Romanian Reports in Physics, Vol. 63, No. 2, P. 587–605, 2011

ASTROPHYSICS

BIANCHI TYPE-V COSMOLOGICAL MODELS OF THE UNIVERSE FOR BULK VISCOUS FLUID DISTRIBUTION IN GENERAL RELATIVITY: EXPRESSIONS FOR SOME OBSERVABLE QUANTITIES

RAVI PRAKASH SINGH[1], LALLAN YADAV[2]

[1]Department of Physics, St. Andrew's College, Gorakhpur-273001, India
E-mail: rpsingh_standrewsgkp@yahoo.in
[2]Department of Physics, D. D. U. Gorakhpur University, Gorakhpur-273009, India
E-mail: lallan_yadav07@yahoo.co.in

Received July 20, 2009

Abstract. A new class of a spatially homogeneous and anisotropic Bianchi type-V cosmological models of the Universe for bulk viscous fluid distribution within the framework of general relativity is investigated by applying the law of variation for the generalized mean Hubble's parameter that yields a constant value of deceleration parameter. The variation for Hubble's parameter generates two types of solutions for the average scale factor one is of power-law type and other is of the exponential form. Using these two forms, Einstein's field equations are solved separately that correspond to singular and non-singular models of the Universe respectively. It is observed that for positive value of deceleration parameter q of the Universe decelerates whereas for negative value of q the Universe accelerates. Expressions for look-back time-redshift, neoclassical tests (proper distance $d(z)$), luminosity distance red-shift and event horizon are derived and their significance are described in detail. Some physical and geometrical properties of the models are also discussed.

Key words: Cosmology, Bianchi type-V bulk viscous Universe, inflationary models, kinematic tests.

1. INTRODUCTION AND MOTIVATION

The study of Bianchi type V cosmological models create more interest as these models contain isotropic special cases and permit arbitrary small anisotropy levels at some instant of cosmic time. This property makes them suitable as model of our Universe. The homogeneous and isotropic Friedman-Robertson-Walker (FRW) cosmological models, which are used to describe standard cosmological models, are particular case of Bianchi type I, V and IX Universes, according to whether the constant curvature of the physical three-space, t = constant, is zero, negative or positive. These models will be interesting to construct cosmological models of the types which are of class one. Present cosmology is based on the FRW model which is completely homogeneous and isotropic. This is in agreement with observational data about the large scale structure of the Universe. However, although homogeneous but anisotropic models are more restricted than the inhomogeneous models, they explain a number of observed phenomena quite satisfactorily. This stimulates the research for obtaining exact anisotropic solution for Einstein's field equations (EFEs) as a cosmologically accepted physical models for the Universe (at least in the early stages). Among different models Bianchi type-V Universes are the natural generalization of the open FRW model, which eventually become isotropic. Roy and Prasad [1] have investigated Bianchi type V Universes which are locally rotationally symmetric and are of embedding class one filled with perfect fluid with heat conduction and radiation. Bianchi type V cosmological models have been studied by several researchers such as Farnsworth [2], Collins [3] Maartens and Nel [4], Wainwright *et al.* [5], Beesham [6], Maharaj and Beesham [7], Hewitt and wainwright [8], Camci *et al.* [9], Meena and Bali [10], Pradhan *et al.* [11, 12]), Aydogdu and Salti [13] in different physical contexts. Christodoulakis *et al.* [14–16] have studied un-tilted diffuse matter Bianchi V Universe with perfect fluid and scalar field coupled to perfect fluid sources obeying a general equation of state. Following the work of Saha [17], Singh and Chaubey [18, 19] have obtained the quadrature form of metric function for Bianchi type-V model with perfect fluid and viscous fluid.

The distribution of matter can be satisfactorily described by a perfect fluid due to the large scale distribution of galaxies in our Universe. However, observed physical phenomena such as the large entropy per baryon and the remarkable degree of isotropy of the cosmic microwave background radiation, suggest analysis of dissipative effects in cosmology. Furthermore, there are several processes which are expected to give rise to viscous effects. These are the decoupling of neutrinos during the radiation era and the decoupling of radiation and matter during the recombination era. Bulk viscosity is associated with the GUT phase transition and string creation. Misner [20] has studied the effect of viscosity on the evolution of cosmological models. The role of viscosity in cosmology has been investigated by Weinberg [21]. Nightingale [22], Heller and Klimek [23] have obtained a viscous Universes without initial singularity. The model studied by Murphy [24] possessed an interesting feature in which big bang type of singularity of infinite space-time curvature does not occur to be a finite past. However, the relationship assumed by Murphy between the viscosity coefficient and the matter density is not acceptable at large density. Thus, we should consider the presence of material distribution other than a perfect fluid to obtain a realistic cosmological models (see Grøn [25] for a review on cosmological models with bulk viscosity). The effect of bulk viscosity on the cosmological evolution has been investigated by a number of authors in the framework of general theory of relativity. This motivates to study cosmological bulk viscous fluid model.

Several authors (Hajj-Boutros [26], Hajj-Boutros and Sfeila [27], Ram [28], Mazumder [29] and Pradhan and Kumar [30]) have investigated the solutions of EFEs for homogeneous but anisotropic models by using some different generation techniques. Bianchi spaces $I-IX$ are useful tools in constructing models of spatially homogeneous cosmologies (Ellis and MacCallum [31], Ryan and Shepley [32]). From these models, homogeneous Bianchi type V Universes are the natural generalization of the open FRW model which eventually isotropize. Camci *et al.* [9] derived a new technique for generating exact solutions of EFEs with perfect fluid for Bianchi type V space-time. Recently Bali and Singh [33], Rao *et al.* [34–37], Tiwari [38], Singh and Baghel [39] and Singh *et al.* [40] have studied Bianchi type V cosmological models in different context.

