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CONSTRAINS ON THE QUINTESSENCE FROM OBSERVATIONAL DATA

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Abstract. We investigated a flat Universe where the quintessence has been approximated by the first order Chebyshev polynomials. We used different cosmological measurements: angular diameter, luminosity distance and the Hubble parameter to constrain it. The first two types of measurements favor a time dependent quintessence with a value of the quintessence at the present epoch higher than in the cosmological constant case. Hubble parameter measurements support a Universe close to the one given by the cosmological constant case.

1. INTRODUCTION

In 1917 Einstein wrote his famous field equations that could describe the evolution of the Universe as a whole. One of his assumptions was the conservation of energy that could relate the energy momentum tensor to the geometrical properties of space-time. In order to create a static, unchanged Universe, he had to introduce a *cosmological constant*. He adjusted the value of the constant so that it would exactly counterbalance the gravitational attraction of matter.

Over the last decade, observations have shown that visible and dark matter combined amount to less than half of the content of the Universe. The remaining is dark energy with a strange and unique feature: it repels. In this way, the Universe is the product of the two tendencies, and the repulsive one prevails.

A common interpretation of the cosmological constant is vacuum energy. However, many cosmologists, are now leaning toward a different idea, known as *quintessence*. The dynamism is what cosmologists find so appealing about quintessence. While vacuum energy is completely inert, maintaining the same density constantly, the quintessence interacts with matter and evolves with time, so it might naturally adjust itself to reach the present observed value.

The sign of the force is determined by the sum of the total energy density plus three times the pressure. If the pressure is positive, as it is for radiation, ordinary matter and dark matter, then the combination is positive and it is attractive. If the pressure is sufficiently negative, the combination is negative and it is repulsive [1].

The exact evolution of the quintessence is not known since it cannot be analytically derived from any equation. For its time variation, different functions have been considered: in [2] the form is oscillating, in [3] it is linear with the scale factor. Another form of quintessence is given by [4]. It has even been considered to be a Chaplygin gas. The main drawback of these functions is that they are empirical, given under the only condition of slow temporal evolution.

Theoretically, geometrical tests can break the degeneracy between the equation of state and time evolution of the quintessence in a direct, model-independent way [5]. This is why in section 2 we will approximate the quintessence using the Chebyshev polynomials (that guaranty the smallest maximum deviation from the true function at any given order) and in section 3 we will constrain the Chebyshev coefficients from different cosmological measurements: angular diameter, luminosity distance and the Hubble parameter at different redshifts. The last section summarizes our main conclusions and presents future work.

2. THE QUINTESSENCE AND OBSERVATIONAL PARAMETERS

One can look at the Universe as a mixture of two perfect fluids: the regular matter (baryonic and dark matter) and the dark energy, each one having its own equation of state and both contributing to the dynamics and evolution of our Universe.

For matter and radiation the equation of state is:

$$p_m = \gamma \rho_m c^2 \tag{1}$$

where γ is equal to 0 for matter and to $1/3$ for radiation. As the Universe is now dominated by non relativistic matter and radiation contribution can be ignored, we will consider from now on that for regular matter $p_m = 0$.

Dark energy is massless and its only effect can be described by a negative pressure. Its equation of state is:

$$p_\Lambda = w \rho_\Lambda c^2 \tag{2}$$

where w is a negative parameter, the quintessence*. The matter density is decreasing with the increase of the volume ($\rho_m = \rho_{m0} R^{-3}$, where the index 0 will always refer

*The quintessence was introduced by [6] that modeled the evolution of Universe and considered that the cosmological term decreases with time as: $\Lambda = 3\alpha R^{-n}$ where α was a constant. They varied n from 0 to 2 and observed that conservation of total energy is achieved if one regards dark energy as a fluid with equation of state $p_\Lambda = c(\frac{n}{3} - 1)\rho_\Lambda$, where c is the speed of light and ρ_Λ is the dark energy density.

to present epoch) while for the dark energy density one assumes that:

$$\rho_\Lambda = \rho_{\Lambda 0} R^{-n(z)}. \tag{3}$$

Note that when $n = 0$ this is *the cosmological constant case*, in which the dark energy density does not change with time.

