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ASESSMENT INSTRUMENT MERLIN – METAL RESISTIVITY ANALYSIS BY LINEARIZATION

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Abstract. In the complex process of developing the potential of every youth's performance through learning, assessment is an important component for which adequate tools are needed. This article describes the design and development of the MERLIN instrument, which is dedicated to the assessment of student performance in physics in an international competition. The article presents the table of specifications for the MERLIN assessment instrument developed on the basis of the Taxonomy Table, an experimental problem created by reference to the table of specifications and an overview of pilot results of this instrument.

Key words: table of specifications, cognitive processes, conceptual knowledge, procedural knowledge, metacognitive knowledge, a model validation through experiment.

1. INTRODUCTION

Youth capable of excelling in the area of performance are considered to be a relatively small target group, but one with an exceptional value to increase the quality of human resources.

In the complex process of developing the potential of every youth's performance through learning [1], assessment is an important component for which adequate tools are needed. Therefore, to assess performance in experimental physics problems, a portfolio of assessment instruments aimed at students of Physics Olympic Team was created.

MERLIN Assessment Instrument (**M**etal **A**nalysis by **R**esistivity **LIN**earization) is one of the tools included in this portfolio, and it was prepared for the tenth edition of the pre-Olympic contest between Romania and Hungary. The contest is held every year just week before the International Physics Olympiad (IPhO) and lasts for a period of five days. Its aim is to prepare students from Romania and Hungary to participate in IPhO. This contest's syllabus relates to [2] and offers participants the opportunity to solve theoretical and experimental problems similar to those in IPhO, under the same environment of international competition.

2. DESIGN OF ASSESMENT INSTRUMENT MERLIN

The experimental problem proposed in the MERLIN assessment instrument focuses on studying temperature dependence of the resistivity of copper wire and to experimentally validate a model of this dependence.

2.1. APPROPRIATE ASSESSMENT OBJECTIVES OF THE MERLIN INSTRUMENT

The MERLIN instrument-specific assessment objectives were aimed at assessing the skills of physics students from the Olympic team as they solved an electricity experimental problem, through the students' analysis and evaluation as they addressed tasks [3, 4, 5, 6, 7].

The following are the general objectives of portfolio assessment for the Physics Olympic team as endorsed by the MERLIN instrument, as well as specific objectives and the task number of experimental problems on which it is intended to assess the physics skills of Olympic team members.

Table 1

The general objectives, specific objectives and tasks of the assessment instrument MERLIN

<table>
<tr><td colspan="3">Ob.I - Solve innovative theoretical and practical problems relating to the proposed experimental activity.</td></tr>
<tr><td>Ob.I.1</td><td>Apply knowledge of the electrical conduction model in metals and metal structure model for the inference of expression of the electrical conductivity.</td><td>Tasks 1a and 1b</td></tr>
<tr><td>Ob.I.2</td><td>Organize information on CFCs crystalline structure and on electrical conductivity in the classical theory of electrical conduction, in order to estimate the values of physical characteristic quantities of the charge carriers participating in electrical conduction in copper.</td><td>Tasks 3a and 3b</td></tr>
<tr><td>Ob.I.3</td><td>Create a knowledge-based linearization strategy for the three proposed models of the linearized expressions of the temperature dependence of electrical resistance.</td><td>Task 4a</td></tr>
<tr><td colspan="3">Ob.II - Registration with appropriate accuracy of the experimental data collected from measurements and processing of data, including error calculation, in order to determine the physical characteristics of the systems.</td></tr>
<tr><td>Ob.II.1</td><td>Apply knowledge about methods and measurement techniques in order to collect important experimental data and to record their values in appropriate tables.</td><td>Task 2a</td></tr>
</table>

Table 1 (continued)

Ob.II.2	Calculate the numerical values of physical quantities characteristic of the analyzed system (including the calculation of errors).	Task 2d
Ob.II.3	Analyze the linearization strategy based on knowledge of the data set plotted in the graph R = R (T), for determining the values of physical characteristic quantities of the analyzed system.	Task 2c
Ob.III - Graphically represent the collected experimental data in appropriate forms.		
Ob.III.1	Apply knowledge about graph construction to plot the dependence R = R (T) for collected experimental data.	Task 2b
Ob.III.2	Create graphs of linearized dependencies of electrical resistance vs. temperature using collected experimental data.	Task 4b
Ob.IV - Evaluate results of the experimental problem.		
Ob.IV.1	Evaluate the linearized dependencies temperature *vs*. electrical resistance in order to validate one of the models.	Task 4c
Ob.IV.2	Select an appropriate subset of the collected data set to estimate Debye temperature for copper.	Task 4d

2.2. TABLE OF SPECIFICATIONS DEVELOPED AT THE DESIGN STAGE OF THE ASSESSMENT INSTRUMENT MERLIN

The MERLIN instrument, dedicated to the assessment of the Olympic performance of physics students from Romanian and Hungarian teams in an international competition was designed with reference to Bloom's taxonomy, revised in 2001 by L. Anderson and D. Krathwohl [8] [9]. The table of specifications was developed based on the Taxonomy Table [8]. This taxonomy has allowed the design and elaboration of a coherent assessment structure and balanced, objective assessment of cognitive processes connecting with the kinds of knowledge and task aligned with those goals.

Addressing the specific tasks in the experimental problems, is necessary to apply cognitive processes of the categories Apply, Analyze, Evaluate, Create to conceptual, procedural and metacognitive knowledge [8]. Therefore, the table of specifications for the MERLIN instrument contains only the categories of cognitive processes and types of knowledge mentioned above (Table 2).

In the design phase assessment instrument has been allocated 7.5% of the total score of the problem tasks whose solution requires the application of the conceptual knowledge of the electrical conduction model in metals and the metal structure model (Ob.I.1), 7.5% of total score of the problem tasks whose solution calls for organizing the information on theories, models, and structures. 15% of the total score of the problem tasks whose solution involves creation based on metacognitive knowledge of the original products - linearized expressions of the temperature dependence of electrical resistance (Ob.I.3).

The application of knowledge about methods and measurement techniques to collect experimental data and record their values in appropriate tables (Ob.II.1) was given 10% of the total score of the problem, and calculating the numerical value of a physical quantity characteristic of the analyzed system (Ob.II.2) was given 2.5% the total score. Benchmark Ob.II.3 is important for assessing the performance of a particular target group, such as the students of Olympic team. Therefore, the metacognitive knowledge-based analysis of the data set plotted to determine the values of physical characteristic quantities of the analyzed system (Ob.II.3) was given 15% of the total score of the problem.

The overall objective assessment Ob.III, graphic representation on appropriate forms of data collected experimentally, is reflected in the MERLIN instrument in the application of procedural knowledge for plotting graphs (Ob.III.1) and creation of the original graphics based on procedural knowledge (Ob.III.2). According to the table of specifications, performing tasks covered by these two specific assessment objectives has been granted 7.5% and 15%, respectively, of the problem score.

Validation by experiment of one of the models proposed in the MERLIN instrument and selecting a suitable subset of data to estimate Debye temperature primarily requires evaluating results of the experimental problem, based on conceptual and procedural knowledge (Ob.IV.1 and Ob IV.2). For performing tasks covered by specific objectives Ob.IV.2 and Ob.IV.1, 15% and 5% of the total score of the problem was assigned, respectively.