Recently Singh, Ram and Zeyauddin [41] have extended the work of Singh and Kumar [42] to spatially homogeneous and totally anisotropic Bianchi type-V models with perfect fluid as source. In this paper, we propose to find Bianchi type V cosmological models in presence of a bulk viscous by using the law of variation for generalized mean Hubble's parameter and we will generalize the solutions of Reference [41]. This paper is organized as follows: The metric and the law of variation for Hubble's parameter for Bianchi type-V model that yields constant value of the deceleration parameter is presented in Section 2. The field equations and the quadrature solutions of the metric functions in terms of average scale factor are presented in Section 3 for two types of cosmologies by using this law. In Section 4, exact solutions of the field equations are presented which correspond to the singular and non-singular cosmological models. Section 4 also contains the geometric and physical properties of the derived models. Section 5 deals with the kinematic tests. Discussions and concluding remarks are given in Section 6.

2. THE METRIC AND THE LAW OF VARIATION FOR HUBBLE'S PARAMETER

We consider the space time metric of the spatially homogeneous and anisotropic Bianchi type-V of the form

$$ds^2 = dt^2 - A^2dx^2 - e^{2mx}\left[B^2dy^2 + C^2dz^2\right], \tag{1}$$

where the metric potentials A, B and C are functions of cosmic time t alone and m is a constant. We define the following physical and geometric parameters to be used in formulating the law and further in solving the Einstein's field equations for the metric (1). The average scale factor a of Bianchi type-V model is defined as

$$a = (ABC)^{\frac{1}{3}}. \tag{2}$$

A volume scale factor V is given by

$$V = a^3 = ABC. \tag{3}$$

We define the generalized mean Hubble's parameter H as

$$H = \frac{1}{3}(H_1 + H_2 + H_3), \tag{4}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are the directional Hubble's parameters in the directions of x, y and z respectively. A dot stands for differentiation with respect to cosmic time t. From Eqs. (2)-(4), we obtain

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right). \tag{5}$$

Since the line element (1) is completely characterized by Hubble parameter H, therefore, let us consider that the mean Hubble parameter H is related to the average scale factor a by the relation

$$H = la^{-n} = l(ABC)^{-\frac{n}{3}}, \tag{6}$$

where $l(>0)$ and $n(\geq 0)$ are constants. Such type of relations have already been considered by Berman [43], Berman and Gomide [44] for solving FRW models. Latter on many authors (see, Singh *et al.* [41,42], Singh and Baghel [39] and references therein) have studied flat FRW and Bianchi type models by using the special law for Hubble parameter that yields constant value of deceleration parameter.

An important observational quantity is the deceleration parameter q, which is defined as

$$q = -\frac{\ddot{a}a}{\dot{a}^2}. \tag{7}$$

From (5) and (6), we obtain

$$\dot{a} = la^{-n+1}, \tag{8}$$

$$\ddot{a} = -l^2(n-1)a^{-2n+1}. \tag{9}$$

Using (8) and (9) in to (7), we obtain

$$q = n - 1. \tag{10}$$

We observe that the relation (10) gives q as a constant. The sign of q indicated whether the model inflates or not. The positive sign of q *i.e.* $(q > 1)$ correspond to "standard" decelerating model whereas the negative sign $- \leq q \leq 0$ for $0 \leq n \leq 1$ indicates inflation. It is remarkable to mention here that though the current observations of SNe Ia and CMBR favours accelerating models ($q < 1$), but both do not altogether rule out the decelerating ones which are also consistent with these observations (see, Vishwakarma [45]). From (8), we obtain the law for average scale factor a as

$$a = (nlt + c_1)^{\frac{1}{n}}, \tag{11}$$

for $n \neq 0$ and

$$a = c_2 \exp(lt), \tag{12}$$

for $n=0$, where c_1 and c_2 are constants of integration. Equation (11) implies that the condition for the expansion of the Universe is $n=q+1>0$.

For $n\neq 0$, the age of the Universe is given by

$$t_0=\frac{1}{n}H_0^{-1}=\frac{1}{1+q}H_0^{-1}, \tag{13}$$

where subscript 0 stands for the present phase. For $n=0$, the age of the Universe is given by

$$t_0=\ln\left(\frac{a_0}{c_2}\right)^{3/l}. \tag{14}$$

A numerical calculation can be made to estimate the present day age of the Universe, the value of deceleration parameter compatible with the Supernovae observations. It is worth mentioned here that relations (11) and (12) independent on the particular gravitational theory being considered. It is approximately valid also for slowly time varying deceleration parameter. If $n>0$, we expect that

$$\lim_{t\to\infty} a=\infty, \quad \lim_{a\to\infty} p=0, \quad \lim_{a\to\infty} \rho=0. \tag{15}$$

The physical quantities of observational interest in cosmology i.e. the expansion scalar θ, the average anisotropy parameter A_m and the shear scalar σ^2 are defined as

$$\theta=u^i_{;i}=\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right), \tag{16}$$

$$\sigma^2=\frac{1}{2}\sigma_{ij}\sigma^{ij}=\frac{1}{2}\left[\frac{\dot{A}^2}{A^2}+\frac{\dot{B}^2}{B^2}+\frac{\dot{C}^2}{C^2}\right]-\frac{\theta^2}{6}, \tag{17}$$

$$A_m=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{\triangle H_i}{H}\right), \tag{18}$$

where $\triangle H_i=H_i-H$, $i=1,2,3$.

3. FIELD EQUATIONS AND GENERATION TECHNIQUE

The stress energy-momentum tensor in presence of bulk stress is given by

$$T_{ij}=(\bar{p}+\rho)u_iu_j-\bar{p}g_{ij}, \tag{19}$$

where

$$\bar{p}=p-\xi u^i_{;i}. \tag{20}$$

Here, ρ, p, $\bar{p}$, ξ and u are, respectively, the energy density, isotropic pressure, effective pressure, bulk viscous coefficient and the fluid four-velocity vector of distribution such that $u^iu_i=1$. In co-moving system of coordinates, we have $u^i=(1,0,0,0)$.