The energy densities for matter and dark energy can be calculated as $\epsilon_i = \rho_i c^2$ and therefore the total energy density is:

$$\epsilon_{tot} = \rho_\Lambda c^2 + \rho_m c^2. \tag{4}$$

Simply enough, the dynamic of the Universe can be described by only two equations (notations from [7]):

$$\left(\frac{\dot{R}}{R}\right)^2 + k\frac{c^2}{R^2} = \frac{8\pi G}{3c^2}\epsilon_{tot} \tag{5}$$

$$2\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 + k\frac{c^2}{R^2} = -\frac{8\pi G}{c^2}p_{tot} \tag{6}$$

where G is the gravitational constant, R is the scale factor and k the curvature constant. p_{tot} is the total pressure (since we exclude radiation and other types of particles like the neutrino, $p_{tot} = p_\Lambda$). The first equation gives the expansion rate and the second gives acceleration.

Equations 5 and 6 can be used together with Bianchi's equation (if one considers conservation of total energy in the Universe):

$$3\frac{\dot{R}}{R} = -\frac{\dot{\epsilon_{tot}}}{\epsilon_{tot} + p_{tot}}, \tag{7}$$

that gives the relation between n and w:

$$(1+z)^{n(z)} = \exp\left(\int_0^z \frac{3(w(z)+1)}{1+z}\,\mathrm{d}z\right). \tag{8}$$

According to recent astrophysical observations, our Universe appears to have undergone a phase of accelerated expansion. This result can be combined with the recent estimates of the average mass density of the Universe $\Omega_0 \approx 0.3$ and with recent measurements of the cosmic microwave background anisotropy peaks, pointing at a nearly critical total energy density, $\Omega_{total} \approx 1$, [8], which makes our Universe flat.

With that in mind, eq. (5) can be expressed in terms of redshift, Hubbles's constant H_0, the matter density parameter at the present time, Ω_0, and the dark energy density parameter, $\Omega_{\Lambda 0}$:

$$H(z) = H_0(\Omega_0(1+z)^3 + \Omega_{\Lambda 0}(1+z)^{n(z)})^{0.5} \tag{9}$$

where $\Omega_0 = \frac{8\pi G\rho_0}{3H_0^2}$ and $\Omega_{\Lambda 0} = \frac{8\pi G\rho_{\Lambda 0}}{3H_0^2}$.

Another important cosmological measurement that can be performed is the angular diameter distance, D_A, which is the ratio between the real size of an object and its apparent diameter on the sky. For a flat Universe, the angular distance can be calculated as [9]:

$$D_A = \frac{c}{H_0}\frac{1}{1+z}\int_0^z \frac{dz}{(\Omega_0(1+z)^3 + \Omega_{\Lambda 0}(1+z)^{n(z)})^{0.5}} \tag{10}$$

Similar, the luminosity distance, D_L, is:

$$D_L = (1+z)^2 \times D_A \tag{11}$$

As can be seen, all important cosmological measurements, eqs. (9)-(11), depend on the time variation of the quintessence through expression 8. This variation cannot be determined analytically. One way to approximate the function $w(x)$ is to consider the Chebyshev expansion:

$$w(x) \approx \sum_{k=0}^{n} w_k T_k(x) \tag{12}$$

where $T_k(x)$ are the called *Chebyshev polynomials* and w_k, *Chebyshev coefficients*.

They give the approximating polynomial which has the smallest maximum deviation from the true function at any given order. The polynomials of order n can be calculated using the recurrence relation [10]:

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x) \tag{13}$$

with $T_0(x) = 1$, $T_1(x) = x$.