Table 2

Table of specifications developed at the design stage of the MERLIN assessment instrument

The Knowledge Dimension	The Cognitive Process Dimension				Total
	Apply	Analyze	Evaluate	Create	
Conceptual Knowledge	Ob.I.1 (1a–2.5%) (1b–5%)	Ob.I.2 (3a–5%) (3b–2.5%)	Ob.IV.1 (4c–15%)	-	30%
Procedural Knowledge	Ob.II.1 (2a–10%) Ob.II.2 (2d–2.5%) Ob.III.1 (2b–7.5%)	-	Ob.IV.2 (4d–5%)	Ob.III.2 (4b–15%)	40%
Metacognitive Knowledge	-	Ob.II.3 (2c–15%)	-	Ob.I.3 (4a–15%)	30%
Total	27.5%	22.5%	20%	30%	100%

The statement of the experimental problem was made following the issue structure determined by table of specifications.

3. PRESENTATION OF THE *MERILN* ASSESMENT INSTRUMENT

In order to pilot and manage the MERLIN assessment instrument, the following were developed:

- Instruction sheet
- MERLIN Experimental problem
- Answer sheet
- Rating scale and scoring
- Detailed solution of the experimental problem

The following is the statement of the experimental problem, along with the detailed solution.

3.1. MERLIN EXPERIMENTAL PROBLEM

3.1.1. About resistivity of metals

Metals are chemical elements whose atoms can easily lose electrons, forming positive ions bound together by metallic bonds [10]. A crystal of metal can be viewed as a network of positive ions immersed in a cloud of delocalized electrons. In the absence of an external electric field, these electrons move freely through the network, randomly – in all directions at high speeds $\left(\approx 10^5\,\mathrm{m}\cdot\mathrm{s}^{-1}\right)$, colliding with the ion network. Between two successive collisions, an electron's trajectory is a straight line segment. In this case, the average speed of electrons is zero and, hence, there is not a net transport of electric charge

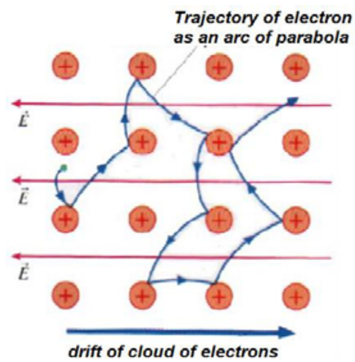

Fig. 1 – The movement of electrons in a fixed network of metal ions in the presence of the electric field.

When applying an external electric field, electron motion is not completely random. All electrons drift in the direction of the applied field, so there is a net flow of electric charge [11, 12]. In an applied electric field, conduction electrons' trajectories are arcs of parabola. The situation is illustrated in Figure 1.

For conduction due to electrons, drift current intensity is the ratio between the charge passing through the conductor and the time that this flow happens. The resistance of a conductor is hindering the free movement of electric charge. It suffered due to electron collisions with ions of the network. As is well known from experimental measurements, the resistance depends on conductor geometry and material's type.

For a metal, the number of quasi-free carriers of electric charge is practically independent of temperature. The resistivity of the material depends on temperature only through the collisions rate of charge carriers in the crystalline network.

Raising temperature increases the amplitude of vibration of the ions in the network, which increases the probability of collisions between electrons and these ions, thus increasing the resistivity. Collective vibrations of the ions (which are sound waves!) do not have a wavelength less than twice the distance between two ions in the network. Maximum possible frequency $\nu_{\max}$ of this wave determines the maximum energy attached to a vibration in the metal $E_{\max} = h \cdot \nu_{\max}$. This energy corresponds to the Debye temperature [13, 14], which is the maximum temperature that can be achieved through the action of a single vibration mode.

$$\theta_D = h \cdot \nu_{\max} / k_B \,. \tag{1}$$

In the expression of equation (1) h is the Planck constant, k_B is the Boltzmann constant, and $\nu_{\max}$ is the Debye frequency, the maximum allowed in a given crystal. The Debye temperature is a material constant, which determines many properties of the material from which its specific heat, electrical resistivity, elastic properties, thermal conductivity, and the width of emitted X-ray spectral lines can be found.

In the theory of electrical resistivity of metals, it is recognized that – around room temperature – the temperature dependence of resistivity is caused by collisions between electrons and collective vibrations of metal ions and is different for temperatures above, below or near the Debye temperature θ_D [15, 16]. Thus, for temperatures much lower than the Debye temperature, the resistivity depends on temperature according to the Matthiessen law

$$\rho = K \cdot \rho_0 \cdot \left(\frac{T}{\theta_D} \right)^5 ; \quad T << \theta_D . \tag{2}$$

In equation (2) ρ_0 has the dimensions of resistivity, and K is a dimensionless constant independent of temperature.

For temperatures near the Debye temperature, $\frac{1}{2}\theta_D < T < 2\theta_D$, the temperature dependence of resistivity is given by

$$\rho = \rho_0 \frac{T}{\theta_D}\left[1 - \frac{1}{18}\left(\frac{\theta_D}{T}\right)^2\right]. \tag{3}$$

At temperatures much higher than θ_D, the temperature dependence of resistivity is given by the linear expression

$$\rho = \rho_0 \cdot \left(\frac{T}{\theta_D}\right); \quad T >> \theta_D . \tag{4}$$

In a very simple approximation, a linear dependence of resistivity on temperature is

$$\rho(t) = \rho(t_0) \cdot \left[1 + \alpha(t - t_0)\right], \tag{5}$$

where α is the thermal coefficient of resistivity.

3.1.2. Experimental setup

On the desk are:

An experimental device that includes a resistance of copper wire with diameter $\Phi = (0.200 \pm 0.001)$ mm and length $\ell = (40.00 \pm 0.01)$ m and an oven made of high resistive nickel alloy wire. The ends of the copper wire are connected to the yellow terminals on the support plate of the device and are printed with text (which is on the yellow portion of the label) Resistance Test Points. The oven is powered by the jack marked with the blue inscription 12V Power Supply – 700mA.

The power of the oven is provided by AC adapter power supply, which is on the desk.

A digital multimeter is used together with a thermocouple to measure temperature. The Selector knob of the temperature measuring instrument should be positioned on $°\mathrm{C}$. The thermocouple is connected to the TEMP terminals of the instrument. The top end of the thermocouple (which detects the temperature) is placed on the central channel of the experimental device. The temperature measured is the indication of the instrument. Instrument error in measuring temperature is $\pm 0.5°\mathrm{C}$.

A digital multimeter is used to measure resistance. The Selector knob must be located on the 200 position in the range marked Ω. The terminals used to measure resistance are, respectively, *COM* and *VΩHz*, which are on the bottom right of the instrument panel. The precision of the measuring instrument is $\pm 0.1\Omega$. The Instrument for measuring the resistance will be connected to the resistance (copper wire) with a provided pair of wires. For experimental data processing, two sheets of graph paper are provided.

Fig. 2 – The experimental device.

3.1.3. Task 1

1a. Derive an expression of electrical conductivity $\sigma = 1/\rho$ as a function of concentration n of charge carriers, the elementary electric charge e, and drift speed v_d of the charge carriers. Assume that the drift speed is proportional to the intensity E of the applied electric field $v_d = \mu \cdot E$. The coefficient of proportionality μ is known as mobility.