The coefficient of bulk viscosity determines the magnitude of viscous stress relative to expansion. We shall use non-causal theory to study the dissipative mechanism. On thermodynamical grounds bulk viscous coefficient ξ is positive, assuming that the viscosity pushes the dissipative pressure towards negative values.

For the energy momentum tensor (19) and Bianchi type V space-time (1), Einstein's field equations

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi T_{ij} \tag{21}$$

yield the following five independent equations

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} = -8\pi G\bar{p}, \tag{22}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{m^2}{A^2} = -8\pi G\bar{p}, \tag{23}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} = -8\pi G\bar{p}, \tag{24}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{3m^2}{A^2} = 8\pi G\rho, \tag{25}$$

$$\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0. \tag{26}$$

The Bianchi identity ($T^{ij}_{;j} = 0$) takes the form

$$\dot{\rho} + (\rho + \bar{p})\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0. \tag{27}$$

It is worth noting here that our approach suffers from a lack of Lagrangian approach. There is no known way to present a consistent Lagrangian model satisfying the necessary conditions discussed in this paper. The field equations $(22) - (25)$ and (27) can be reduced in terms of H, σ^2 and q as follows:

$$8\pi G\bar{p} = H^2(2q-1) - \sigma^2 + \frac{m^2}{A^2}, \tag{28}$$

$$8\pi G\rho = 3H^2 - \sigma^2 + \frac{3m^2}{A^2}, \tag{29}$$

$$\dot{\rho} + 3(\rho + \bar{p}) = 0. \tag{30}$$

The summation of relations (22) – (24) and three times of (25) gives the following equation for a

$$\frac{\ddot{a}}{a} + \frac{2\dot{a}^2}{a^2} = 4\pi G(\rho - \bar{p}) + \frac{2m^2}{a^2}. \tag{31}$$

From Eq. (31), we observe that $m = 0$ and $A = B = C$, the equation describes the flat FRW model, in that case when $\bar{p} = 0$ and $\rho > 0$, the Universe decelerates.

$$A^2 = BC. \tag{32}$$

We now describe the quadrature form of the Einstein's field equations (22) $-$ (26). Subtracting (24) from (23), one finds the following relation between A and B

$$\frac{A}{B} = d_1 \exp\left(k_1 \int \frac{dt}{a^3}\right). \tag{33}$$

Analogically, we find the other following relations

$$\frac{B}{C} = d_2 \exp\left(k_2 \int \frac{dt}{a^3}\right), \tag{34}$$

and

$$\frac{C}{A} = d_3 \exp\left(k_3 \int \frac{dt}{a^3}\right), \tag{35}$$

where d_1, d_2, d_3, k_1, k_2 and k_3 are constants of integration, satisfying

$$d_1 d_3 = d_2^{-1}, \quad k_1 + k_2 + k_3 = 0. \tag{36}$$

In view of (36) and following the approach of Saha and Rikhvitsky [46], Singh and Chaubey [19] and Singh *et al.* [41, 42], we obtain the metric functions from (33) $-$ (35) explicitly as

$$A(t) = (\ell_1)^{1/3} a \exp\left(\frac{K_1}{3} \int \frac{dt}{a^3}\right), \tag{37}$$

$$B(t) = (\ell_2)^{1/3} a \exp\left(\frac{K_2}{3} \int \frac{dt}{a^3}\right), \tag{38}$$

$$C(t) = (\ell_3)^{1/3} a \exp\left(\frac{K_3}{3} \int \frac{dt}{a^3}\right), \tag{39}$$

where

$$K_1 = k_1 - k_3, \;\; K_2 = -2k_1 - k_3, \;\; K_3 = k_1 + 2k_3, \tag{40}$$

and

$$\ell_1 = \left(\frac{d_1}{d_3}\right)^{1/3}, \;\; \ell_2 = \left(\frac{1}{d_1^2 d_3}\right)^{1/3}, \;\; \ell_3 = (d_1 d_3^2)^{1/3}. \tag{41}$$

It is observed from (37) $-$ (39) that for $a = (nlt + c_1)^{\frac{1}{n}}$ with $n > 0$, the exponent tends to unity for large t, and anisotropic model becomes isotropic. From Eqs. (32) and (37) $-$ (41), we obtain

$$K_1 = 0, \;\; K_2 = -K_3 = K, \;\; \ell_1 = 1, \;\; \ell_2 = \ell^{-1} = M^3, \tag{42}$$

where K and M are constants. Accordingly Eqs. (37) $-$ (39) lead to

$$A(t) = a, \tag{43}$$

$$B(t) = Ma \exp\left(\frac{K}{3}\int \frac{dt}{a^3}\right), \tag{44}$$

$$C(t) = M^{-1} a \exp\left(-\frac{K}{3}\int \frac{dt}{a^3}\right). \tag{45}$$

Thus, the metric functions are represented explicitly in terms of average scale factor a. Once we obtain the value of a, we can find the metric functions.

4. SOLUTIONS OF FIELD EQUATIONS

As already described in introduction many authors have tried to find the solutions of the quadrature equations (43) – (45) by using different techniques. In present paper, we solve these equations by using the average scale factor as obtained in (11) and (12) for $n \neq 0$ and $n = 0$, respectively by assumption of (6), which has physical interest to describe the decelerating and accelerating Universes.

