ON COULOMB DISSOCIATION OF HALO NUCLEI

R.A. IONESCU, C. HATEGAN

Institute of Atomic Physics, Bucharest, Romania

Received May 27, 2010

The Coulomb dissociation of neutron halo nuclei is studied as an excitation process into the continuum. This way we proved that the dissociation of the halo nuclei in high energy collisions on heavy targets is a powerful way to extract structure informations. The results presented can be used to compare the theoretical differential cross sections for different models of the two-neutron halo with the experimental data.

1. INTRODUCTION

Fragmentation reactions are powerful tools to investigate halo nuclei [1]. Different models for one-neutron halo breakup [2–4] as well as for two-neutron halo [5–7] have been discussed in the literature. In general the breakup of weakly bound nuclear systems is governed both by the short range nuclear attraction and the long range Coulomb repulsion. We will focus in what follows on the processes governed by the Coulomb repulsion.

The basic features of heavy ion reactions can be understood in terms of an interaction potential between the centers of mass of the two colliding nuclei consisting of a Coulomb repulsion and a short-range nuclear attraction. The interaction potential displays a maximum (the Coulomb barrier) and the two colliding nuclei must have an energy of relative motion which exceeds this barrier in order to form a composite system. The Coulomb barrier increases with the atomic number of the two colliding nuclei. If two heavy ions scatter with an energy of relative motion below the Coulomb barrier the main interaction between them is the point Coulomb interaction which gives rise to a hyperbolic (Rutherford) trajectory of relative motion. Due to the finite extensions of the interacting nuclei, the electric field may cause excitations in both systems. Because of the simplicity of the Coulomb interaction, the experimental results from Coulomb excitation can be analyzed in a model independent way with great accuracy, providing unambiguous information about the electromagnetic properties of nuclear states.

During the last forty years this process has been extensively studied with lighter ions and has played a crucial role in establishing the properties of collective (vibrational and/or rotational) low-lying states of nuclei throughout the mass table [8].

Because of the slow variation of the Coulomb field in space, it can only excite modes of low multipolarity. Because of the slow variation of the Coulomb field in time, it is impossible to excite states of high energy, since the process in this limit becomes adiabatic.

Even below the Coulomb barrier the elastic scattering deviates strongly from the Rutherford cross section; this strong deviation does not imply any significant change in the trajectory of relative motion from the classical hyperbolic orbit, but is solely due to the depopulation of the ground state through Coulomb excitation.

With the weakly bound nuclear systems which becomes available recently in the experimental facilities around the world we expect that the Coulomb interaction may excite the system into continuum as the necessary energy is small and moreover there is no excited bound states available in the system.

In the following we will study the general properties of the Coulomb dissociation of the neutron halo nuclear systems.

2. MULTIPOLE EXPANSION OF COULOMB INTERACTION FOR NEUTRON HALO PROJECTILE

The Coulomb potential acts only on the charged core and one has in the target frame of reference

$$V_C = \frac{e^2}{4\pi\epsilon_0}\frac{Z_T Z_P}{|\vec{\rho}+\vec{r}|},$$

where $\vec{\rho}$ is the projectile center of mass position vector, $\vec{r}$ is the core position in the projectile center of mass frame and Z_T and Z_P are the atomic numbers of the target and projectile, respectively. One can expand the Coulomb potential in power series in $\vec{r}$ and in the first order (dipole approximation) one obtains

$$V_C \simeq \frac{e^2}{4\pi\epsilon_0}\frac{Z_T Z_P}{\rho} - \frac{e^2}{4\pi\epsilon_0}\frac{Z_T Z_P}{\rho^3}\vec{\rho}\vec{r}.$$

The first term acts on the position $\vec{\rho}$ of the projectile center of mass and leads to an overall scattering of the halo nucleus. As we are only interested in the part that acts on the relative position of the core and valence neutrons and cause dissociation we have to consider only the second term and denote it by V_{dip}.