1b. Derive an expression of electrical conductivity as a function of n concentration of charge carriers, e the elementary electric charge, electron mass m and relaxation time τ (mean time interval between two collisions of the electron in the lattice).

3.1.4. Task 2

2a. Feed the electric oven by plugging in the power supply. Measure the electrical resistance R of the copper wire and the corresponding temperature T in the electric oven. Perform at least 35 measurements for a range of temperatures between room temperature and $95°\text{C}$. Create an appropriate table and fill it in with collected pairs of numerical data values (resistance, temperature). If the temperature rises too fast when you make measurements, you can occasionally stop heating by switching-off the electric oven supply.

2b. Using the experimental data collected, make a graph with the value of $R = R(T)$ on the available graph paper sheet. In the graph, mark the experimental points and their measurement errors. Indicate two sources of experimental errors.

2c. Using graph drawn in Task 2b, determine the values for the electrical resistance of copper wire at a temperature of $0°\text{C}$ and the thermal coefficient of electrical resistivity of copper. Write their values and the errors determined in the appropriate boxes in the Answer Sheet.

2d. Calculate the value of the electrical resistivity of copper at $0°C$ and the error of this value.

3.1.5. Task 3

3a. Copper crystallizes in the FCC structure (represented in Fig. 3). The side of the cube is $a = 3.6 \times 10^{-10}$ m. Keep in mind that a "corner" atom participate in eight cubes, that a "face" atom participates in two cubes, and each copper atom participates with one electron in the electrical conduction. Estimate the concentration of charge carriers participating in electrical conduction in the copper crystal.

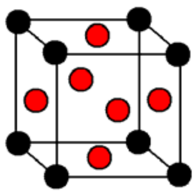

Fig. 3 – Face-centered cubic FCC.

3b. Estimate the value of relaxation time for charge carriers in copper at a temperature of $0°C$.

3.1.6.Task 4

4a. Using relations (2), (3) and (4) on temperature dependencies of electrical resistivity for the three indicated ranges, set up a linearized expression of the electrical resistance of copper wire dependence on temperature for each of the three ranges. In interpreting the data, ignore measurement errors and temperature resistance.

4b. Using the appropriate coordinate systems, draw the three linearized plots established in Task 4a in three separate diagrams.

4c. Specify whether the temperature range where measurements have been carried out is under, over or near the Debye temperature and justify your answer.

4d. Select an appropriate subset of the data set collected during the measurements and estimate the value of the Debye temperature value for copper.

If necessary you can use formulas required for slope (B) *and ordinate at the origin* (A) *of the linear regression line* $\overline{y} = A + B \cdot \overline{x}$

$$\begin{cases} A = \dfrac{\left(\sum y_i\right)\cdot\left(\sum x_i^2\right)-\left(\sum x_i\right)\cdot\left(\sum x_i\cdot y_i\right)}{n\cdot\left(\sum x_i^2\right)-\left(\sum x_i\right)^2} \\ B = \dfrac{n\cdot\left(\sum x_i\cdot y_i\right)-\left(\sum x_i\right)\cdot\left(\sum y_i\right)}{n\cdot\left(\sum x_i^2\right)-\left(\sum x_i\right)^2} \end{cases}.$$

The σ, *standard deviation, the measure of how values are spread over a sample average value, can be calculated using the formula*

$$\sigma = \sqrt{\frac{n\cdot\left(\sum x_i^2\right)-\left(\sum x_i\right)^2}{n\cdot(n-1)}}.$$

3.2. SOLUTION

3.2.1. Task 1

1a. If n denotes the concentration of charge carriers and e denotes the load of each charged particle, then the total load transported during time t is

$$Q = v_d \cdot t \cdot A \cdot n \cdot e. \tag{6}$$

The current intensity passing through the conductor has the expression

$$I = \frac{Q}{t} = v_d \cdot A \cdot n \cdot e, \tag{7}$$

and the density of current is

$$j = \frac{I}{A} = v_d \cdot n \cdot e. \tag{8}$$

If it is assumed that the drift speed is proportional to the applied electric field strength

$$v_d = \mu \cdot E \tag{9}$$

(where μ is the mobility), relation (8) can be written as

$$j = v_d \cdot n \cdot e = \mu \cdot n \cdot e \cdot E. \tag{10}$$

Comparing relation (10) with Ohm law written as

$$j = \sigma \cdot E, \tag{11}$$

this means that

$$\sigma = ne\mu, \tag{12}$$

if charge carriers are electrons with the electric load e .

1b. Consider a carrier having electric charge e that moves inside a solid under the influence of an electric field intensity E_x. The electric field exerts an electric force F_x on the charge carrier that determines its acceleration a_x.

$$\begin{cases} F_x = eE_x \\ a_x = \dfrac{eE_x}{m}. \end{cases} \tag{13}$$

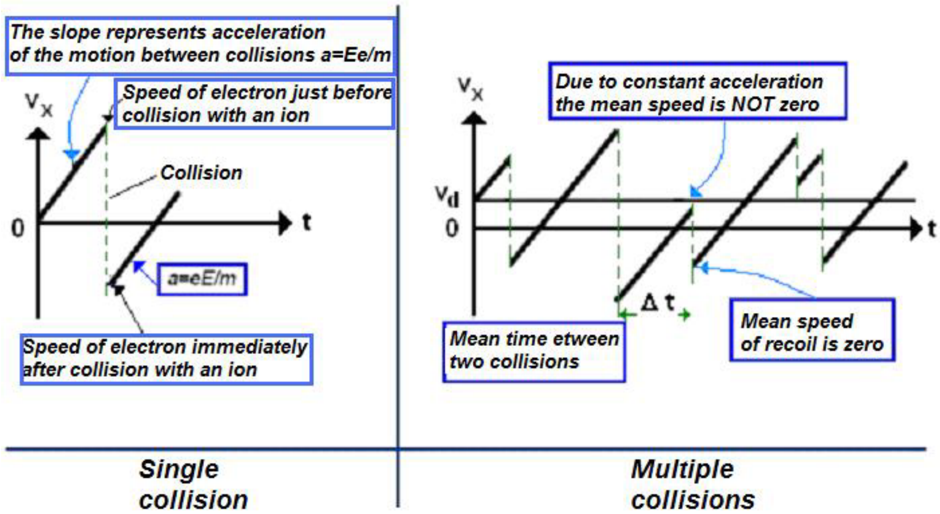

Fig. 4 – Motion of the electron under the influence of an external electric field in a metal lattice.

If the average time between two collisions is Δt, the maximum speed reached by the charge carriers is

$$v_x = v_{0x} + \frac{eE_x}{m}\Delta t . \tag{14}$$

When a charge carrier collides with an ion, it changes its direction of motion. After the collision, it has the same acceleration due to electric field action. Figure 4 presents the speed dependence of the charge carriers for a single collision, as well as a sequence of collisions. The average speed (in time) of the charge carriers' motion through a succession of collisions is the drift velocity v_{drift}.

$$v_{drift} = \overline{v_x} = \overline{v_{0x}} + \frac{eE_x}{m}\overline{\Delta t} . \tag{15}$$

Since the average velocity is zero during collision – as in the absence of electric field – the drift velocity has the expression

$$\begin{cases} v_{drift} = \dfrac{eE_x}{m}\overline{\Delta t} \\ v_{drift} = \dfrac{eE_x}{m}\tau. \end{cases} \tag{16}$$

In this expression, τ is the average time between two collisions, called the relaxation time.