4.1. CASE 1: when $n \neq 0$

Using (11) into (43) - (45), the solutions for metric functions can be written as

$$A(t) = (nlt + c_1)^{1/n}, \tag{46}$$

$$B(t) = M(nlt + c_1)^{1/n} \exp\left[\frac{K(nlt + c_1)^{(n-3)/n}}{3l(n-1)}\right], \tag{47}$$

$$C(t) = M^{-1}(nlt + c_1)^{1/n} \exp\left[-\frac{K(nlt + c_1)^{(n-3)/n}}{3l(n-1)}\right], \tag{48}$$

where $n \neq 3$.
After using suitable transformation of the co-ordinates, the model (1) reduces to the form

$$ds^2 = \frac{dT^2}{n^2 l^2} - T^{\frac{2}{n}} dX^2 - e^{2mX} T^{\frac{2}{n}} \times$$
$$\left[M^2 \exp\left(\frac{2KT^{\frac{(n-3)}{n}}}{3l(n-3)}\right) dY^2 + M^{-2} \exp\left(-\frac{2KT^{\frac{(n-3)}{n}}}{3l(n-3)}\right) dZ^2\right] \tag{49}$$

Putting the values of relevant quantities into (28) and (29), we obtain the expressions for effective pressure $\bar{p}$ and energy density ρ for the model (49)

$$8\pi G\bar{p} = (2n-3)l^2 T^{-2} - \frac{K^2}{9} T^{-\frac{6}{n}} + m^2 T^{-\frac{2}{n}}, \tag{50}$$

$$8\pi G\rho = 3l^2 T^{-2} - \frac{K^2}{9} T^{-\frac{6}{n}} - 3m^2 T^{-\frac{2}{n}}. \tag{51}$$

We assume for the specification of ξ, that the fluid obeys an equation of state of the form

$$p = \gamma\rho, \tag{52}$$

where $\gamma(0 \le \gamma \le 1)$ is a constant.

Thus, given $\xi(t)$, we can solve the system for the physical quantities. In most of investigations involving bulk viscosity is assumed to be a simple power function of the energy density (Pavon *et al.* [47]; Maartens [48]; Zimdahl [49]; Santos *et al.* [50])

$$\xi(t) = \xi_0 \rho^w, \tag{53}$$

where ξ_0 and w are real constants. For small density, w may even be equal to unity as used in Murphy's work [51] for simplicity. If $w = 1$, Eq. (28) may correspond to a radiative fluid (Weinberge [52]). Near the big bang, $0 \le w \le \frac{1}{2}$ is a more appropriate assumption (Belinskii and Khalatnikov [53]) to obtain realistic models. From (50) and (53), we obtain

$$8\pi G(p - \xi_0\rho^w\theta) = (2n-3)l^2T^{-2} - \frac{K^2}{9}T^{-\frac{6}{n}} + m^2T^{-\frac{2}{n}}, \tag{54}$$

where θ is the scalar of expansion calculated for the flow vector u^i and is given by

$$\theta = \frac{3l}{(nlt + c_1)} = \frac{3l}{T}. \tag{55}$$

For simplicity and realistic models of physical importance, we consider the following two cases ($w = 0, 1$):

4.1.1. Model I: Solution for $w = 0$

When $w = 0$, Eq. (53) reduces to $\xi = \xi_0$ = constant. Hence in this case Eq. (54), with the use of (51), (52) and (55), leads to

$$8\pi Gp = 8\pi G\gamma\rho = \frac{24\pi G\xi_0 l}{T} + \frac{(2n-3)l^2}{T^2} + \frac{K^2}{9T^{\frac{6}{n}}} + \frac{m^2}{T^{\frac{2}{n}}}. \tag{56}$$

4.1.2. Model II: Solution for $w = 1$

When $w = 0$, Eq. (53) reduces to $\xi = \xi_0\rho$. Hence in this case Eq. (54), with the use of (51), (52) and (55), leads to

$$8\pi Gp = 8\pi G\gamma\rho = \frac{l^2(3\theta\xi_0 + 2n - 3)}{T^2} - \frac{K^2(\xi_0\theta + 1)}{9T^{\frac{6}{n}}} - \frac{m^2(3\xi_0\theta - 1)}{T^{\frac{2}{n}}}. \tag{57}$$

4.1.3. Some Geometric and Physical Properties of the Models

The expressions for the magnitude of shear σ^2, the average anisotropy parameter A_m and proper volume V^3 for the model (49) are given by

$$\sigma^2 = \frac{K^2}{9} T^{-\frac{6}{n}}, \tag{58}$$

$$A_m = \frac{2K^2}{27l^2} T^{\frac{2n-6}{n}}, \tag{59}$$

$$V^3 = \sqrt{-g} = T^{\frac{9}{n}}. \tag{60}$$

The rotation ω is identically zero.

The rate of expansion H_i in the direction of x, y and z are given by

$$H_x = \frac{\dot{A}}{A} = \frac{l}{T}, \tag{61}$$

$$H_y = \frac{\dot{B}}{B} = \frac{l}{T} + \frac{K}{3T^{\frac{3}{n}}}, \tag{62}$$

$$H_z = \frac{\dot{C}}{C} = \frac{l}{T} - \frac{K}{3T^{\frac{3}{n}}}. \tag{63}$$

Hence the average generalized Hubble's parameter is given by

$$H = \frac{l}{T}. \tag{64}$$

The scalar curvature is calculated and given as

$$R = \frac{2K^2}{9T^{\frac{6}{n}}} - \frac{6l^2(n-2)}{T^2} - \frac{6m^2}{T^{\frac{2}{n}}}. \tag{65}$$

Here we observe that the relations (30) and (31) are identically satisfied by putting therein (51), (56) or (57) and (64).