The dipolar potential can be expressed alternatively in terms of the valence neutron position using the properties of the center of mass. We have the core position in the center of mass frame, $\vec{r}$, for one-neutron halo

$$\vec{r} = -\frac{\vec{r}_n}{A_P - 1} = -\frac{\vec{\rho}_n}{A_P},$$

where A_P is the projectile mass number, $\vec{r}_n$ is the neutron position in the projectile center of mass frame and $\vec{\rho}_n$ is the neutron position in the core frame of reference. For a two-neutron halo nucleus the core position in the projectile center of mass frame, $\vec{r}$, is given by

$$\vec{r} = -\frac{\vec{r}_{n1} + \vec{r}_{n2}}{A_P - 2} = -\frac{\vec{\rho}_{n1} + \vec{\rho}_{n2}}{A_P},$$

where $\vec{r}_{n1}$ and $\vec{r}_{n2}$ are the two valence neutrons positions in the projectile center of mass frame and $\vec{\rho}_{n1}$ and $\vec{\rho}_{n2}$ are the positions in the core frame of reference.

One has to remark that the projectile position in the target frame of reference, $\vec{\rho}$, is given in terms of the projectile position in the center of mass of the projectile plus target system, $\vec{R}$, by the relation

$$\vec{\rho} = \left(1 + \frac{A_P}{A_T}\right)\vec{R},$$

with A_P and A_T the mass numbers of projectile and target, respectively.

Therefore, the dipolar potential in the one-neutron halo system will be

$$V_{dip} = \frac{e^2}{4\pi\epsilon_0}\frac{Z_T Z_P}{A_P}\frac{\vec{\rho}\cdot\vec{\rho}_n}{\rho^3}$$

and for a two-neutrons halo

$$V_{dip} = \frac{e^2}{4\pi\epsilon_0}\frac{Z_T Z_P}{A_P}\frac{\vec{\rho}\cdot(\vec{\rho}_{n1} + \vec{\rho}_{n2})}{\rho^3}.$$

In general one has to deal with a coupling of the relative projectile-target motion (given by $\vec{R}$ or $\vec{\rho}$) and the evolution of the internal state of the weakly bound projectile (described by $\vec{r}$ or $\vec{\rho_n}$). For the projectile structure one has a spectrum which, in general, consists of both discrete and continuum part. The above dipole interaction may produce changes of the internal structure of the projectile. For usual stable nuclei the important part is the Coulomb excitation of higher energy bound states. This problem was considered from the beginning of nuclear structure studies [8] and the general expressions for the Coulomb excitation amplitude has been derived by Winther and Alder [9]. For weakly bound systems there are no other excited bound states and the Coulomb interaction will produce transitions in the continuum, *i.e.* dissociation of the nuclear system.

The simplest approximation to describe the coupled relative and internal motion is to consider that the dynamics of the projectile-target motion is decoupled from the changes of the projectile internal state; the relative dynamics is governed mainly by the monopole part of the Coulomb interaction which results into Rutherford scattering. As we are interested only on the processes related to the Coulomb interaction at relatively high bombarding energies (above the Coulomb barrier) one has to look only at peripheral collisions. In this case one can safely neglect the quantum effects on the relative motion and, moreover, adopt the straight-line approximation

$$\vec{\rho}(t) = b\vec{e}_x + vt\vec{e}_z,$$

where b is the classical impact parameter, v is the projectile velocity in the target frame of reference, and t is the time. This is a reasonable approximation for a wavelength small compared to the impact parameter and to the projectile dimensions. As the impact parameter is supposed large compared to the projectile dimensions, the dipole approximation is also justified. In this case the matrix element of the time-dependent dipolar potential between the ground state $|i>$ and an excited state which can belong to the continuum part of the spectrum $|f>$ becomes [10]

$$< f|V_{dip}|i >= \frac{e^2}{4\pi\epsilon_0}\frac{Z_T Z_P}{A_P}\left\{\frac{vt}{[(vt)^2+b^2]^{3/2}} < f|\rho_n^z|i > + \right.$$
$$\left. + \frac{b}{[(vt)^2+b^2]^{3/2}} < f|\rho_n^x|i >\right\}$$