Using equation (16) with (9), the result is

$$\mu = \frac{e}{m}\tau, \tag{17}$$

so that the conductivity of the material can be written as

$$\sigma = ne\mu = \frac{ne^2}{m}\tau. \tag{18}$$

3.2.2. Task 2

2a. The presented data set was chosen randomly from among the sets collected during piloting.

Table 3

Experimental data temperature-resistance

Nr.	t [°C]	R [Ω]	Nr.	t [°C]	R [Ω]
1	25.0	22.9	19	61.0	26.8
2	27.0	23.0	20	63.0	26.9
3	29.0	23.2	21	65.0	27.2
4	31.0	23.4	22	67.0	27.3
5	33.0	23.6	23	69.0	27.4
6	35.0	23.7	24	71.0	27.6
7	37.0	23.9	25	73.0	27.8
8	39.0	24.0	26	75.0	27.8
9	41.0	24.2	27	77.0	28.0
10	43.0	25.1	28	79.0	28.2
11	45.0	25.4	29	81.0	28.3
12	47.0	25.6	30	83.0	28.4
13	49.0	25.7	31	85.0	28.6
14	51.0	25.9	32	87.0	28.7
15	53.0	26.1	33	89.0	28.8
16	55.0	26.3	34	91.0	28.9
17	57.0	26.5	35	93.0	29.0
18	59.0	26.6			

According to the problem statement, the error of the temperature measuring instrument is $\mathrm{Er}(T) = \pm 0.5°C$, and the error of the resistance measuring instrument is $\mathrm{Er}(R) = \pm 0.1\Omega$.

2b.

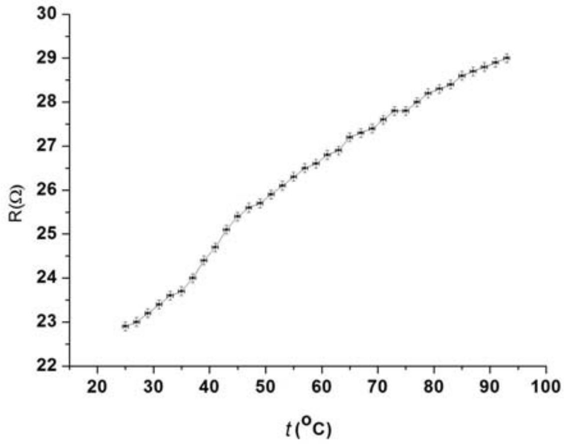

Fig. 5 – Temperature dependence of measured resistance. Measurement errors, the same for all points, are represented by vertical and horizontal end-marked segments.

Measurement errors are known. Two uncontrollable sources of error in the experiment are:

•The difference between the temperature measured by thermocouple and oven temperature.

• The possibility of the existence of zones with different temperatures along the oven and consequently along the thread whose resistance is measured.

2c. By multiplying relation (5) with constant value $4 \cdot \ell / (\pi \cdot \phi^2)$ an expression is obtained for temperature dependence of resistance R of the wire as a function of its resistance value R_0 at $0°\mathrm{C}$ and the thermal coefficient of resistivity, α.

There are two ways of determining the values of electrical resistance of copper wire at a temperature of $0°\mathrm{C}$ and thermal coefficient of electrical resistivity of copper:

I. Law of variation of electrical resistance with temperature can be written – for two close temperatures as

$$R_i = R_{i-1} \cdot \left(1 + \alpha \cdot \left(t_i - t_{i-1}\right)\right), \tag{19}$$

so that

$$\begin{cases} \alpha = \dfrac{R_i - R_{i-1}}{R_{i-1} \cdot \left(t_i - t_{i-1}\right)} = \dfrac{1}{R_{i-1}} \cdot \dfrac{\Delta R_i}{\Delta t_i} \\ R_0 = \dfrac{R_i}{1 + \alpha \cdot t_i}. \end{cases} \tag{20}$$

Considering successive pairs of data from Table 3, successive values of thermal coefficient of resistance values corresponding to a resistance wire can be obtained as in Table 4.

Table 4

The thermal coefficient of resistance values corresponding to a resistance of wire

Nr.	α	$R(0)$	R [Ω]	Nr.	α	$R(0)$	R [Ω]
1	0.00218	21.1	22.9	18	0.00375	22.0	26.6
2	0.00434	21.0	23.0	19	0.00186	22.0	26.8
3	0.00431	21.1	23.2	20	0.00557	22.2	26.9
4	0.00427	21.1	23.4	21	0.00183	22.1	27.2
5	0.00211	21.2	23.6	22	0.00183	22.1	27.3
6	0.00632	21.1	23.7	23	0.00365	22.1	27.4
7	0.00833	21.2	23.9	24	0.00181	22.1	27.6
8	0.00614	21.5	24.0	25	0.00180	22.0	27.8
9	0.00809	21.6	24.2	26	0.00359	22.1	27.8
10	0.00597	21.8	25.1	27	0.00357	22.1	28.0
11	0.00393	21.9	25.4	28	0.00177	22.0	28.2
12	0.00195	22.0	25.6	29	0.00176	22.0	28.3
13	0.00389	21.9	25.7	30	0.00352	22.0	28.4
14	0.00386	22.0	25.9	31	0.00174	22.0	28.6
15	0.00383	22.0	26.1	32	0.00174	22.0	28.7
16	0.00380	22.1	26.3	33	0.00173	21.9	28.8
17	0.00188	22.1	26.5	34	0.00173	21.9	28.9

The average value of determined values of thermal coefficient of resistance is

$$\overline{\alpha} = 0.00348 \quad \Omega/°\text{C}. \tag{21}$$

Individual values obtained deviate pretty much from the average.

The maximum deviation below the average is 0.00175, and the maximum deviation above the average is 0.00484. Standard deviation [17, 18] of the thermal coefficient of resistance value – calculated by the formula

$$\sigma = \sqrt{\frac{n \cdot \left(\sum x_i^2\right) - \left(\sum x_i\right)^2}{n \cdot (n-1)}} \tag{22}$$

is

$$\sigma_\alpha = 0.00185 \quad \Omega/°\text{C} \tag{23}$$

The average of values determined for the resistance at $0°\text{C}$ is

$$\overline{R_0} = 21.8\Omega , \quad (24$$

and standard deviation of values determined for the resistance at $0°\text{C}$ is set at

$$\sigma_{R_0} = 0.4\Omega . \tag{25}$$

The presented method does not consider the experimental data as a whole; to the value of the "table" of the thermal coefficient of copper resistivity $\alpha_{table} = 0.004 \quad \Omega/°\text{C}$, the result differs by 15%.