From the above results, it can be seen that the spatial volume is zero at $T=0$. This shows that the Universe starts evolving with zero volume at $T=0$ and expands with cosmic time T. In this case the condition of expansion for the model is $n \geq 0$. From equations (61) – (63), we observe that all the three directional Hubble parameters are zero at $T=0$. In both models I and II, the energy density and pressure tend to infinity at $T=0$. The models have the point-type singularity at $T=0$. The rate of the expansion, the mean anisotropy parameter for $n<3$ and shear scalar all diverse at $T=0$. The scalar curvature R tends to infinity at the point of singularity. As $T \to \infty$, the scale factors $A(t)$ and $B(t)$ tend to infinity whereas $C(t)$ tends to indeterminate. The energy density and pressure becomes zero as $T \to \infty$. The space time exhibit pancake-type singularity at $T=\infty$ for $K>0$. The expansion scalar, mean anisotropy parameter and shear scalar all tend to zero as $T \to \infty$. This shows that the Universe is expanding with the increase of cosmic time but the rate of expansion and shear scalar decrease to zero and tend to isotropic. We also see that $R \to 0$ as $T \to \infty$. At the initial stage of expansion, when p and ρ are large, the Hubble parameter is also large and with the expansion of the Universe H, θ decrease as does ρ. Since $\lim_{T\to\infty} \frac{\sigma^2}{\theta^2} \neq 0$ provided $n \geq 3$, the models do not approach isotropy for large values of T. The cosmological evolution of Bianchi type-V space-time is expansionary, with all the three scale factors monotonically increasing function of time. The dynamics of the mean anisotropy parameter depends on the value of n. For $n < 3$, A_m has singular state, with infinite energy density and zero scale factors. Since $\lim_{T\to\infty} \frac{\sigma^2}{\theta^2} = 0$ provided $n < 3$, the models approach isotropy for large values of T. For small time, measure of anisotropy increases and in the large time limits, it ends in a homogeneous and isotropic state. Our both models satisfy all conditions of homogeneity and isotropization according to the formal definitions given by Collins and Hawking [54] when $n < 3$. By putting $\xi = 0$ in the present solutions, we obtain the solution obtained by Singh, Ram and Zeyauddin [41].

4.2. CASE 2: when $n = 0$

Using (12) into (43) – (45), the solutions for metric functions can be written as

$$A(t) = c_2 \exp(lt), \tag{66}$$

$$B(t) = Mc_2 \exp\left[lt - \frac{K}{3c_2^3 l}\exp(-lt)\right], \tag{67}$$

$$C(t) = M^{-1}c_2 \exp\left[lt + \frac{K}{3c_2^3 l}\exp(-lt)\right]. \tag{68}$$

After using suitable transformation of the co-ordinates, the model (1) reduces to the form

$$ds^2 = \frac{d\tau^2}{l^2} - c_2^2 \exp(2\tau)dX^2 - c_2^2 e^{2mX} \times$$
$$\left[M^2 \exp 2(\tau - \chi\exp(-\tau))dY^2 + M^{-2}\exp 2(\tau + \chi\exp(-\tau))dZ^2\right], \tag{69}$$

where $\chi = \frac{K}{3c_2^3 l}$.

Putting the values of relevant quantities into (28) and (29), we obtain the expressions for effective pressure $\bar{p}$ and energy density ρ for the model (49)

$$8\pi G\bar{p} = -3l^2 - \chi^2 \exp(-2\tau) + \kappa\exp(-2\tau), \tag{70}$$

$$8\pi G\rho = 3l^2 - \chi^2 \exp(-2\tau) - 3\kappa\exp(-2\tau), \tag{71}$$

where $\frac{m^2}{c_2^2} = \kappa$. From (70) and (53), we obtain

$$8\pi G(p - \xi_0\rho^w\theta) = -3l^2 - \chi^2 \exp(-2\tau) + \kappa\exp(-2\tau), \tag{72}$$

where θ is the scalar of expansion calculated for the flow vector u^i and is given by

$$\theta = 3l. \tag{73}$$

For simplicity and realistic models of physical importance, we consider the following two cases ($w = 0, 1$):

4.2.1. Model I: Solution for $w = 0$

When $w = 0$, Eq. (53) reduces to $\xi = \xi_0$ = constant. Hence in this case Eq. (72), with the use of (71), (52) and (73), leads to

$$8\pi G p = 8\pi G \gamma \rho = 24\pi G \xi_0 l - 3l^2 - \chi \exp(-2\tau) + \kappa \exp(-2\tau). \tag{74}$$

4.2.2. Model II: Solution for $w = 1$

When $w = 0$, Eq. (53) reduces to $\xi = \xi_0 \rho$. Hence in this case Eq. (72), with the use of (71), (52) and (73), leads to

$$8\pi G p = 8\pi G \gamma \rho = 3l^2(3l\xi_0 - 1) - \chi^2 \exp(-2\tau)(3l\xi_0 + 1) + \kappa \exp(-2\tau)(1 - 9l\xi_0). \tag{75}$$

4.2.3. Some Geometric and Physical Properties of the Models

The expressions for the magnitude of shear σ^2, the average anisotropy parameter A_m, deceleration parameter q and proper volume V^3 for the model (69) are given by

$$\sigma^2 = l^2 \chi^2 \exp(-2\tau), \tag{76}$$

$$A_m = 6\chi^2 \exp(-2\tau), \tag{77}$$

$$q = -1, \tag{78}$$

$$V^3 = \sqrt{-g} = c_2^9 \exp(9\tau). \tag{79}$$

The rotation ω is identically zero.

The rate of expansion H_i in the direction of x, y and z are given by

$$H_x = \frac{\dot{A}}{A} = l, \tag{80}$$

$$H_y = \frac{\dot{B}}{B} = l + l\chi \exp(-\tau), \tag{81}$$

$$H_z = \frac{\dot{C}}{C} = l - l\chi \exp(-\tau). \tag{82}$$

Hence the average generalized Hubble's parameter is given by

$$H = l. \tag{83}$$

In this case, the scalar curvature is calculated as

$$R = 12l^2 + (2l\chi - 6\kappa)\exp(-2\tau). \tag{84}$$

From equations (80) – (83), we observe that the directional Hubble parameters are time dependent while the average Hubble parameter is constant. As $\tau \to \infty$, all the spatial scale factors diverge whereas the directional Hubble's parameters become constant and uniform. It is remarkable to mention here that our solution identically satisfy the conservation equation (30) and the field equation (31).