The quintessence can be approximated by a first order expansion with Chebyshev polynomials as:

$$w(x) \approx w_0 + w_1 T_1(x). \tag{14}$$

when x is defined in the range $(-1,1)$. If observational data are in the interval $z \in (0, z_{max})$, the first order expansion for the quintessence becomes:

$$w(z) = (w_0 - w_1) + 2w_1 \frac{z}{z_{max}} \tag{15}$$

3. METHOD AND RESULTS

In the following section we will present 1σ and 2σ joint confidence levels for degeneracy in the power coefficients $w_0 - w_1$ and $2w_1/z_{max}$ of the quintessence (15) by performing the χ^2 test. The test has the form:

$$\chi^2 = \sum_{k=1}^{nb.data.points} \left(\frac{X_{obs}(z_{obs,k}) - X_{calc}(z_{obs,k})}{\sigma_{X_{obs,k}}} \right)^2 . \quad (16)$$

The theoretical expressions for X_{calc} are given by eqs.(9)-(11) and $X_{obs}(z_{obs})$ can be one of the following observational data sets:

- angular diameter distance $D_{A_{obs}}(z_{obs})$ for 38 redshifts given in [11];
- luminosity distance $D_{L_{obs}}(z_{obs})$ calculated from 30 supernovae of type Ia (GOLD sample) given in [12];
- Hubble parameter $H_{obs}(z_{obs})$ for 9 redshifts given in [13].

In calculations we will consider a flat Universe. We will first set the expansion rate at $H_0 = 70$ km/s/Mpc. For each individual data set we will vary the matter density in the interval $0.25 - 0.31$ and determine the minimum χ^2 value. The results are given in Table 1.

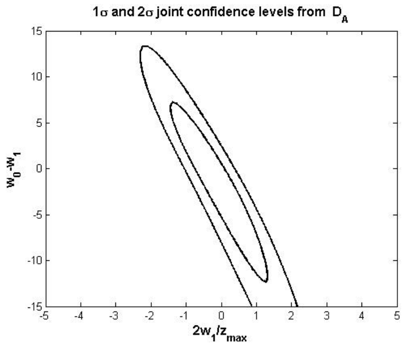
Table 1.χ^2 for different values of the matter density from different observational data sets when $H_0 = 70$ km/s/Mpc

χ^2	$\Omega_0 = 0.26$	$\Omega_0 = 0.27$	$\Omega_0 = 0.28$	$\Omega_0 = 0.29$	$\Omega_0 = 0.30$
from D_L	29.6572	29.6084	29.5765	**29.5612**	29.5768
from D_A	36.2629	36.2627	36.2628	36.2614	**36.2607**
from H	8.8937	**8.8645**	8.9257	8.8874	8.9452

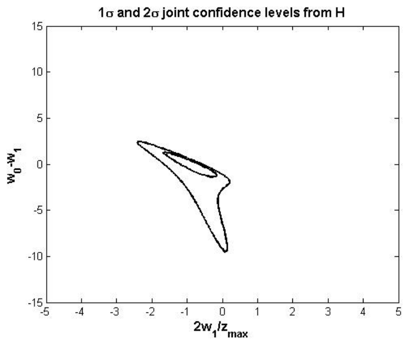
For $\Omega_0 = 0.29$, the degeneracy in the quintessence coefficients are presented in Fig. 1. A similar trend is distinguished, as all observations favor a value of $\frac{2w_1}{z_{i,max}}$ (the coefficient of z) close to 0 - a time independent quintessence. The free term $(w_0 - w_1)$ can only be constrained from the measurements on H and its central value is close to -1. The values of $w_0 - w_1$ and $\frac{2w_1}{z_{i,max}}$ that minimize χ^2 are presented in Table 2 for $\Omega_0 = 0.27$, $\Omega_0 = 0.29$ and $\Omega_0 = 0.30$.

For better results we will combine the measurements. First we will use the Hubble parameter together with the luminosity distance (Fig. 2(a)). We will then add the angular diameter distance - Fig. 2(b). The total χ^2 will be:

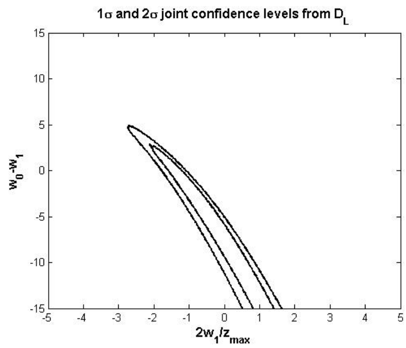
$$\chi^2 = \sum_{k=1}^{30} \left(\frac{\mu_{obs,SN} - \mu_{calc,SN}}{\sigma_{SN}} \right)^2 + \sum_{k=1}^{38} \left(\frac{D_{A,obs} - D_{A,calc}}{\sigma_{DA}} \right)^2 + \sum_{k=1}^{9} \left(\frac{H_{obs} - H_{calc}}{\sigma_H} \right)^2 \quad (17)$$



