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THE TEMPORAL EVOLUTION AND THE SPECTRAL STRUCTURE OF THE STOKES PARAMETERS OF THE LIGHT MODULATED BY LONGITUDINAL POCKELS EFFECT IN CRYSTALS OF CLASS $\bar{4}2m$ *

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Abstract. This article analyses the temporal evolution and the spectral structure of the Stokes parameters of the light modulated by longitudinal Pockels effect in $\bar{4}2m$ crystals, for low modulating voltage applied to the modulator. The temporal evolution and the spectral structure of Stokes parameters of the modulated light are analysed for an arbitrary d.c. bias. Two cases important in applications – no d.c. bias voltage and quarter wave bias voltage – are particularized and experimentally ilustrated.

Key words: Pockels effect, light modulation, spectral analysis, Stokes parameters.

1. INTRODUCTION

The description of the interaction between the polarized light and various dynamical polarization devices is a question of linear algebra and can be handled in various matrix (Jones and Mueller) [1, 2] or pure operatorial [3] formalisms.

Time-varying Jones and Stokes vectors were introduced in analysing the time evolution of the state of optical polarization in the output of anisotropic lasers [4-7]. In what concernes the polarization dynamics of the various kind of modulators (*e.g.* electrooptic or photoelastic) the analysis has been performed only in the frame of the Jones matrix formalism[1, 2, 8].

Complementary, a spectral analysis of the time-varying state of optical polarization was performed. The polarization-spectral content of the anisotropic lasers output [5, 6] or of the emerging light from some electro-optical modulators

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[2, 9, 10] was discussed and the homodyne (beat-signal) spectrum of the output light was displayed [2, 8-10, 11].

In this article we analyse the temporal evolution and the spectral structure of the Stokes parameters of the light modulated by longitudinal Pockels effect in a KDP crystal, for low modulating voltage applied the modulator.

2. THEORETICAL BACKGROUND

The KDP crystal has a fourfold axis of symmetry (C_4), the optic axis Z, as well as two mutually orthogonal twofold axes of symmetry that lie in the plane normal to Z. These are designated as X and Y axes. The symmetry group of this crystal is $\bar{4}2m$. By applying an electric field parallel to the optical axis of the crystal, Z, it becomes biaxial, with the plane of the optic axis at 45° to the X and Y crystallographic axes (the new induced principal axes $X^'$ and $Y^'$ will be rotated at an angle of 45° with respect to the crystallographic axes) [12].

Let us consider the modulation arrangement presented in Fig. 1, where K is a KDP longitudinal electooptic modulator. In the usual arrangement, the incoming light is linearly polarized along X axis at 45° to the electrically induced axes of the crystal. We will perform the calculi in the reference system OXY.

The refractive indices for the light polarized along the induced principal axes in a KDP longitudinal electrooptic modulator, are [13]:

$$n_{x'} = n_0 + \frac{n_0^3}{2} r_{63} E_z , \quad n_{y'} = n_0 - \frac{n_0^3}{2} r_{63} E_z , \tag{1}$$

where n_0 is the unperturbed crystalline index of refraction, r_{63}, the electrooptic coefficient for the electric field applied along the crystallographic Z axis and E_z electric field applied to the crystal. If the voltage across the crystal has a d.c. component U_0, as well as a harmonically-varying one, $U_m \sin \Omega t$, the phase shift between linearly polarized components along the principal axes $X^'$ (slow axis) and $Y^'$ (fast axis) is:

$$\Phi = 2(\Phi_0 + \Gamma \sin \Omega t) , \tag{2}$$

where: $\Phi_0 = \frac{\pi}{\lambda_0} n_0{}^3 r_{63} U_0$, $\Gamma = \frac{\pi}{\lambda_0} n_0{}^3 r_{63} U_m$, λ_0 being the vacuum wavelength of the incident light.

The KDP electrooptic modulator behaves as a time varying phase shift plate. The characteristic angles of the wave plate are: θ – the fast axis azimuth and Φ – the phase shift introduced by the plate between the $E_{x'}$ and $E_{y'}$ components of the electric field.