Therefore the problem is reformulated as a problem of transitions between the states of the halo projectile under the action of a time dependent interaction. It is a standard result that the amplitude to find the projectile in the state $|f>$ after passage in the target field, a_f, is given in the first-order perturbation theory by

$$a_f = -\frac{i}{\hbar}\int_{-\infty}^{\infty} dt e^{i\frac{E_f-E_i}{\hbar}t} < f|V_{dip}(t)|i >$$
$$= -\frac{i}{\hbar}\frac{e^2}{4\pi\epsilon_0}\frac{Z_T Z_P}{A_P}\int_{-\infty}^{\infty} dt \frac{e^{i\frac{E_f-E_i}{\hbar}t}}{[(vt)^2+b^2]^{3/2}}\{vt < f|\rho_n^z|i > + b < f|\rho_n^x|i >\}.$$

With the changes of variables

$$\beta = \frac{(E_f-E_i)b}{\hbar v}$$
$$s = \frac{v}{b}t,$$

this may be written

$$a_f = -\frac{i}{\hbar v b}\frac{e^2}{4\pi\epsilon_0}\frac{Z_T Z_P}{A_P}\left\{< f|\rho_n^z|i > \int_{-\infty}^{\infty} ds \frac{s e^{i\beta s}}{(1+s^2)^{3/2}} + \right.$$
$$\left. + < f|\rho_n^x|i > \int_{-\infty}^{\infty} ds \frac{e^{i\beta s}}{[1+s^2]^{3/2}}\right\}.$$

The two integrals can be expressed using the integral representation of the modified Hankel function [10]

$$\int_{-\infty}^{\infty} ds \frac{\cos\beta s}{(1+s^2)^{3/2}} = \beta K_1(\beta),$$

and the probability amplitude is

$$a_f = -\frac{2}{\hbar v b}\frac{e^2}{4\pi\epsilon_0}\frac{Z_T Z_P}{A_P}\left\{ < f|\rho_n^z|i > (-\beta K_0(\beta)) + i < f|\rho_n^x|i > \beta K_1(\beta)\right\}.$$

For small values of β we have

$$\beta K_1(\beta) \to 1$$
$$K_0(\beta) \to C + \ln\frac{2}{\beta},$$

with $C = 0.5772...$ the Euler constant. When β is small (this means high energy collisions, *i.e.* large velocity v and not too large impact parameter b) the transition amplitudes are dominated by the transverse term $< f|\rho_n^x|i >$. In the following we will study the implications of the above formulae using different models for the internal structure.

3. COULOMB DISSOCIATION OF ONE-NEUTRON HALO NUCLEI

For one-neutron halo nuclei (*e.g.* ^{11}Be) one can consider that the continuous spectrum is given in terms of the plane waves

$$|f> = |\vec{k}> \sim e^{i\vec{k}\vec{\rho}_n}.$$

A better approximation will be to work with distorted wave functions. Nevertheless, because there is no resonance close to the threshold in the *p*-wave and the halo state is a *s*-state, the plane wave approximation gives a good description for the momentum dependence of the probability of dissociation as the Coulomb interaction couples the *s*-wave ground state only with the *p*-wave continuous spectrum in the dipolar approximation. Because the dissociation amplitude is dominated by the behavior of the wave functions at large distance, the energy dependence will be well described, even if the absolute values might differ by a factor two [11].

The amplitude to find the valence neutron with momentum $\vec{k}$ in the core rest frame (*e.g.* ^{10}Be rest frame) can be written

$$a_{\vec{k}} \sim \int d^3\rho_n e^{-i\vec{k}\vec{\rho}_n}\rho_n^x \frac{e^{-\alpha\rho_n}}{\rho_n} \sim \frac{d}{dk_x}\int d^3\rho_n e^{-i\vec{k}\vec{\rho}_n}\frac{e^{-\alpha\rho_n}}{\rho_n}.$$

The amplitude is related to the Fourier transform of the ground state wave function which is supposed to be $\frac{e^{-\alpha\rho_n}}{\rho_n}$, *i.e.* a Yukawa function in the core rest frame. The Fourier transform of the Yukawa function is elementary and we get the probability density to obtain a neutron with momentum $\vec{k}$

$$\frac{dP_{\vec{k}}}{d^3k} = |a_{\vec{k}}|^2 \sim \left(\frac{d}{dk_x}\frac{1}{k^2+\alpha^2}\right)^2 \sim \frac{k^2}{(k^2+\alpha^2)^4}\sin^2\theta\cos^2\phi.$$

Integrating over the $\vec{k}$ direction one obtains the neutron-core relative energy dependence of the cross section

$$\frac{d\sigma}{dE} \sim \frac{d\sigma}{kdk} \sim k\frac{dP_{\vec{k}}}{d^3k} \sim \frac{k^3}{(k^2+\alpha^2)^4} \sim \frac{E^{3/2}}{(E+|E_{gs}|)^4},$$

which has a maximum at $3/5 \cdot |E_{gs}|$.