We also observe that the geometric and physical quantities such as H, H_x, H_y, H_z, p, ρ, σ^2, θ, V^3 and A_m are constant at $\tau = 0$. This indicates that the Universe starts evolving with constant volume and expands with exponential rate. From Eq. (83), we observe that the expansion scalar becomes constant through out the time of evolution. Thus the models represent uniform expansion and volume grows exponentially with time. These solutions help to resolve several cosmological problems (flatness, horizon, monopole) associated with the standard model.

The mean anisotropic parameter A_m and shear scalar σ decrease with time and tend to zero as $\tau \to \infty$. Since $\lim_{\tau\to\infty} \frac{\sigma^2}{\theta^2} = 0$, the models approach isotropy for large values of τ. Here we see that $R \to 12l^2$ (constant) as $\tau \to \infty$. In this case we find the the deceleration parameter $q = -1$ which implies $\frac{dH}{dt} = 0$. Hence we obtain the greatest value of Hubble parameter and fastest rate of expansion of the Universe. Obviously, the negative value of q would accelerate and increase the age of the Universe. By putting $\xi = 0$ in the present solutions, we obtain the solution obtained by Singh, Ram and Zeyauddin [41].

5. KINEMATICAL TESTS

The values of $a(t)$ derived in (11) and (12) may be used in formulating the kinematics tests for any arbitrary large red-shifts. We now study the consistency of the derived models for both cases with the observational parameters through kinematics tests.

5.1. WHEN $n \neq 0$

5.1.1. Look Back Time

The time in the past at which the light we now receive from a distant object was emitted is called the look back time. How *long ago* the light was emitted (the look back time) depends on the dynamics of the Universe.

The radiation travel time (or look-back time) $T - T_0$ for photon emitted by a source at instant T and received at T_0 is given by

$$T - T_0 = \int_a^{a_0} \frac{da}{\dot{a}}, \tag{85}$$

Equation (11) can be rewritten as

$$a = T^{\frac{1}{n}}. \tag{86}$$

This follows that

$$\frac{a_0}{a} = 1 + z = \left(\frac{T_0}{T}\right)^{\frac{1}{n}}, \tag{87}$$

where a_0 is the present scale factor. The above equation gives

$$T = T_0(1+z)^{-n}. \tag{88}$$

From Eqs. (13) and (88), we obtain

$$T_0 - T = \frac{H_0^{-1}}{n}\left[1-(1+z)^{-n}\right], \tag{89}$$

which is

$$H_0(T_0 - T) = \frac{1}{n}\left[1-(1+z)^{-n}\right], \tag{90}$$

where H_0 is Hubble's constant at present in km/s/Mpc and its value is believed to be somewhere between 50 and 100 km/s/Mpc. However, the reciprocal of Hubble's constant is called the Hubble time T_H, $T_H = H_0^{-1}$, where T_H is expressed in s and H_0 in s^{-1}. For small z one obtain

$$H_0(T_0 - T) = \frac{1}{n}\left[nz - \frac{n(n-1)}{2}z^2 + \ldots\right]. \tag{91}$$

The above relation can be transformed by using $q = (n-1)$ into

$$H_0(T_0 - T) = z - \frac{q}{2}z^2 + \ldots. \tag{92}$$

Taking limit $z \to \infty$ in (90), the present age of the Universe (the extrapolated time back to the bang) is

$$T_0 = \frac{H_0^{-1}}{n} = \frac{H_0^{-1}}{1+q}, \tag{93}$$

which is same as expected in (13). Putting $n = \frac{3}{2}$ in (93), we obtain the result of well-known Einstein-de-Sitter Universe

$$H_0(T_0 - T) = \frac{2}{3}\left[1-(1+z)^{-\frac{3}{2}}\right]. \tag{94}$$

It is remarkable to mention here that relation (93) is used to describe look-back time in Einstein-de-Sitter Universe. In the limit $z \to \infty$, Eq. (94) is reduced to

$$T_0 = \frac{2}{3}H_0^{-1} = \frac{2}{3}T_H. \tag{95}$$

5.1.2. Neoclassical Tests (Proper Distance $d(z)$)

A photon emitted by a source with coordinate $r = r_1$ and $T = T_1$ and received at a time T_0 by an observer located at $r = 0$. The emitted radiation will follow null geodesics on which $(\theta_1, \theta_2, ...\theta_n)$ are constant.

The proper distance between the source and observer is given by

$$d(z) = a_0 \int_a^{a_0} \frac{da}{a\dot{a}}, \qquad r_1 = \int_{T_1}^{T_0} \frac{dT}{a} = \frac{a_0^{-1} H_0^{-1}}{(n-1)} \left[1 - (1+z)^{1-n}\right]. \tag{96}$$

Hence

$$d(z) = r_1 a_0 = \frac{H_0^{-1}}{n-1} \left[1 - (1+z)^{1-n}\right], \tag{97}$$

where $(1+z) = \frac{R_0}{R}$ = red-shift and a_0 is the present scale factor of the Universe. For small z, (97) reduces to

$$H_0 d(z) = z - \frac{1}{2n} z^2 + \ldots. \tag{98}$$

By using (13) in above equation, we obtain

$$H_0 d(z) = z - \frac{1}{2}(1+q) z^2 + \ldots. \tag{99}$$

From Eq. (97), it is observed that the distance d is maximum at $z = \infty$. Hence

$$d(z = \infty) = H_0^{-1} \left(\frac{1}{n-1}\right). \tag{100}$$

Eq. (97) gives the Freese *et al.* [55] results for the proper distance if we choose $n = 2$.