In our experiment the incident light is linearly polarized along the X axis, having the Stokes vector (we will take the imput Stokes vector normed to 1, *i.e.* the imput intensity equal to 1):

$$S_{in} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \tag{3}$$

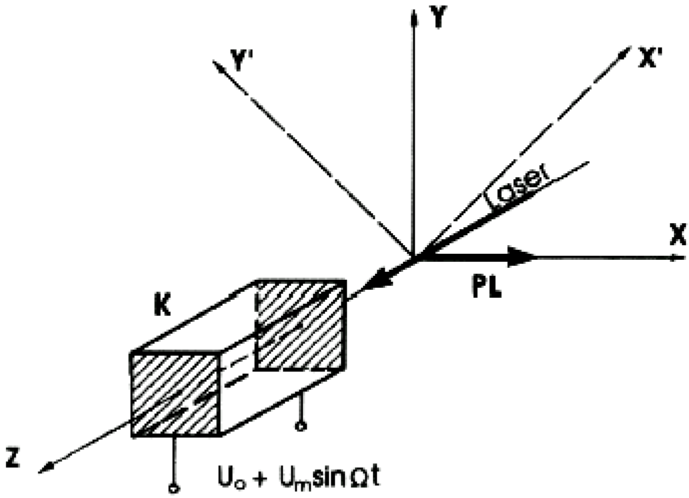

Fig. 1 – The longitudinal Pockels effect in a KDP crystal.

In this case the fast axis azimuth, θ is 135° (Fig. 1). Thus, the Mueller matrix of the modulator is [14]:

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\Phi & 0 & \sin\Phi \\ 0 & 0 & 1 & 0 \\ 0 & -\sin\Phi & 0 & \cos\Phi \end{pmatrix}. \tag{4}$$

The Stokes vector, S_{out}, of the emergent light from the modulator will be given by:

$$S_{out} = MS_{in} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\Phi & 0 & \sin\Phi \\ 0 & 0 & 1 & 0 \\ 0 & -\sin\Phi & 0 & \cos\Phi \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ \cos\Phi \\ 0 \\ -\sin\Phi \end{pmatrix} = \begin{pmatrix} 1 \\ \cos 2(\Phi_0 + \Gamma \sin \Omega t) \\ 0 \\ -\sin 2(\Phi_0 + \Gamma \sin \Omega t) \end{pmatrix} \quad (5)$$

The output Stokes vector has two constant components (S_0 and S_2) and two harmonic components (S_1 and S_3). The KDP crystal being non-absorbant, the S_0 parameter which signifies the total output intensity is constant, namely equal to one. Since the incident light on the KDP modulator is linearly polarized along the X axis, at 45° to X' and Y' induced axes, the S_2 parameter is equal to zero.

We use the decomposing formulas [15]:

$$\begin{aligned} \cos\left(2\Gamma \sin \Omega t\right) &= J_0(2\Gamma) + 2\sum_{k=1}^{\infty} J_{2k}\left(2\Gamma\right)\cos\left(2k\Omega t\right) \cong J_0(2\Gamma) + \\ &\quad + 2J_2(2\Gamma)\cos\left(2\Omega t\right) \\ \sin\left(2\Gamma \sin \Omega t\right) &= 2\sum_{k=1}^{\infty} J_{2k-1}\left(2\Gamma\right)\sin\left[\left(2k-1\right)\Omega t\right] \cong 2J_1\left(2\Gamma\right)\sin(\Omega t) \end{aligned} \quad (6)$$

for $\Gamma \ll 1$, where $J(2\Gamma)$ are Bessel functions of the first kind.

Using (3) we obtain the following expressions for the Stokes parameters S_1 and S_3:

$$\begin{aligned} S_1 &= J_0(2\Gamma)\cos 2\Phi_0 - 2J_1(2\Gamma)\sin(\Omega t)\sin 2\Phi_0 + 2J_2(2\Gamma)\cos\left(2\Omega t\right)\cos 2\Phi_0, \\ S_3 &= -J_0(2\Gamma)\sin 2\Phi_0 - 2J_1(2\Gamma)\sin(\Omega t)\cos 2\Phi_0 - 2J_2(2\Gamma)\cos\left(2\Omega t\right)\sin 2\Phi_0. \end{aligned} \quad (7)$$

The Stokes parameters S_1 and S_3 have a steady term, a harmonic term with the frequency equal to the modulation frequency, Ω, and a harmonic term with the double of the frequency of modulation. The phase difference between the harmonic components of the Stokes parameters is not constant in time, because their frequencies are different. The amplitudes of various harmonics are dependent upon the corresponding order Bessel functions of 2Γ. The harmonic structure of the S_1 and S_3 Stokes parameters depends on the d.c. bias in such way, that, for appropriate values of the U_0, all the odd order or all the even order harmonics may be suppressed in block.