The neutron-core relative energy distribution after the Coulomb dissociation of the ^{11}Be halo nucleus is well described by the above simple model [11]. In general the Coulomb dissociation of one-neutron halo nuclei is better understood than the two-neutron halo case. Our main goal is to study the possibility to extract structure informations from the Coulomb dissociation in the case of two-neutron halo. In the following section we will see which observables are sensitive to the structure of the initial state.

4. COULOMB DISSOCIATION OF TWO-NEUTRON HALO NUCLEI

For the Coulomb dissociation of a two-neutron halo nucleus like ^{11}Li one can consider that the continuum part of the spectrum is given in terms of the plane waves

$$|f>=|\vec{k}_1,\vec{k}_2>\sim e^{i\vec{k}_1\vec{\rho}_{n1}}e^{i\vec{k}_2\vec{\rho}_{n2}}.$$

The amplitude to find the valence neutrons with momentum $\vec{k}_1$ and $\vec{k}_2$ in the core rest frame can be written

$$\begin{aligned} a_{\vec{k}_1,\vec{k}_2} &\sim \int d^3\rho_{n1}d^3\rho_{n2}e^{-i\vec{k}_1\vec{\rho}_{n1}}e^{-i\vec{k}_2\vec{\rho}_{n2}}(\rho^x_{n1}+\rho^x_{n2})\varphi(\vec{\rho}_{n1},\vec{\rho}_{n2}) \\ &\sim \left(\frac{d}{dk_{1x}}+\frac{d}{dk_{2x}}\right)\int d^3\rho_{n1}d^3\rho_{n2}e^{-i\vec{k}_1\vec{\rho}_{n1}}e^{-i\vec{k}_2\vec{\rho}_{n2}}\varphi(\vec{\rho}_{n1},\vec{\rho}_{n2}), \end{aligned}$$

where $\varphi(\vec{\rho}_{n1},\vec{\rho}_{n2})$ is the wave function of the two-neutron halo nuclear system. Thus, the amplitude is given by the Fourier transform of the bound state wave function, *i.e.* it is the halo state wave function in the momentum representation. If we assume that the wave function is a product of two Yukawa functions (independent weakly bound valence neutrons) one obtains

$$\frac{dP_{\vec{k}_1,\vec{k}_2}}{d^3k_1d^3k_2} = |a_{\vec{k}_1,\vec{k}_2}|^2 \sim \frac{k_1^2}{(k_1^2+\alpha^2)^4}\frac{k_2^2}{(k_2^2+\alpha^2)^4}\sin^2\theta_1\cos^2\phi_1\sin^2\theta_2\cos^2\phi_2.$$

From the above expression of the double differential cross section one can derive different relative momentum distributions measured in experiments integrating over some momentum variables. For instance, one obtain the energy distribution of the two neutrons in the core rest frame by integrating over the angular variables θ_1, ϕ_1,

θ_2, ϕ_2 and the energy of one of the neutrons

$$\frac{d\sigma}{dE}(E) \sim \int_0^E dE_1 \frac{E_1^{3/2}}{(E_1+\alpha^2)^4} \frac{(E-E_1)^{3/2}}{(E-E_1+\alpha^2)^4}.$$

One can use other bound state wave functions as well to obtain the relevant momentum distributions of the emitted neutrons after the Coulomb dissociation. Some alternative wave functions could be the wave functions obtained in terms of the scattering lengths characterizing the zero-energy interaction in the two-body subsystems in the hyperspherical formalism [12, 13], or using the Fermi pseudopotentials [14, 15].