(a) Chebyshev parameters from D_A



(b) Chebyshev parameters from $H(z)$

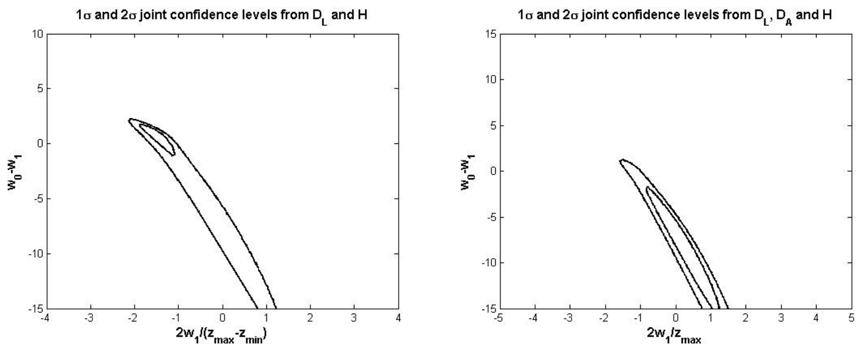


(c) Chebyshev parameters from D_L

Fig. 1 – 1σ and 2σ joint confidence levels for $w_0 - w_1$ and $\frac{2w_1}{z_{i,max}}$ (the power coefficients) from different data sets. The values for χ^2_{min} are $\chi^2_{minD_A} = 36.26$, $\chi^2_{minD_L} = 29.58$ and $\chi^2_{minH} = 8.95$ in a flat geometry with $\Omega_0 = 0.29$ and with $H_0 = 70$ km/s/Mpc.

Table 2. Best fit parameters for the power coefficients. The errors represent 1σ uncertainty

	Ω_0	$\frac{2w_1}{z_{max}}$	$w_0 - w_1$
from D_A	0.27	$-0.783^{+4.93}_{-5.42}$	$-0.18^{+0.722}_{-0.722}$
	0.29	$-0.90^{+5.18}_{-6.02}$	$-0.18^{+0.84}_{-0.84}$
	0.30	$-0.9^{+5.3}_{-6.38}$	$-0.18^{+0.84}_{-0.84}$
from D_L	0.27	$-2.59^{+3.73}_{-6.14}$	$-0.66^{+1.08}_{-0.82}$
	0.29	$-3.19^{+4.45}_{-10.49}$	$-0.66^{+1.8}_{-0.96}$
	0.30	$-3.55^{+4.57}_{-11.44}$	$-0.66^{+1.8}_{-0.96}$
from H	0.27	$0.06^{+0.60}_{-0.72}$	$-0.78^{+0.36}_{-0.36}$
	0.29	$0.06^{+0.84}_{-0.72}$	$-0.78^{+0.36}_{-0.48}$
	0.30	$0.06^{+0.72}_{-0.96}$	$-0.9^{+0.48}_{-0.36}$



(a) Chebyshev param. from D_L & H (b) Chebyshev param. from D_A,D_L& $H(z)$

Fig. 2 – 1σ and 2σ joint confidence levels for $w_0 - w_1$ and $\frac{2w_1}{z_{i,max}}$ from different combinations of data sets. The values for χ^2_{min} are $\chi^2_{D_L,H} = 41.52$, $\chi^2_{minD_A,D_L,H} = 85.48$, for 37, 75 degrees of freedom, in a flat geometry with $\Omega_0 = 0.3$ and with $H_0 = 70$ km/s/Mpc.

with 75 degrees of freedom. One should note that the minimum value for χ^2 when the Hubble parameter is combined with the luminosity distance is obtained when $\Omega_0 = 0.29$, that is $\chi^2 = 41.49$. The best fit values for $\frac{2w_1}{z_{max}}$ and $w_0 - w_1$ that minimize χ^2, in a flat Universe, are summarized in Table 3. The large errors in observational data are clearly affecting the results.