Particular cases :

a) $U_0 = 0$. For no d.c. bias voltage applied to the crystal, $\Phi_0 = 0$, and from (7), it is obtained:

$$S_1 = J_0(2\Gamma) + 2J_2(2\Gamma)\cos\left(2\Omega t\right)$$
$$S_3 = -2J_1(2\Gamma)\sin(\Omega t) \tag{8}$$

The parameter S_1 has a frequency doubled with respect to that of the parameter S_3 and, implicitely that of the modulation frequency.

b) $U_0 = U_{\lambda/4}$. The modulator is biased to the middle point of the linear portion of the intensity transmission characteristic by applying a d.c. voltage or by inserting a quarter-wave plate in the light path, before or after the modulator, $2\Phi_0 = \frac{\pi}{2}$. From (7), it is obtained:

$$S_1 = -2J_1(2\Gamma)\sin(\Omega t)$$
$$S_3 = -J_0(2\Gamma) - 2J_2(2\Gamma)\cos\left(2\Omega t\right). \tag{9}$$

In this case the parameter S_3 has the double frequency related to the parameter S_1 and implicit related to the modulation frequency. The temporal evolution and the spectrum of the parameter S_1 is the same as the spectrum the parameter S_3 for no d.c. bias voltage applied to the crystal. The temporal evolution and the spectrum the parameter S_3 has the harmonic component in antiphase with the harmonic component of the parameter S_1 for no d.c. bias voltage applied to the crystal.

3. EXPERIMENTAL SETUP AND RESULTS

The optical scheme of the experimental setup used in the acquisition of the Stokes parameters is presented in Fig. 2.

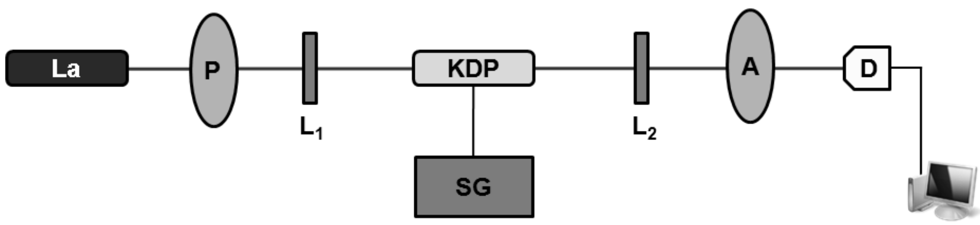

Fig. 2 – The experimental setup.

The experimental setup is formed of the He-Ne laser source ($\lambda = 632,8\text{nm}$) *La*, the linear polarizer *P*, the *KDP* modulator, connected to the harmonic voltage generator *SG* (whose frequency is Ω), the polarization state analyzer, the photodetector *D* and a computer. The linear polarizer *P* polarize light given by He-

Ne laser source along the crystallographic axis X of the modulator. The KDP crystal, modulates the light by means of longitudinal Pockels effect. To obtain quarter wave bias voltage, is inserted a quarter-wave plate L_1 in the light path, before or after the KDP modulator. The quarter-wave plate has neutral lines parallel to the induced principal axes of the KDP modulator. The polarization state analyzer contains the linear polarizer *A* and the quarter-wave plate L_2.

The photodetector *D* provides an electrical signal proportional to the intensity of the emergent light to the polarization state analyzer. The computer is used for the acquisition and processing experimental data taken from the photodetector. The Stokes parameters have been measured for low modulating voltage ($\Gamma \ll 1$) applied to the modulator. The spectral structure of the parameters S_1 and S_3 is obtained by means of fast Fourier transform.