II. Figure 6 (a) is the experimental data regression line – without taking measurement error into account.

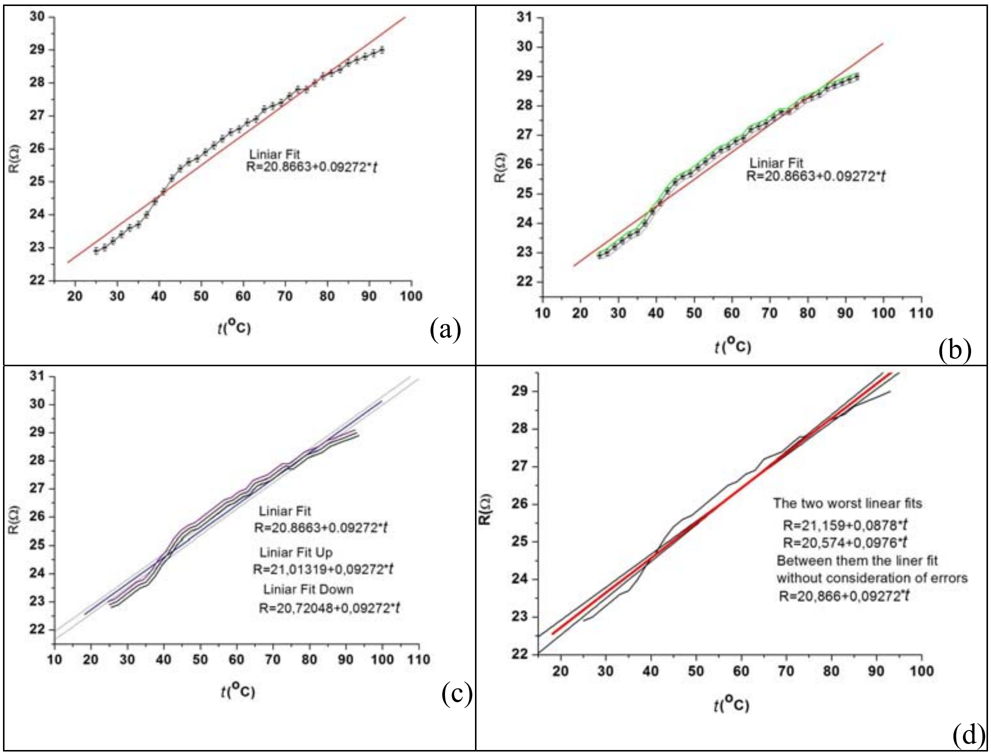

Fig. 6 – a) Linear regression for the data in Table 1; b) representation of the "band" for the true experimental values; c) best-fit lines for the band edges' bounding values of "true" data; d) best-fit lines for experimental points considering the most unfavorable errors.

Using the relations given in the problem statement, the following expression for the regression line results:

$$R = 20.8 + 0.0927 \cdot T\ , \tag{26}$$

which allows the determination of the values

$$\begin{aligned} &R_0 = 20.8 \quad \Omega, \\ &\alpha = 0.00445 \quad \Omega / °\mathrm{C}. \end{aligned} \tag{27}$$

The “true” values of the experimental data are in a band surrounding the points representing the measurement results regardless of errors. The width of this band is determined by the error values, as shown in Fig. 6b. For the top and bottom edges of this band, best fit lines (Fig. 6c) can be used to determine the most disadvantageous best fit lines for data analysis (Fig. 6d). The analysis of equations of straight lines fit the “most imprecise” results value of thermal coefficient of resistance

$$\alpha = \left(0.0044 \pm 0.0003\right) \quad \Omega/°\mathrm{C}\ . \tag{28}$$

Copper wire resistance at $0°C$ results in the value

$$R_0 = \left(20.8 \pm 03\right) \quad \Omega\ . \tag{29}$$

Construction of the regression line [18, 19] is a good way to take all the experimental data into consideration.

2d. Since

$$R = \rho \frac{4\ell}{\pi \cdot \Phi^2}\ , \tag{30}$$

so that

$$\rho = \frac{R \cdot \pi \cdot \Phi^2}{4\ell} \tag{31}$$

and because the differential resistivity is expressed as

$$\mathrm{d}\rho = \rho \cdot \left(\frac{\mathrm{d}R}{R} - \frac{\mathrm{d}\ell}{\ell} + 2\frac{\mathrm{d}\Phi}{\Phi} \right), \tag{32}$$

resistivity error is

$$\Delta\rho = \rho \cdot \sqrt{\left(\frac{\mathrm{d}R}{R}\right)^2 + \left(\frac{\mathrm{d}\ell}{\ell}\right)^2 + 2 \cdot \left(\frac{\mathrm{d}\Phi}{\Phi}\right)^2}\ . \tag{33}$$

From the data provided in the problem statement and relationship (32) results

$$\begin{cases} R_0 = (20.8 \pm 0.3) \ \ \Omega \\ \Phi = (0.200 \pm 0.001) \ \ \text{mm} \\ \ell = (40.00 \pm 0.01)\,\text{m}. \end{cases} \tag{34}$$

The numerical value of resistivity at $0°\text{C}$ is

$$\rho = \frac{20.75 \cdot \pi \cdot \left(0.2 \times 10^{-3}\right)^2}{4 \times 40} = (1.63 \pm 0.03) \times 10^{-8}\,\Omega\text{m}. \qquad (35$$

3.2.3. Task 3

3a. According to the problem statement, the crystal structure of copper has CFC, with the cube side measuring $a = 3.6 \times 10^{-10}\,\text{m}$. Each atom contributes to electrical conduction with a free electron. Each "corner" atom participate in 8 cubic cells. Therefore it can be considered to contribute with a "participation rate" of $1/8$. Each "face" atom participating in two cubic cells has a "participation rate" $1/2$. Consequently, the concentration of charge carriers in the copper crystal is expressed as

$$n = \frac{8 \times (1/8) + 6 \times (1/2)}{a^3} = \frac{4}{a^3}. \tag{36}$$

By entering the appropriate numeric data in (36), the results for the concentration of electrical charge carriers has the numerical value

$$n = \frac{4}{\left(3.6 \times 10^{-10}\right)^3} \cong 8.57 \times 10^{28}\,\text{m}^{-3}. \tag{37}$$

3b. Using the expression derived from the task 1b, one can determine the numerical value of relaxation time for copper at room temperature.

For $m = 9.1 \times 10^{-31}\,\text{kg}$, $e = 1.6 \times 10^{-19}\,\text{C}$ and a value of resistivity of $\rho_0 = 1.63 \times 10^{-8}\,\Omega \cdot \text{m}$ (or a conductivity of $\sigma_0 = 6.13 \times 10^7\,\Omega^{-1}\text{m}^{-1}$) relaxation time is set at

$$\begin{cases} \tau = \dfrac{\sigma_0 \cdot m}{ne^2} \\ \tau = \dfrac{6.13 \times 10^7 \cdot 9.1 \times 10^{-31}}{\left(1.6 \times 10^{-19}\right)^2 \cdot 8.57 \times 10^{28}} \cong 2.54 \times 10^{-14}\,\text{s} \end{cases}. \tag{38}$$

3.2.4. Task 4

4a. By multiplying the given dependencies of resistivity on temperature by $4 \cdot \ell / \left(\pi \cdot \phi^2\right)$, the expressions for the temperature dependence of the wire resistance R can be obtained for different temperature ranges. Linearization expressions of the temperature behavior of resistance are used to find new variables in the relationship of linear dependence. Thus:

• If the temperature is much higher than Debye temperature , $T > \theta_D$ electrical resistance has $R = R_0 \left[\dfrac{T}{\theta_D}\right]$. Denoting

$$A = \frac{R_0}{\theta_D}, \tag{39}$$

the above expression has a linear dependency

$$R = A \cdot T . \tag{40}$$

Therefore, if measurements are performed well above the Debye temperature, the temperature dependence of resistance chart should be a straight line passing through the origin.