5.1.3. Luminosity Distance

Luminosity distance is the another important concept of theoretical cosmology of a light source. The luminosity distance is a way of expanding the amount of light received from a distant object. It is the distance that the object appears to have, assuming the inverse square law for the reduction of light intensity with distance holds. The luminosity distance is *not* the actual distance to the object, because in the real Universe the inverse square law does not hold. It is broken both because the geometry of the Universe need not be flat, and because the Universe is expanding. In other words, it is defined in such a way as generalizes the inverse-square law of the brightness in the static Euclidean space to an expanding curved space [56].

If d_L is the luminosity distance to the object, then

$$d_L = \left(\frac{L}{4\pi l}\right)^{\frac{1}{2}}, \tag{101}$$

where L is the total energy emitted by the source per unit time, l is the apparent luminosity of the object. Therefore one can write

$$d_L = r_1 a_0 = d(1+z). \tag{102}$$

Using Eq. (97) in (102) reduces to

$$H_0 d_L = \frac{(1+z)}{n-1}\left[1-(1+z)^{1-n}\right]. \tag{103}$$

For small value of z, Eq. (103) gives

$$H_0 d_L = z + \frac{1}{2}(1-q)z^2 + \ldots. \tag{104}$$

The luminosity distance depends on the cosmological model we have under discussion, and hence can be used to tell us which cosmological model describe our Universe. Unfortunately, however, the observable quantity is the radiation flux density received from an object, and this can only be translated into a luminosity distance if the absolute luminosity of the object is known. There is no distant astronomical objects for which this is the cases. This problem can however be circumvented if there are a population of objects at different distances which are believed to have the same luminosity; even if that luminosity is not known, it will appear merely as an overall scaling factor.

5.2. WHEN $n = 0$

5.2.1. Look Back Time

In this case we obtain

$$H_0(\tau_0 - \tau) = \log(1+z). \tag{105}$$

For small z, we have

$$H_0(\tau_0 - \tau) = \left[z - \frac{z^2}{2} + \ldots\right]. \tag{106}$$

5.2.2. Luminosity Distance

Here we obtain

$$d_L = \frac{1}{H_0}(z + z^2), \tag{107}$$

which shows that the luminosity distance increases faster with red-shift z for $q = -1$.

5.2.3. Event Horizon

The solution exists an event horizon which is given by

$$r_E = a(t_0)\int_{\tau_0}^{\infty} \frac{d\tau}{a(\tau)} = \frac{1}{c_2 H}. \tag{108}$$

This value of the limit gives the event horizon where no observer beyond a proper distance r_E at $\tau = \tau_0$ can communicate with another observer.

6. DISCUSSION AND CONCLUDING REMARKS

In this paper we have generalized the solutions obtained by Singh, Ram and Zeyauddin [41]. The proposal of a law of variation for Hubble's parameter that yields a constant value of deceleration parameter is discussed in homogeneous and anisotropic Bianchi type-V space-time in general relativity. The law of variation for Hubble's parameter defined in (6) for Bianchi type-V space-time model gives two types of cosmologies, (i) first form (for $n \neq 0$) shows the solution for positive value of deceleration parameter indicating the power law expansion of the Universe whereas (ii) second one (for $n = 0$) shows the solution for negative value of deceleration parameter, which shows the exponential expansion of the Universe. Exact solutions of Einstein's field equations for this model of the Universe have been obtained by using the two forms of average scale factor. The power law solutions represent the singular model where the spatial scale factors and volume scalar vanish at $T = 0$. The energy density and pressure are infinite at this initial epoch. As $T \to \infty$, the scale factors diverge and p, ρ both tend to zero. A_m and σ^2 are very large at initial time but decrease with cosmic time and vanish as $t \to \infty$. The model shows isotropic state in later time of its evolution. The exponential solutions represent singularity free model of the Universe. In this case as $T \to -\infty$, the scale factors tend to zero which indicates that the Universe is infinitely old and has exponential inflationary phase. All the parameters such as scale factor, p, ρ, θ, σ^2 and A_m are constant at $T = 0$. The rate of expansion of the Universe is uniform through out the evolution.

Under the law of variation for Hubble's parameter defined in (6), it has been shown that the two classes of solutions lead to the conclusion that, if $q > 0$ the model expands but always decelerate whereas $q < 0$ provides the exponential expansion and later accelerates the Universe. The evolution of the Universe in such a scenario is shown to be consistent with the present observations predicting an accelerated expansion. We have also described the well-known astrophysical phenomena, namely look-back time, neoclassical tests, luminosity distance and event horizon with red-shift. It has been observed that such models of the Universe are compatible with present observations. It is observed that luminosity distance increases linearly with red-shift for $q = 1$ whereas it increases faster with red-shift z for $q = 0$ and $q = -1$. The solutions obtained in the present paper could give an appropriate description of the evolution of Universe. In summary, we have extended the law of variation of Hubble parameter proposed by Berman [43] to Bianchi type V space-time to investigate the exact solutions of Einstein's field equations.

The effect of bulk viscosity is to produce a change in perfect fluid and hence exhibit essential influence on the character of the solution. We also observe that Murphy's conclusion [24] about the absence of a big bang type singularity in the infinite past in models with bulk viscous fluid, in general, is not true. The results obtained in Myung and Cho [57] also show that, it is, in general, not valid, since for some cases big bang singularity occurs in finite past.