The Stokes parameter S_1 is measured with the linear polarizer *A* set at azimuth 0° and 90° with respect to X axis, in the absence the quarter-wave plate L_2. In order to measure the Stokes parameter S_3 the quarter-wave plate L_2 must be inserted into optical path with the linear polarizer *A* set at 45° with respect to X axis.

The temporal evolution and the spectral structure of the parameters S_1 and S_3 are presented both in case no d.c. bias voltage and in case quarter wave bias voltage applied to the KDP modulator in the Figs. 3 and 4. The experimental results are consistent with the theoretical analysis.

4. CONCLUSIONS

In the present paper we have analysed the temporal evolution and the spectral structure of the Stokes parameters of the light modulated by longitudinal Pockels effect in KDP crystals, for low modulating voltage ($\Gamma \ll 1$) applied to the modulator. The principal features of the Stokes parameters of the modulated light can be summarized as follows:

The first parameter S_0 is equal to one, the crystal being non-absorbant, the third parameter S_2 is equal to zero, the incident light on the KDP modulator being linearly polarized along the X axis, at 45° to X' and Y' induced axes. The parameters S_1 and S_3 have a steady term, a harmonic term with the frequency equal to the modulation frequency and the another harmonic term with the double of the modulating frequency related to the frequency of modulation. The amplitudes of both harmonics are dependent on the corresponding order Bessel functions of 2Γ. The harmonic structure of the S_1 and S_3 Stokes parameters depends on the d.c. bias in such way, that, for appropriate values of the U_0, all the odd order or all the even order harmonics may be suppressed in block.