In particular, using the wave function proposed in [14]

$$\varphi(\vec{\rho}_{n1}, \vec{\rho}_{n2}) \sim \left[\frac{e^{-\alpha\rho_{n1}}}{\rho_{n1}} + \frac{e^{-\alpha\rho_{n2}}}{\rho_{n2}}\right] \delta(|\vec{\rho}_{n1} - \vec{\rho}_{n2}| - R),$$

where α and R are parameters, the integrals can be done analytically and one obtains

$$|a_{\vec{k}_1,\vec{k}_2}|^2 \sim \left\{ \left(\frac{d}{dk_{1x}} + \frac{d}{dk_{2x}}\right) \frac{1}{|\vec{k}_1+\vec{k}_2|^2+\alpha^2} \left[\frac{\sin(k_1 R)}{k_1 R} + \frac{\sin(k_2 R)}{k_2 R}\right] \right\}^2.$$

The above formula could be used to test the wave function of the two-neutron halo state against the data obtained in the dissociation on a heavy target when the dominant mechanism is governed by the Coulomb interaction. For light target we should look only at the peripheral collision events to separate the contribution of the Coulomb dissociation from the nuclear breakup.

5. CONCLUSIONS

We discussed the problems raised by the extraction of the structure informations on the neutron halo states from the Coulomb dissociation data. The main result is that the Coulomb dissociation may be described similar to the old problem of Coulomb excitation in a heavy ion collision if we consider that the excited states are into the continuum.

We presented a detailed discussion of the simple case of the one-neutron halo looking at the energy dependence of the emitted neutron. This energy dependence is governed only by the large distance behavior of the wave function which is, in any case, of the Yukawa type. The absolute value of the cross section depends also on the wave function normalization, which is related to the small distance behavior of the wave function. The small distance behavior depends on the details of the nuclear interaction beyond its scattering length. It might be possible to extract the effective range of the interaction looking at the absolute values of the Coulomb dissociation cross section, but this point needs further study.

For the more complicated case of a two-neutron halo nuclei we presented the general formulae and the relevant particularization to the case in which the wave function has the form proposed in one of our previous works on the description of two-neutron borromean nuclei.

For neutron halo states the main contribution to the Coulomb dissociation comes from the dipolar term. For proton weakly bound states it is possible that the higher order multipolar terms, especially the quadrupolar term, have a significant contribution.

Acknowledgements. This work was partially supported by CNCSIS-UEFISCSU, project number PNII-IDEI 734/2007.

REFERENCES

1. A.S. Jensen, K. Riisager, D.V. Fedorov - Rev. Mod. Phys. **76** (2004) 215
2. Y. Suzuki, Y. Tosaka - Nucl. Phys. **A517** (1990) 599
3. C.A. Bertulani, L.F. Canto - Nucl. Phys. **A549** (1992) 163
4. P. Banerjee, J.A. Tostevin, I.J. Thomson - Phys. Rev. **C58** (1998) 1337
5. A. Cobis, D.V. Fedorov, A.S. Jensen - Phys. Rev. **C58** (1998) 1403
6. E. Garrido, D.V. Fedorov, A.S. Jensen, K. Riisager - Phys. Rev. Lett.**86** (2001) 1986
7. E. Garrido, D.V. Fedorov, A.S. Jensen - Phys. Lett. **B499** (2001) 109
8. R.A. Broglia, A. Winther - *Heavy Ion Reactions*, Lecture Notes vol. 8, *The elementary processes*, Addison-Wesley, Redwood City, California, 1991
9. A. Winther, K. Alder - Nucl. Phys. **A319** (1979) 518
10. S. Flügge - *Practical quantum mechanics*, Springer-Verlag, Berlin, 1999
11. R. Palit *et al.* - Phys. Rev. **C68** (2003) 034318
12. R.A. Ionescu, C. Hategan - Europhys. Lett. **45** (1999) 296
13. R.A. Ionescu, C. Hategan - Rom. J. Phys. **47** (2002) 107
14. C. Hategan, R.A. Ionescu - Proc. Rom. Acad. **A6** (2005) 137
15. C. Hategan, R.A. Ionescu, H.H. Wolter - Mod. Phys. Lett. **A22** (2007) 1469