REFERENCES

1. Roy S. R. and Prasad, A., Gen. Rel. Grav., **26**, 939 (1994).
2. Farnsworth, D. L., J. Math. Phys., **8**, 2315(1967).
3. Collins, C. B., Comm. Math. Phys., **39**, 131 (1974).
4. Maartens, R. and Nel, S. D., Comm. Math. Phys., **59**, 273 (1978).
5. Wainwright, J., Ince, W. C. W. and Marshman, B. J., Gen. Rel. Grav. **10**, 259 (1979).
6. Beesham, A., Astrophys. Space Sci., **125**, 99 (1986).
7. Maharaj, S. D. and Beesham, A., S. Afr. J. Phys., **17**, 34 (1988).
8. Hewitt, C. G. and Wainwright, J., Phys. Rev. D, **46**, 4242 (1992).
9. Camci, U., Yavuz, I., Baysal, H., Tahran, I. and Yilmaz, I., Astrophys. Space Sci., **275**, 391 (2000).
10. Meena, B. L. and Bali, R., Pramana – Journal of Phys., **62**, 1007 (2004).
11. Pradhan, A. and Rai, A., Astophys. Space Sci., **291**, 149 (2004).
12. Pradhan, A., Yadav, L. and Yadav, A. K., Czech. J. Phys., **54**, 487 (2004).
13. Aydogdu, O. and Salti, M., Czech. J. Phys., **56**, 789 (2006).
14. Christodoulakis, T, Kofinas, G. and Zarikas, V., Phys. Lett. A, **275**, 182 (2000).
15. Christodoulakis, T., Kofinas, G. and Papadopoulos, G. O., Phys. Lett. B, **514**, 149 (2001).
16. Christodoulakis, T., Grammenos, T., Helias, C., Kevrekidis, P. G. and Spanou, A., J. Math. Phys., **47**, 042505 (2006).
17. Saha, B., Astrophys. Space Sci., **302**, 83 (2006).
18. Singh, T. and Chaubey, R., Pramana – Journal of Phys., **67**, 415 (2006).
19. Singh, T. and R. Chaubey, Pramana – Journal of Phys., **68**, 721 (2007).
20. Misner, C. W., Nature, **214**, 40, (1967).
    Misner, C. W., Astrophys. J., **151**, 431 (1968).
21. Weinberg, S., Astrophys. J., **168**, 175 (1971).
22. Nightingale, J. P., Astrophys. J., **185**, 105 (1973).
23. Heller, M. and Klimek, Z. Astrophys. Space Sci., **33**, 37 (1975).
24. Murphy, G. L., Phys. Rev. D, **8**, 4231 (1973).
25. Grøn, Ø., Astrophys. Space Sci. **173**, 191 (1990).
26. Hajj-Boutros, J., *Lecture Notes in Physics , Gravitation, Geometry and Relativistic Physics*, Springer, Berlin, **212**, 51, (1984).
    Hajj-Boutros, J., J. Math. Phys., **26**, 2297 (1985).
    Hajj-Boutros, J., Class. Quant. Grav. **3**, 311 (1986).
27. Hajj-Boutros, J. and Sfeila, J., Int. J. Theor. Phys., **26**, 97 (1987).
28. Ram, S., Int. J. Theor. Phys., **29**, 901 (1990).
    Ram, S., Gen. Rel. Grav., **21**, 697 (1989).
29. Mazumder, A., Gen. Rel. Grav., **26**, 307 (1994).
30. Pradhan, A. and Kumar, A., Int. J. Mod. Phys. D, **10**, 291 (2001).
31. Ellis, G. F. R. and MacCallum, M. A. H., Comm. Math. Phys., **12**, 108 (1969).
32. Ryan, M. P. Jr. and Shepley, L. C., *Homogeneous Relativistic Cosmology*, Princeton University Press, Princeton, (1975).
33. Bali, R. and Singh, D. K., Astrophys. Space Sci., **300**, 387 (2005).

34. Rao, V. U., Vinutha, T. and Shanthi, M. V., Astrophys. Space Sci., **312**, 189 (2007).
35. Rao, V. U., Vinutha, T. and Shanthi, M. V., Astrophys. Space Sci., **314**, 213 (2008).
36. Rao, V. U., Vinutha, T., Shanthi, M. V. and Sireesha, K. V. S., Astrophys. Space Sci., **315**, 211 (2008).
37. Rao, V. U., Vinutha, T., Shanthi, M. V. and Kumari, G. S. D., Astrophys. Space Sci., **317**, 89 (2008).
38. Tiwari, R. K., Fizika B, **18**, 9 (2009).
39. Singh, J. P. and Baghel, P. S., Int. J. Phys., **48**, 449 (2009).
40. Singh, C. P., Zeyauddin, M. and Ram, S., Int. J. Theor. Phys., **47**, 3162 (2008).
41. Singh, C. P., Ram, S. and Zeyauddin, M., Astrophys. Space Sci., **315**, 181 (2008).
42. Singh, C. P. and Kumar, S., Int. J. Mod. Phys. D, **15**, 419 (2006).
43. Berman, M. S., Il Nuovo Cim. B, **74**, 182 (1983).
44. Berman, M. S. and Gomide, R. M., Gen. Relativ. Gravit., **20**, 1217 (1988).
45. Vishwakarma, R. G., Mon. Not. R. Astron. Soc., **345**, 545 (2003).
46. Saha, B. and Rikhvitsky, V., Physica D, **219**, 168 (2006) 168.
47. Pavon, D., Bafaluy, J. and Jou, D., Class. Quant. Grav. **8**, 347 (1991).
48. Maartens, R., Class. Quant. Grav., **12**, 1455 (1995).
49. Zimdahl, W., Phys. Rev. D, **53**, 5483 (1996).
50. Santos, N. O., Dias, R. S. and Banerjee, A., J. Math. Phys., **26**, 878 (1985).
51. Murphy, G. L., Phys. Rev. D, **8**, 4231 (1973).
52. Weinberg, S., *Gravitation and Cosmology*, Wiley, New York, (1972).
53. Belinskii, U. A. and Khalatnikov, I. M., Sov. Phys. JETP, **42**, 205 (1976).
54. Collins, C. B. and Hawking, S. W., Astrophys. J., **180**, 317 (1973).
55. Freese, K., Adams, F. C., Frieman, J. A. and Mottola, E., Nuc. Phys. B, **287**, 797 (1987).
56. Waga, I., Astrophys. J., **414**, 436 (1993).
57. Myung, S. and Cho, B. M., Mod. Phys. Letter A, **1**, 37 (1986).