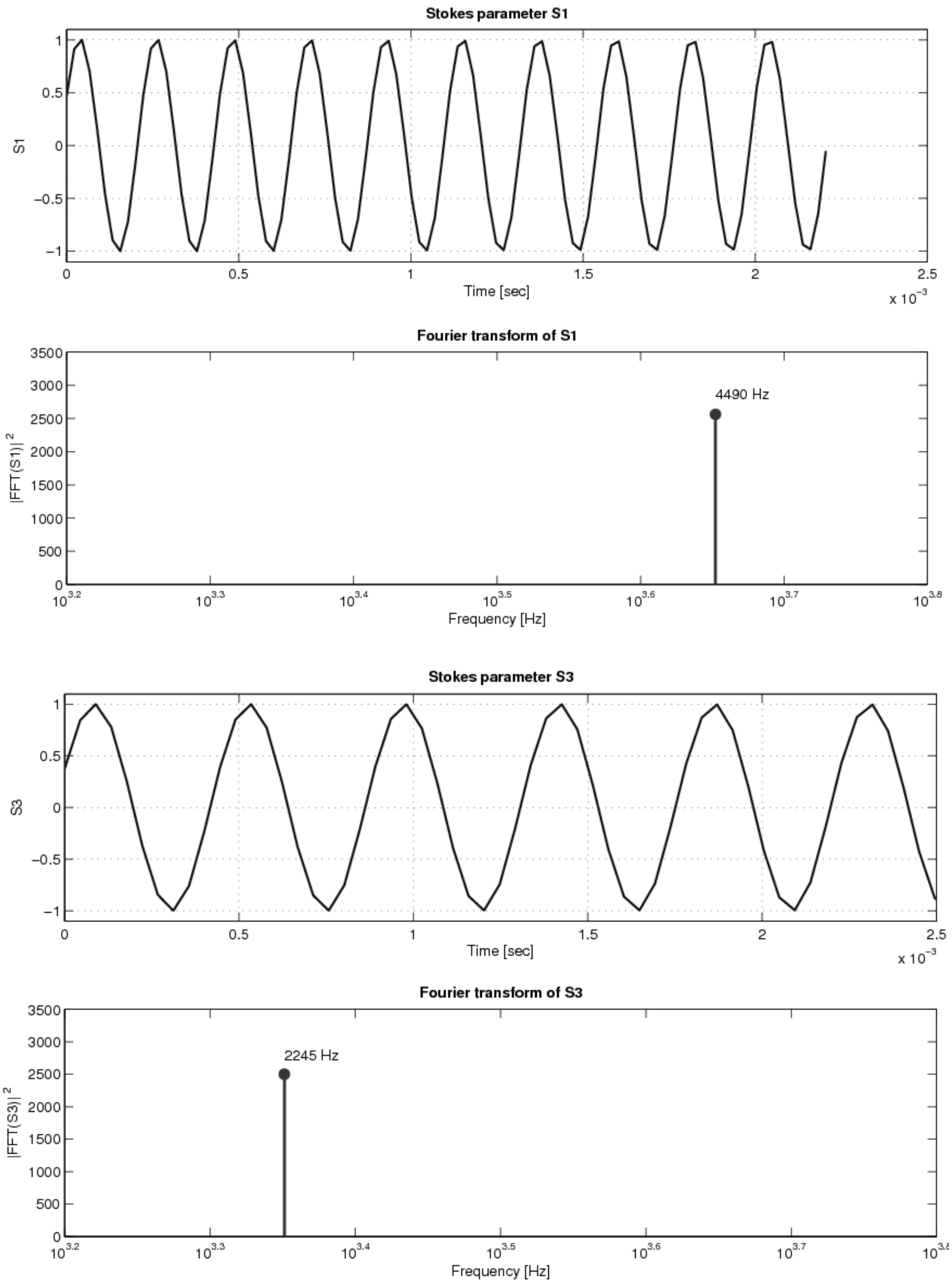

Fig. 3 – $U_0 = 0\text{V}$. The temporal evolution and the spectral structure of the Stokes parameters S_1 and S_3 .

