(z) \phi_G(z) \delta_m(\hat{n}, z),
\] (4)

where \(b(z)\) is the redshift dependent bias function relating the galaxy overdensity to the total matter overdensity, and \(\phi_G(z)\) is the galaxy selection function, which model the redshift distribution of the galaxy sample.

The quantity probing the late ISW effect is the angular cross-correlation function of the galaxy overdensity and CMB temperature anisotropy, defined as:

\[
w_{TG}^{\text{ISW}}(\theta) = \langle \Delta_T^{\text{ISW}}(\hat{n}_1) \delta_G(\hat{n}_2) \rangle,
\] (5)

where the average is over all the direction pairs \((\hat{n}_1, \hat{n}_2)\) separated by the angle \(\theta\). The angular cross-correlation function can be expanded, in a Legendre polynomial basis, as:

\[
w_{TG}^{\text{ISW}}(\theta) = \sum_{l} \frac{2l + 1}{4\pi} p_l(\cos \theta) C_{TG}^{\text{ISW}}(l),
\]

where \(C_{TG}^{\text{ISW}}(l)\) is the corresponding angular power spectrum of the CMB anisotropy-galaxy overdensity cross-correlation. Using the Poisson equation, which relate the Newtonian gravitational potential to the matter distribution, \(C_{TG}^{\text{ISW}}(l)\) can be written as [8, 9]:

\[
C_{TG}^{\text{ISW}}(l) = \frac{4}{(2l + 1)^2} \int dz W_{ISW}(z) W_G(z) \frac{H(z)}{c} P(k),
\] (6)

with
\[
W_{\text{ISW}}(z) = 3\Omega_m(H_0 / c)^2 \frac{d[D(z)(1+z)]}{dz}, \quad W_G(z) = b(z)\phi_G(z)D(z),
\]

where \( H_0 \) is the Hubble expansion rate today, \( c \) is the speed of light, \( H(z) \) is the Hubble rate at redshift \( z \), \( \Omega_m \) is the matter energy density parameter, \( P(k) \) is the matter power spectrum today, defined by the total matter overdensities in Fourier space as \( P(k)\delta^3(\hat{k} - \hat{k}') = \langle \delta_m(\hat{k}) \delta_m(\hat{k}') \rangle \), being related to the matter power spectrum at a given redshift, \( z \), through the growth factor \( D(z) \)
\[
P(k,z) = D^2(z)P(k).
\]

The wave number \( k \) in Eq. (6) is established, for each multipole order \( l \), by the conformal distance to redshift \( z \), through the relation
\[
k = \frac{l+1/2}{r(z)},
\]
where \( r(z) \) is the conformal distance to redshift \( z \).

### 2.2 COMPUTATIONAL APPROACH

In this work we implemented the numerical algorithm for the computation of the theoretical angular cross-correlation power spectrum, \( C_{\text{TG}}^{\text{ISW}}(l) \), by modifying the public software package CAMB [10], originally designed to calculate the total CMB anisotropies angular power spectra and the matter power spectrum for a given cosmological model. Our modified version of CAMB calculate, in addition to the CMB total anisotropy power spectra and the total matter density perturbations power spectrum, the angular CMB anisotropy – galaxy overdensity cross-correlation power spectrum, for a galaxy sample with a known redshift distribution. In order to compute with CAMB, using Eq. (6), the angular cross-correlation power spectrum, \( C_{\text{TG}}^{\text{ISW}}(l) \), we used the theoretical matter power spectrum \( P(k) \), computed by the original CAMB for the default range of wavenumbers \( k \), obtaining by interpolation the matter power spectrum \( P(k) \) for each wavenumber \( k \) in the integral from Eq. (6). Also, we used additional routines for the numerical computation, for a given cosmological model, of the conformal distance to redshift \( z \), \( r(z) \), of the Hubble expansion rate at redshift \( z \), \( H(z) \), of the growth factor at redshift \( z \), \( D(z) \) and of the derivative \( \frac{d[D(z)(1+z)]}{dz} \), for each redshift in the integration.

The galaxy selection function \( \phi_G(z) \) has been modeled, following [8], as
\[
\phi_G(z) \propto (z-z_c)^2 \exp\left(-\frac{z-z_c}{z_0-z_c}\right)^{3/2} \quad \text{for} \quad z > z_c \quad \text{and} \quad \phi_G(z) = 0 \quad \text{otherwise},
\]
where \( z_0 \) specifies the median redshift \( z_m=1.4z_0 \) and \( z_c \) the cut redshift of a galaxy sample.

For testing the modified CAMB software we calculated the theoretical CMB-galaxy cross-correlation power spectra using the galaxy selection functions.
describing the redshift distribution of two selected galaxy samples included in the publicly available photometric data from Sloan Digital Sky Survey – Data Release 4 [11]. The galaxy samples have been selected by [8] from SDSS-DR4 photometric data as satisfying a magnitude criterion, for the $r=20–21$ galaxy sample, respectively a color criterion for the Luminous Red Galaxy (LRG) sample. Following [8], we used a galaxy selection function with $z_c \approx 0$ and $z_0 \approx 0.2$ to describe the redshift distribution of the $r=20–21$ SDSS-DR4 galaxy sample, respectively $z_c \approx 0.37$ and $z_0 \approx 0.45$ for the SDSS-DR4 LRG galaxy sample. We used the angular cross-correlation functions of these two galaxy samples and the WMAP – 3 years CMB anisotropy map, given by [8] in Table 1, in comparison with our theoretical angular cross-correlation functions, obtained with the modified CAMB. We adopted the constant values $b(z)=1.24$ and $b(z)=1.43$ for the corresponding bias of the $r=20–21$ and LRG galaxy samples respectively, which are the average values quoted by [8].