• If measurements are made at temperatures near the Debye temperature, $\frac{1}{2}\theta_D < T < 2\theta_D$, electrical resistance depends on temperature according to relation (3) rewritten for resistance, $R = R_0 \dfrac{T}{\theta_D}\left[1 - \dfrac{1}{8}\left(\dfrac{\theta_D}{T}\right)^2\right]$. Noting

$$y = \frac{R}{T} \quad , \quad x = \frac{1}{T^2} \quad , \quad A = \frac{R_0}{\theta_D} \quad , \quad B = \frac{R_0 \cdot \theta_D}{8}, \tag{41}$$

the above expression written as

$$\frac{R}{T} \cong R_0 \left[\frac{1}{\theta_D}\right] \cdot \left(1 - \frac{\theta_D^2}{8}\left(\frac{1}{T}\right)^2\right) = \frac{R_0}{\theta_D} - \frac{R_0 \cdot \theta_D}{8} \cdot \frac{1}{T^2} \tag{42}$$

is a linear dependency

$$y = A - B \cdot x \tag{43}$$

with a negative slope. Therefore, if measurements are made around the Debye temperature, the temperature dependence of resistance written as in (42) should be a straight line with negative slope.

• If measurements are performed at temperatures that are well below the Debye temperature $T < \theta_D$, the temperature dependence of electrical resistance can be written as an expression using relation (2) with $R = K \cdot R_0 \cdot \left(\frac{T}{\theta_D}\right)^5$. Noting

$$x = \ln T, \quad y = \ln R, \quad a = \ln\left(K \cdot R_0\right) - 5\ln\left(\theta_D\right). \tag{44}$$

The above can be written in the form

$$\ln R \cong a_0 + 5\ln\left[\frac{T}{\theta_D}\right], \tag{45}$$

which is a linear form

$$y = 5x + a \tag{46}$$

having a slope value of 5.

4b. Experimental data collected are processed for graphical representation [19]. Results are shown in Table 5. To simplify operations, judgments are made directly on the measured resistance.

Table 5

Experimental data processed for linearization

Nr.	t°C	$R[\Omega]$	$T(\mathrm{k})$	$In(T)$	$\ln(R)$	R/T	$1/T^2 \times 10^5$
1	25	22.9	298	5.697	3.131	0.0768	1.126
2	27	23	300	5.704	3.135	0.0767	1.111
3	29	23.2	302	5.71	3.144	0.0768	1.096
4	31	23.4	304	5.717	3.153	0.077	1.082
5	33	23.6	306	5.724	3.161	0.0771	1.067
6	35	23.7	308	5.73	3.165	0.0769	1.054
7	37	23.9	310	5.737	3.174	0.0771	1.040
8	39	24	312	5.743	3.178	0.0769	1.027
9	41	24.2	314	5.749	3.186	0.0771	1.014
10	43	25.1	316	5.756	3.223	0.0794	1.001
11	45	25.4	318	5.762	3.235	0.0799	0.988
12	47	25.6	320	5.768	3.243	0.08	0.976
13	49	25.7	322	5.775	3.246	0.0798	0.964
14	51	25.9	324	5.781	3.254	0.0799	0.952
15	53	26.1	326	5.787	3.262	0.0801	0.940
16	55	26.3	328	5.793	3.27	0.0802	0.929
17	57	26.5	330	5.799	3.277	0.0803	0.918
18	59	26.6	332	5.805	3.281	0.0801	0.907
19	61	26.8	334	5.811	3.288	0.0802	0.896
20	63	26.9	336	5.817	3.292	0.0801	0.885
21	65	27.2	338	5.823	3.303	0.0805	0.875

Table 5 (continued)

22	67	27.3	340	5.829	3.307	0.0803	0.865
23	69	27.4	342	5.835	3.311	0.0801	0.854
24	71	27.6	344	5.841	3.318	0.0802	0.845
25	73	27.8	346	5.846	3.325	0.0803	0.835
26	75	27.8	348	5.852	3.325	0.0799	0.825
27	77	28	350	5.858	3.332	0.08	0.816
28	79	28.2	352	5.864	3.339	0.0801	0.807
29	81	28.3	354	5.869	3.343	0.0799	0.797
30	83	28.4	356	5.875	3.346	0.0798	0.789
31	85	28.6	358	5.881	3.353	0.0799	0.780
32	87	28.7	360	5.886	3.357	0.0797	0.771
33	89	28.8	362	5.892	3.36	0.0796	0.763
34	91	28.9	364	5.897	3.364	0.0794	0.754
35	93	29	366	5.903	3.367	0.0792	0.746

In order to verify the validity of models, the three areas are plotted in the three graphs (Fig. 7).

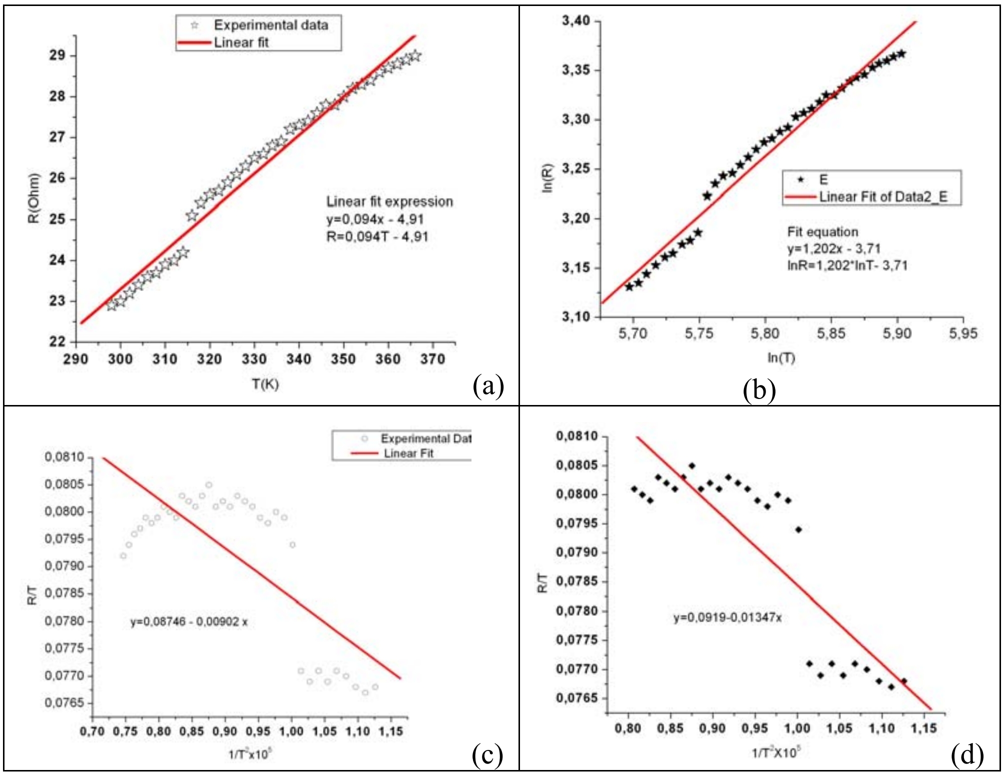

Fig. 7 – Graphical testing of models for dependencies R (T).