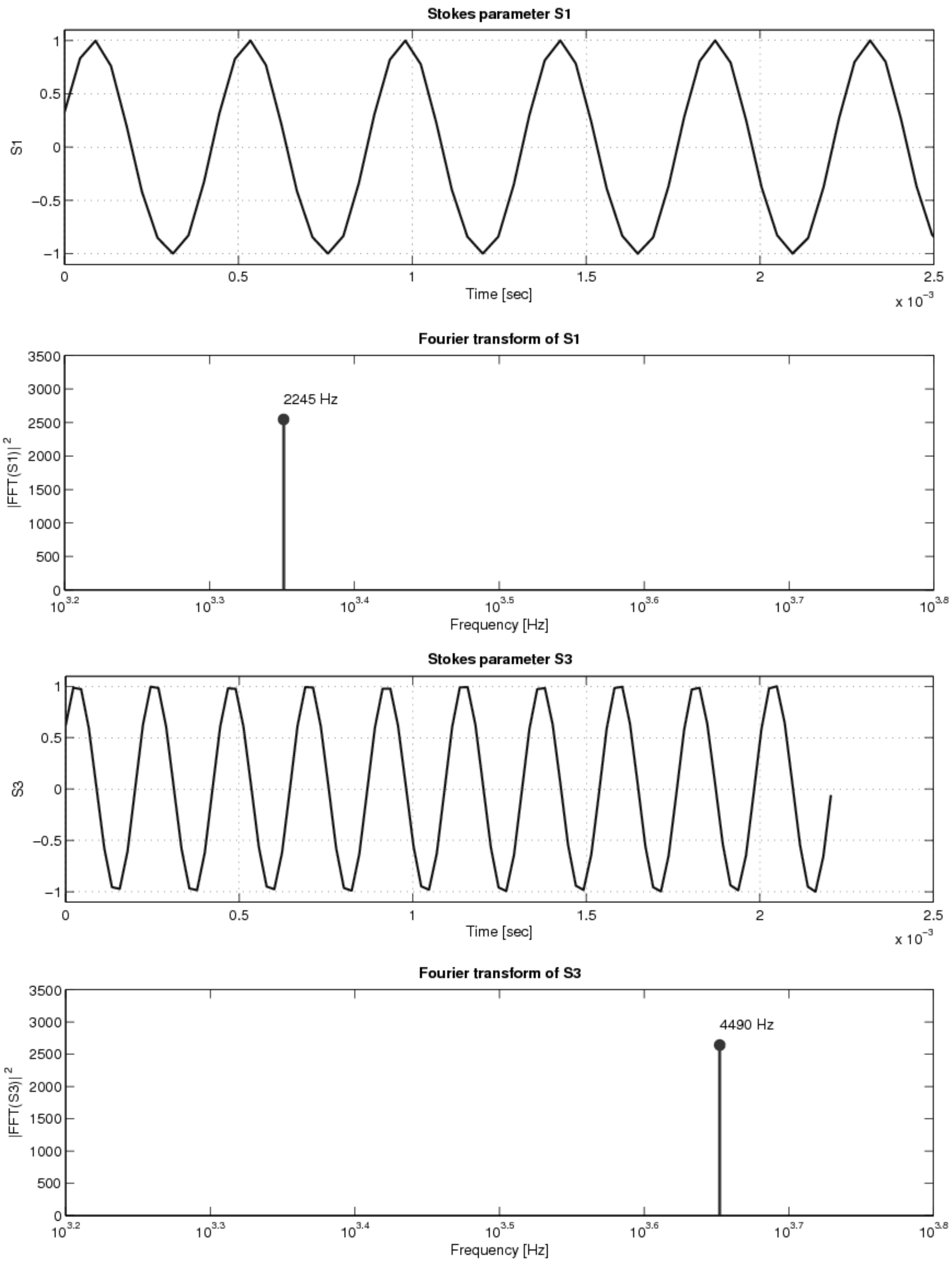

Fig. 4 – $U_0 = U_{\lambda/4}$. The temporal evolution and the spectral structure of the Stokes parameters S_1 and S_3 .

REFERENCES

1. T. Tudor, *Spectral analysis of the devices operators in polarization dynamics*, Journal of Modern Optics, **48**, *11*, 1169–1689 (2001).
2. T. Tudor, *Harmonic structure of light modulated by longitudinal electrooptic effect in crystals of class 42m*, J. Optics (Paris), **14**, *3*, 161–168 (1983).
3. T. Tudor, *Pauli algebraic analysis of polarized light modulation*, Appl. Opt., **47**, *14*, 2721–2727, (2008).
[4. L.P. Svirina, *A four-frequency anisotropic-cavity ring gas laser*, Quantum Semiclass. Opt., **10,** 213–222 (1998).
5. L.P. Svirina, Gudelev V G, Zhurik Yu P, *Spontaneous pulsations in gas class-A lasers with weakly anisotropic cavities*, Phys. Rev. A, **56**, 5053–5064 (1997).
6. A.P. Voitovich, A.M. Kul'Minskii, V.N. Severikov, *Nonlinear dynamics of laser systems at intracavity modulation of polarization*, Optics Communications, **126**, 152–166 (1996).
7. P. Paddon, A. D. May, E. Sjerve, G.Stephan, *Theory of the polarization states of quasi-isotropic lasers: A reformulation in terms of Stokes parameters*, **75**, *3*, 167–180 (1997).
8. J. Badoz, M. P. Silverman, J. C. Canit, *Wave propagation through a medium with static and dynamic birefringence: theory of the photoelastic modulator*, J. Opt. Soc. Am. A, **7**, 672–682 (1990).
9. T. Tudor, *The evolution of the light polarization state, along a KDP electro-optical modulator*, Optik, **109**, *1*, 27–34 (1998).
10. T. Tudor and I. Vinkler, *Time-varying coherency matrices and spectral coherency matrices*, Pure Appl., **7**, 1451–1457 (1998).
11. A.D. May, P. Paddon, E. Sjerve, G. Stéphan, *An alternative interpretation of the Zeeman and Faraday laser*, Phys. Rev. A, **53**, 2829–2841 (1996).
12. B.H. Billings, *The electo-optic effect in uniaxial crystals of the type* XH_2PO_4, J. Opt. Soc. Am., **37**, 10, 197-808 (1949).
13. Ajoy K. Ghatak, K. Thyagarajan, *Optical electronics*, Cambridge University Press, Cambridge, 495-497 (1989).
14. D.H. Goldstein, *Polarized light,* Marcel Dekker, New York, 2003, p. 103.
15. N.W McLachlan, *Bessel Functions for Engineers*, Oxford University Press, London, 1941.