In Fig. 1 we present the theoretical ISW-galaxy angular cross-correlation functions for the $r=20–21$ (upper panel) and LRG (lower panel) galaxy samples, compared with the experimental data published in [8].

![Fig. 1 – The ISW-galaxy experimental angular cross-correlation functions for $r=20–21$ (top) and LRG (bottom) SDSS-DR4 galaxy samples with WMAP-3 years CMB data and error bars, from [8]. The continuous lines shows the models computed with our modified CAMB software.](image-url)
The cosmological model assumed for the theoretical angular cross-correlation functions computed with our modified CAMB version is the best fit model obtained by [8] and is described by the following cosmological parameters: the reduced Hubble expansion rate today $h = 0.71$, the energy density parameter for baryonic matter $\Omega_b = 0.022 / h^2$, the energy density of the dark energy $\Omega_{\Lambda} = 0.83$, flat cosmological model with $\Omega_k = 0$, the spectral index for the scalar primordial perturbations $n_s = 0.938$ and the normalization of the matter power spectrum given by $\sigma_8 = 0.75$.

### 3. DATA ANALYSIS

Our purpose has been to test the ability of the ISW cross-correlation signal from CMB anisotropy and galaxy overdensity maps to constrain the cosmological model, with a special attention to the energy density parameter of dark energy. We used the CMB-galaxy angular cross correlation data published by [8] for constraining the cosmological parameters. For this purpose we have used the COSMOMC software package [12], which explore the cosmological parameters space by a fast Markov – Chain Monte-Carlo method and compare each cosmological model with one or more data sets through the likelihood function. The original version of COSMOMC package use the CAMB software to calculate the CMB anisotropies power spectra and the total matter power spectrum of the cosmological models when CMB or LSS data are analyzed. In order to compare the CMB anisotropy – galaxy overdensity cross correlation data sets with the theoretical predictions for different cosmological models, we included in the public COSMOMC package our modified version of CAMB software, and used it to compute the theoretical ISW angular cross-correlation function for the selected galaxy samples. Also, we extended the original COSMOMC modules, by including the evaluation of the likelihood function for the new added CMB-galaxy cross-correlation data.

Running the modified COSMOMC we obtained the parameters of the $\Lambda$CDM cosmological model which match with the considered ISW cross-correlation data. We compared these results with those obtained by running the COSMOMC on the WMAP-5 years CMB anisotropy data.

In Fig. 2 we compare the probability distributions, obtained by marginalizing over the other parameters, for the cosmological parameters $\Omega_{\Lambda}$ and $\Omega_m = 1 - \Omega_{\Lambda}$, obtained from running COSMOMC on the WMAP-5 years CMB data and on the cross-correlation of WMAP-3 years CMB map with the galaxy overdensity maps corresponding to $r = 20–21$ and LRG galaxy samples from SDSS-DR 4.
Fig. 2 – Probability distributions for the cosmological parameters \( \Omega_\Lambda \) and \( \Omega_m \) obtained from ISW-galaxy cross-correlation data (dashed lines) and from WMAP-5 years CMB anisotropy data (continuous lines).

Fig. 3 – 2D contours at 68% and 95% CL in the plane \( \Omega_\Lambda - \Omega_m \) from ISW cross-correlation data (dashed lines) and from WMAP-5 years CMB anisotropy data (continuous lines).
The confidence intervals at 95% CL obtained from these simulations, for the parameters $\Omega_\Lambda$ and $\Omega_m$ are: $0.594 < \Omega_\Lambda < 0.910$ and $0.090 < \Omega_m < 0.406$ from CMB anisotropy – galaxy overdensity cross-correlation and $0.681 < \Omega_\Lambda < 0.798$ and $0.202 < \Omega_m < 0.319$ from WMAP-5 years CMB data.

Figure 3 show the 2D contours in the plane $\Omega_\Lambda - \Omega_m$ at 68% and 95% CL, obtained from the same COSMOMC runs.

4. DISCUSSION

In this work we tested if the ISW signal from the cross-correlations of CMB anisotropies with galaxy samples may be used to constrain the parameters of the cosmological model. Therefore, we implemented in the publicly available CAMB and COSMOMC software packages the numerical algorithms for computing the theoretical predictions for the ISW-galaxy angular cross-correlation function for galaxy samples with known redshift distributions and for a given cosmological model and compared these predictions with experimental data.

Using the experimental data published by [8] for the cross-correlation of WMAP-3 years CMB anisotropy data with the galaxy overdensities corresponding to two galaxy samples selected from SDSS-DR4, we constrained the parameters of the $\Lambda$CDM cosmological model, comparing the results with those obtained from the analysis of CMB anisotropy data from WMAP-5 years.

We focused on the ability of ISW cross-correlation data to constrain the dark energy density parameter $\Omega_\Lambda$ of the $\Lambda$CDM flat cosmological model. It can be observed, from Fig. 2, that cross-correlation data prefer a higher best fit value for $\Omega_\Lambda$ comparing to WMAP-5 years data and, also, a larger confidence interval. Nevertheless, it can be observed from Fig. 2 and Fig. 3, that the confidence intervals for $\Omega_\Lambda$ obtained from the analysis of ISW cross-correlation data and from CMB data overlap, emphasizing the excellent agreement between these two probes.

While ISW effect seen in the cross-correlation of CMB anisotropy and galaxy overdensity maps does not constrain the cosmological model better than CMB alone or other cosmological probes, it can be seen as an independent probe for the dark energy, having the advantage of probing directly the epoch when the dark energy started to dominate the Universe.

We hope that, from the ISW effect seen by future better CMB and LSS data, we will have a contribution in improving our understanding of the nature and dynamics of the dark energy.
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