The graph in Fig. 7a is drawn using the data in Table 5, considering that the measurements were made at temperatures much higher than the Debye temperature.

The graph in Fig. 7b is drawn using data from Table 5, considering that the measurements were made within a range of temperatures located well below the Debye temperature.

The graph in Fig. 7c is drawn using data from Table 5, considering that the measurements were made near the Debye temperature.

4c. Given the correct answer to the task 4b, the hypothesis that the measurement were performed in a range near Debye temperature, $\frac{1}{2}\theta_D < T < 2\theta_D$, may be accepted.

Using the best fit line parameters, $y = A - B \cdot x$ (slope and ordinate at the origin) can be written as

$$y = \frac{R}{T}, \quad x = \frac{1}{T^2}, \quad A = \frac{R_0}{\theta_D}, \quad B = \frac{R_0 \cdot \theta_D}{8}, \tag{47}$$

$$\begin{cases} A = \dfrac{R_0}{\theta_D} = 0.08746 \\ B = \dfrac{R_0 \cdot \theta_D}{8} = 902 \end{cases}, \tag{48}$$

and therefore, in a first approximation

$$\theta_D = \sqrt{\frac{8B}{A}} \approx 290\text{K}. \tag{49}$$

4d. Considering the subset of experimental points below $80°\text{C}$ (to keep only the points for which the linear dependence has a negative slope), the graph becomes like in Fig. 7d and the best fit line changes its parameters.

For the new situation, the calculation leads to the value of Debye temperature

$$\theta_D = \sqrt{\frac{8B}{A}} \approx \sqrt{\frac{8 \times 1347}{0.0919}} \approx 342\text{K}. \tag{50}$$

4. PILOTING STAGE OF *MERLIN* ASSESSMENT INSTRUMENT

In the piloting stage, the MERLIN assessment instrument was applied to a representative group, whose composition was determined on the basis of a study dedicated to choosing the appropriate sample. As a result of this study, a group of 22 third-year students in the Faculty of Physics, University of Bucharest were selected, along with five former winners of international contests (during the piloting stage, they were already students) and nine outstanding high school students who were not participating in the contest for which the assessment instrument was prepared.

Validity of the experimental problem [20, 21] in the Merlin assessment instrument was based on several measurements. Thus content validity was established by consensus of an expert group, and construct validity was evaluated based on expert group discussions and individual interviews conducted with each group member participating in instrument piloting.

4.1. OVERVIEW OF PILOTING RESULTS OF THE MERLIN ASSESSMENT INSTRUMENT

When piloting the MERLIN assessment instrument, answer sheets, scoring memorandums with rating scale and scoring, videos from the experimental exam, observer notes of the experimental sample, and videos of interviews with individual members of the piloting group were collected.

A complete analysis [22, 23, 24, 25] was made based on this collected data, which led to a refinement of the experimental problem statement.

Since the group for which the instrument was built is a special one composed of young members of the Olympic teams capable of high performance who were competing in an international competition, it was envisaged that the assessment instrument was well calibrated. Therefore, the piloting of this instrument was a comprehensive study on the average degree of performance of a task, on the experimental problem discrimination and difficulty, and on the time allocation for activities and tasks [26, 27, 28].

4.1.1. The average performance of a task

A comprehensive analysis of the results recorded on each task was used as an indicator of the average performance G_{med} of each task. It was defined as the ratio between the average score P_{med} achieved for a task (expressed as the arithmetic average of the scores given for that work load) and maximum score $P_{\max}$ in the rating scale provided for scoring of that task

$$G_{med} = \frac{P_{med}}{P_{\max}} \cdot 100\% \tag{51}$$

Table 6 and Fig. 8 show the average performance for each task, in the piloting stage of MERLIN.

Table 6

The average performance for each task

Task	Average performance	Task	Average performance
1a	95.00%	3a	76.94%
1b	71.67%	3b	72.22%
2a	84.17%	4a	77.41%
2b	83.52%	4b	56.02%
2c	71.94%	4c	37.50%
2d	68.33%	4d	30.56%

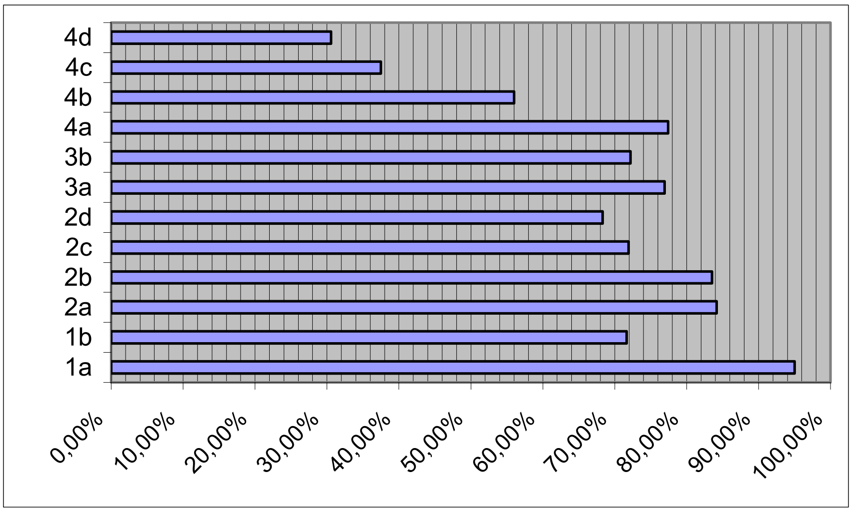

Fig. 8 – The average performance for each task.

The task 1a, whose solution is based on the conceptual knowledge model for electrical conduction in metals and metal structure model had the highest average performance (95.00%). Tasks 2a and 2b had average degrees of performance, 84.17% and 83.52% respectively. These two tasks involved the application of procedural knowledge about methods and measurement techniques to collect experimental data and record them in appropriate tables and constructing graphs, respectively.

The lowest average levels of performance were 37.50% and 30.56%, corresponding to the tasks 4c and 4d, whose solution involves evaluation based on conceptual and procedural knowledge of the results.

4.1.2. Discrimination and difficulty of experimental problem

In order to determine discrimination and difficulty of the experimental problem, the scores obtained by piloting group members were ordered decreasingly in a list. The difficulty of the experimental problem was determined by relationship (52),

$$\text{Difficulty} = \frac{S_1 + S_2}{2 \cdot n \cdot (\text{max score achieved})}, \tag{52}$$

where S_1 is the sum of the scores corresponding to the first $25\ \%$ of positions in the list, S_2 is the sum of scores for the last $25\ \%$ of positions in the list, and n indicates the value obtained by calculating $25\ \%$ of the total number of students / pupils participating in piloting stage.

Discrimination was calculated using the expression in equation (53)

$$\text{Discrimination} = \frac{S_1 - S_2}{n \cdot (\text{max score achieved})}. \tag{53}$$

For the MERLIN assessment instrument, the difficulty of experimental problem was found to be 66.78%, and the discrimination was 52.85%.

4.1.3. Table of specifications of MERLIN made after piloting of the assessment instrument

Based on the results of the piloting, the table of specifications developed in the MERLIN instrument design phase was completed with new data fields.

In the table's cells, the percentage of the total score allocated to each task was given. For comparison, the percentages of the total score as performed on average by the piloting group on each task are also included.