Table 3. Best fit parameters for the power coefficients. The errors represent 1σ uncertainty

	Ω_0	$\frac{2w_1}{z_{max}}$	$w_0 - w_1$
from H and D_L	0.26	$1.02^{+0.48}_{-0.72}$	$-1.38^{+0.24}_{-0.24}$
	0.27	$0.90^{+0.60}_{-0.72}$	$-1.38^{+0.24}_{-0.24}$
	0.28	$1.14^{+0.48}_{-1.08}$	$-1.50^{+0.36}_{-0.24}$
	0.29	$1.02^{+0.60}_{-1.32}$	$-1.50^{+0.36}_{-0.24}$
	0.30	$0.90^{+0.84}_{-1.92}$	$-1.50^{+0.36}_{-0.36}$
from D_A, H and D_L	0.26	$0.42^{+0.72}_{-0.84}$	$-1.14^{+0.24}_{-0.24}$
	0.27	$0.42^{+0.72}_{-7.10}$	$-1.14^{+1.32}_{-0.24}$
	0.28	$0.18^{+0.84}_{-12.28}$	$-1.14^{+2.04}_{-0.24}$
	0.29	$-6.56^{+7.34}_{-7.22}$	$0.06^{+0.93}_{-1.45}$
	0.30	$-7.77^{+6.02}_{-7.22}$	$0.18^{+0.96}_{-0.96}$

For the last set of simulations we will put a prior on H_0. In order to do that, we will consider two important redshifts: the first corresponds to the time when the expansion becomes accelerated, z_{infl}, (that can be determined from the condition $\ddot{R} = 0$ in eq. 6). The second to the time when the matter density equals the density of dark energy, z_{eq}. This can be determined by setting $\rho_m = \rho_\Lambda$ in eq. 5.

Both redshifts must be larger than 0 so we can use this as a prior when calculating the degeneracy in the Chebyshev coefficients from observational data. We will vary Ω_0 between 0.26 (favored by [14]) and 0.3, and for each value we will let H_0 vary between 60 and 80 km/s/Mpc, in a flat Universe. From measurements of D_L, the minimum χ^2 is obtained as $\Omega_0 = 0.30$; from D_A as $\Omega_0 = 0.29$ and from H at the smallest value, $\Omega_0 = 0.26$.