The new form of the table of specifications is presented in Table 7.

Based on data collected at piloting, a correlation was investigated [29] of the scores achieved at tasks 4a and 4b, whose solution requires a cognitive process of creation, and the scores achieved at the 4c and 4d, the solving of which involves a cognitive process of evaluating the results obtained from tasks 4a and 4b.

Table 7

The table of specifications of MERLIN after piloting

The Knowledge Dimension	The Cognitive Process Dimension: Apply		Analyze		Evaluate		Create		Total	
Conceptual Knowledge	Ob.I.1 (1a,1b)		Ob.I.2 (3a,3b)		Ob.IV.1 (4c)		-		30.00%	17.24%
	7.50%	5.96%	7.50%	5.65%	15.00%	5.63%				
Procedural Knowledge	Ob.II.1 (2a)		-		Ob.IV.2 (4d)		Ob.III.2 (4b)		40.00%	26.32%
	10.00%	8.42%			5.00%	1.53%	15.00%	8.40%		
	Ob.II.2 (2d)									
	2.50%	1.71%								
	Ob.III.1 (2b)									
	7.50%	6.26%								
Metacognitive Knowledge	-		Ob.II.3 (2c)		-		Ob.I.3 (4a)		30.00%	22.40%
			15.00%	10.79%			15.00%	11.61%		
Total	27.50%	22.35%	22.50%	16.44%	20.00%	7.16%	30.00%	20.01%	100.00%	

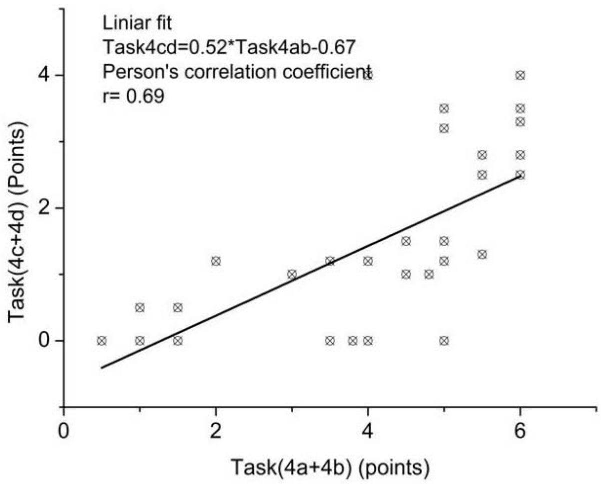

Fig. 9 – Correlation between scores for the tasks 4a and 4b, and 4c and 4d, respectively.

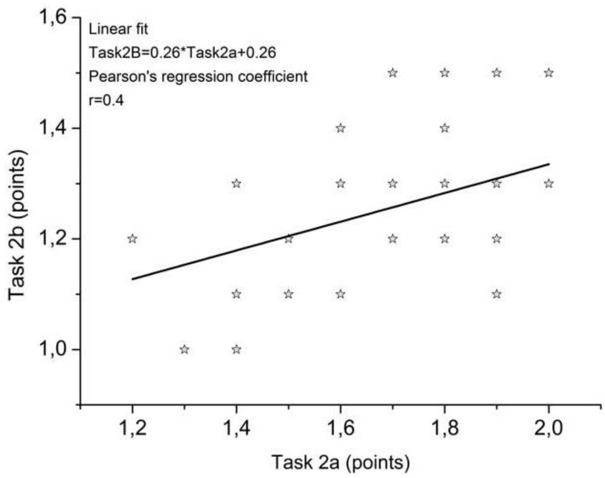

Fig. 10 – The correlation between scores obtained for 2a and 2b tasks.

The result – shown in Fig. 9 – shows a correlation coefficient of 0.69, showing that a majority of the piloting participants were able to evaluate the original results that were produced through a cognitive process of creation.

The correlation between scores on tasks 2a and 2b were also investigated. These tasks' solutions required the application of procedural knowledge to collect experimental data (task 2a) and plotting $R = R(T)$ based on the data collected (task 2b). The result indicates a weaker correlation between scores obtained on the two and, consequently, between the capacity to collect experimental data and draw a graphic dependence based on the measured data (Fig. 10).

4.1.4. The average time of performing the tasks

Also included in the piloting procedure of the MERLIN instrument was a study concerning the time needed to solve each task and the assigned time to complete the various activities necessary to solve the problem (reading the statement, mounting experimental set-up, writing the solution in the Answer Sheet, etc.). For this purpose laboratory test was video recorded and an observer was present during the test in laboratory.

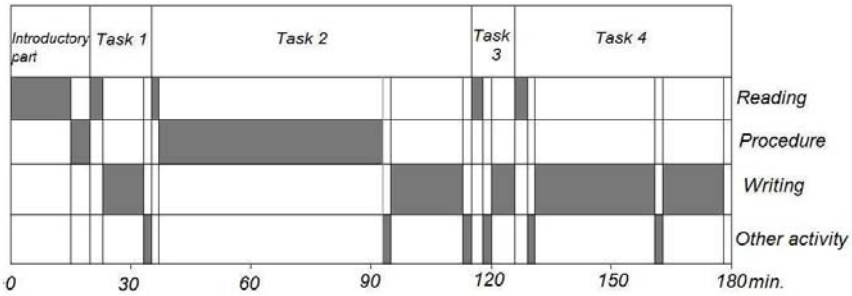

Fig. 11 – Time line during MERLIN piloting.

The chart in Fig. 11 summarizes the results of this study, while highlighting the development of experimental activities and time intervals during the test allocated on average to each activity and each type of task. The four types of activities listed in the chart (Reading, Procedure, Writing, Other activity) have the following meanings:

Reading	Reading the experimental problem statement.
Procedure	Identifying the experimental procedure, setup mounting, and data collection.
Writing	Record keeping, calculation of physical quantities, drafting, and completion of the answer sheet.
Other activity	Any activity other than those mentioned above, but related to solving the experimental problem.

The results of this study allowed for the time calibration of the application assessment instrument.

5. CONCLUSIONS

MERLIN is part of a portfolio of assessment instruments addressing students of the Physics Olympic team. It is developed through reference to the International Physics Olympiad syllabus and aims to assess student performance in the international competition. The target group for which MERLIN was created is a group of young members of the Olympic teams from Romania and Hungary capable high performance.

This assessment instrument has a coherent structure and balanced objectives connecting types of knowledge and cognitive processes and tasks in line with these objectives. This is done based on modern assessment strategy and refers to Bloom's taxonomy revised in 2001 by L. Anderson and D. Krathwohl. The table of specifications for the MERLIN instrument was developed based on the Taxonomy Table.

The experimental problem proposed in the MERLIN instrument was focused on studying the temperature dependence of resistivity of copper wire and experimentally validating a model of this dependence. Addressing the specific tasks is necessary to apply cognitive processes of the categories Apply, Analyze, Evaluate, create to conceptual, procedural and metacognitive knowledge.

During piloting, the MERLIN instrument was applied to a representative group. In order to correct calibration of the MERLIN instrument after piloting, item analysis was completed and a comprehensive study on the average degree of performance of a task, of discrimination and difficulty of the problem and of allocating time on activities and task was also done. The results of the survey items and analysis on the global application indicated the opportunity to apply the MERLIN assessment instrument during the tenth edition of the Romania-Hungary Pre-Olympic Competition.

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