Table 4. Best fit values (that minimize χ^2) for the coefficients and H_0

		$\frac{2w_1}{z_{max}}$	$w_0 - w_1$	H_0 [km/s/Mpc]
$\Omega_0 = 0.26$	from D_L	$-1.96^{+4.54}_{-13.03}$	$-1.06^{+1.81}_{-1.51}$	$73.33^{+6.66}_{-13.33}$
	from D_A	$3.18^{+5.15}_{-9.39}$	$-1.66^{+2.72}_{-0.90}$	$80^{+0}_{-17.37}$
	from H	$-0.45^{+1.21}_{-0.60}$	$-0.15^{+0.30}_{-1.51}$	$61.01^{+18.98}_{-1.01}$
	from D_A and D_L	$-6.81^{+3.33}_{-5.75}$	$0.15^{+0.60}_{-1.21}$	$71.31^{+7.87}_{-6.06}$
	from D_A, H and D_L	$-5.90^{+4.54}_{-5.75}$	$-1.06^{+0.60}_{-0.60}$	$80^{+0.00}_{-2.22}$
$\Omega_0 = 0.27$	from D_L	$-1.96^{+4.24}_{-13.03}$	$-1.06^{+1.81}_{-1.51}$	$72.72^{+7.27}_{-12.72}$
	from D_A	$3.48^{+5.45}_{-9.09}$	$-1.66^{+2.42}_{-0.90}$	$80^{+0}_{-16.56}$
	from H	$-0.45^{+1.51}_{-0.60}$	$-0.15^{+0.30}_{-1.81}$	$60.60^{+19.39}_{-0.60}$
	from D_A and D_L	$-7.12^{+3.33}_{-6.66}$	$0.15^{+0.60}_{-0.60}$	$71.11^{+6.86}_{-6.66}$
	from D_A, H and D_L	$-6.51^{+4.84}_{-8.48}$	$-1.06^{+0.90}_{-0.60}$	$79.79^{+0.20}_{-3.23}$
$\Omega_0 = 0.29$	from D_L	$-3.18^{+4.54}_{-11.81}$	$-0.45^{+1.51}_{-1.81}$	$67.87^{+11.51}_{-7.87}$
	from D_A	$5.30^{+5.75}_{-11.21}$	$-1.96^{+2.72}_{-0.90}$	$80^{+0.00}_{-16.16}$
	from H	$-0.75^{+1.81}_{-0.90}$	$-0.15^{+0.30}_{-1.81}$	$64.24^{+15.75}_{-4.24}$
	from D_A and D_L	$-8.33^{+3.63}_{-6.66}$	$0.45^{+0.90}_{-0.90}$	$69.29^{+6.26}_{-6.26}$
	from D_A, H and D_L	$-9.24^{+6.36}_{-5.75}$	$-0.45^{+0.60}_{-0.91}$	$76.76^{+3.23}_{-2.82}$
$\Omega_0 = 0.30$	from D_L	$-3.1^{+4.24}_{-11.81}$	$-0.45^{+1.51}_{-1.81}$	$65.25^{+11.31}_{-5.25}$
	from D_A	$5.30^{+6.06}_{-11.51}$	$-1.96^{+2.72}_{-0.90}$	$80^{+0}_{-16.16}$
	from H	$-0.75^{+1.51}_{-0.90}$	$-0.15^{+0.30}_{-1.81}$	$63.83^{+15.15}_{-3.83}$
	from D_A and D_L	$-9.54^{+4.24}_{-5.45}$	$0.75^{+0.90}_{-0.90}$	$67.87^{+6.66}_{-5.65}$
	from D_A, H and D_L	$-8.93^{+5.15}_{-6.06}$	$-0.45^{+0.90}_{-0.60}$	$75.75^{+2.82}_{-3.03}$

It can be observed again, the D_A measurements favor the largest Hubble constant and the H ones, the smallest. We can see that the $2\frac{w_1}{z_{max}}$ coefficient is close to a zero value which makes the quintessence constant with redshift. The free term $w_0 - w_1$ (the value of the quintessence at present) is negative but smaller than in the cosmological constant case. The measurements on D_A result in a smaller value for the free term.

4. CONCLUSIONS

In this paper we have constrained the coefficients of the time dependent quintessence approximated to the first order using Chebyshev polynomials. Of course, the precision would increase if higher order expansion would be considered but in order to do this, one needs much more observational data with higher precision.

When it comes to observational data, the redshift interval (0.5,1) is a good restricter for the Hubble parameter. In that region the maximum deviation from the cosmical constant case occurs therefore more observations in that region would help.

It is imperative to have many more observations also at higher redshifts. Of course, the data quality should be improved, as the given errors are still very high. Special attention should be given to SNe Ia as the results are highly empirical and the physical models describing them are not accurate yet. We hope that the Supernovae Legacy Survey (SNLS), that when complete expects $\sim$ 700 SNe Ia measurements in the redshift range $0 < z < 1.7$ [15], will solve this problem.

Using the individual observations and fixed values of H_0 and Ω_0 (that is 70 km/s/Mpc and 0.29), one can see that the measurements of the Hubble parameter are the ones that result in coefficients closest to the cosmological constant case, as the coefficient of z is 0.06 and the free one is 0.9. The greatest deviation from it is archived in the case of the angular diameter which gives $\frac{2w_1}{z_{max}} = -3.55^{+4.57}_{-11.44}$ and $w_0 - w_1 = -0.66^{+1.8}_{-0.96}$ (but again with high error bars).

When we let H_0 and Ω_0 vary, one can see that measurements of the angular and luminosity distance favor a Universe with a higher matter parameter content, in contrast to the Hubble parameter measurements. The latter also supports a coefficient of z close to 0 similar to the cosmological constant case. Both distance measurements give a value of the quintessence at the present epoch ($w_0 - w_1$) that is higher than in the cosmological constant case. They also favor a time dependent quintessence.

We hope that more observations at a higher precision analyzed with a higher order Chebyshev expansion will lead to a better estimation of the quintessence.

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