Romanian Reports in Physics, Vol. 63, Supplement, P. 1105–1119, 2011

Dedicated to Professor Marin Ivaşcu's 80th Anniversary

CORRELATIONS OF LOW-ENERGY EXCITED STATES IN EVEN-EVEN NUCLEI

D. BUCURESCU[1,a], G. CĂTA-DANIL[2], M. IVAŞCU[1], N.V. ZAMFIR[1]

[1]"Horia Hulubei" National Institute for Physics and Nuclear Engineering , 30 Reactorului str., RO-077125, Bucharest - Măgurele, Romania

[a]Email: *bucurescu@tandem.nipne.ro*

[2]Physics Department, University Politehnica, Bucharest, Romania

Received August 1, 2011

Abstract. Simple and compact correlations are reported for the excitation energies of the lowest 0_2^+, 0_3^+, 2_2^+, 2_3^+, 3_1^+ and 4_2^+ states in all even-even nuclei, both with the energy ratio $R(4/2) = E(4_1^+)/E(2_1^+)$ from the yrast band, and between different such energies. The most compact such correlations are presented and discussed.

Key words: Even-even nuclei, low-energy excited states, energy correlations.

PACS: 21.10.-k, 21.10.Re, 21.60.Ev

1. INTRODUCTION

The way the nuclear structure evolves with the number of nucleons is an old subject, which is still actual as new experimental data accumulate for nuclei situated farther away from the valley of stability. This evolution has been mainly studied in the collective even-even nuclei, that display a relatively "standard" level structure at low excitation energies. One way to follow the structure evolution is to observe the systematics of collective observables, and correlations between them, such as, for example, the correlation of the energies of the first excited 4^+ and 2^+ states (denoted by 4_1^+ and 2_1^+, respectively), that led to the discovery of the spherical-deformed ground-state phase transitions in nuclei $[1,2]$. Interesting correlations were also found between ratios of energies in the quasi-band structures of collective even-even nuclei. Initially, this kind of correlation was observed for the ground state quasiband [4], but it was later shown that it is "universal", that is, it is obeyed by all the band structures in all (even-even, odd-mass, and odd-odd) nuclei [5]. Finally, another direction of interest is to study the intrinsic excitation modes and their evolution (for example, find out the nature of the 0_2^+ state – the first excited 0^+ state [3]).

The first kind of investigations is usually related with some signature of the nuclear collectivity (or shape), such as the ratio $R(4/2) = E(4_1^+)/E(2_1^+)$ defined for the ground state (or yrast) structure. The correlation(s) between intrinsic excitations and collective observables (such as those related to the nuclear equilibrium shape, like $R(4/2)$), are poorly studied. An example of such an attempt is Ref. [6], where a simple and compact correlation was found between the excitation energy (normalized to that of the 2_1^+ state) of the 0_2^+ state and the yrast energy ratio $R(4/2)$.

In this article we investigate correlations between the excitation energies of the lowest excited states of the even-even nuclei. Besides the usual 2_1^+ and 4_1^+ states, we consider the following excited states: 0_2^+, 0_3^+, 2_2^+, 2_3^+, 3_1^+ and 4_2^+, which are experimentally known in a large number of nuclei. Because we study only these rather common excitations, all of positive parity, in the following we drop the parity and simply refer to, *e.g.*, the 2_2^+ state as 2_2. The correlations discussed here represent preliminary, phenomenological results, and are discussed here only at an empirical level. An attempt to better understand these correlations will be made in a forthcoming publication.

2. THE ANALYZED DATA SET

The experimental excitation energies for the mentioned states were extracted from the latest version (as of November 2010) of the ENSDF database [7]. All even-even nuclei with Z between 8 and 100 and having at least one more known excited state from the set $S = \{0_2^+, 0_3^+, 2_2^+, 2_3^+, 3_1^+, 4_2^+\}$ besides the 2_1^+ and 4_1^+ states, were considered. The "non-collective" nuclei (that is, those with $R(4/2) < 2.0$) were also included, thus covering the whole range of nuclear structures. In constructing the correlations between the energies of any two states from this set, entered all nuclei having that pair of states experimentally known.

The energies of the excited nuclear states vary widely with both the mass number and the structure of the nuclei (such as, *e.g.*, nuclei with closed shells or collective nuclei, usually in the middle of a major shell); thus, as an example, the energy of the 2_1^+ state varies from several tens of keV in the collective actinides, to about 6-7 MeV in the light nuclei. In order to be able to compare such very widely different nuclei, we will measure the energies on the scale of the $E(2_1^+)$ values, that is, instead of the energies of a state from the considered set, we will use the ratio between the energy of that state and the $E(2_1^+)$ energy: the notation will be, for example, $R(2_3) = E(2_3^+)/E(2_1^+)$. In this way, the scale is "standardized" for all nuclei, and, as concerns the $R(4/2)$ ratio, it has a range of variation only from about 1.0 (for "shell model" – closed shell nuclei), to 2.0 (for spherical vibrators) and 3.33 (for rigid rotors), also passing, within this range, through values typical to different critical shape-phase transition points.

3. ENERGY RATIO CORRELATIONS

Figure 1 shows the observed correlations between the energy ratios of the 0_2^+, 2_2^+, 2_3^+, and 3_1^+ states and the $R(4/2)$ ratio. The correlation $R(0_2)$ versus $R(4/2)$ was already observed and discussed in Ref. [6]. The other three states represented in this figure show similar correlation patterns, and the same is valid for the other excited states from our set S not explicitly shown in Fig. 1. One can see that these correlations, implying all nuclear structures, from closed-shell nuclei to rigid rotors, follow relatively compact trajectories. To emphasize this fact, for the first two correlations in the graph we have fitted smooth curves to the hyperbola-like data point trajectories. We found that the cosech function gives a good description to the data, therefore the formula used was the following:

$$y = \begin{cases} a_1 + b_1 \operatorname{cosech}(c_1 - x): & \text{for } x \leq x_0 \\ a_2 + b_2 \operatorname{cosech}(c_2 - x): & \text{for } x \geq x_0. \end{cases}, \tag{1}$$

where y represents the energy ratio of the given state, and x is $R(4/2)$, the two hyperbolic cosecant functions being smoothly joined at some point x_0. For the two fits shown in Fig. 1, the parameters are given in Table 1.

Table 1.

The values of the coefficients from (1), for the two correlations from (1), for the two correlations from Fig. 1 fitted with this expression. The values a_1, b_1, c_1, c_2, x_0 are determined from fit, while a_2 and b_2 are determined from the smooth joining condition.

Energy ratio	a_1	b_1	c_1	c_2	a_2	b_2	x_0
$R(0_2)$	0.60(10)	2.33(20)	3.47(2)	3.33(1)	8.46	0.23	3.27
$R(2_2)$	0.96(11)	1.87(19)	3.45(2)	3.33(1)	7.57	0.22	3.27

This type of correlations shows that the "normalized" nuclear level schemes (that is, scaled by $E(2_1^+)$, follow simple and rather compact trajectories. On the other hand, one should emphasize that we did not consider at all the possible nature of the excitations considered, but just sorted all of them in the same "class" (such as, the 0_2^+ states) according to their energy.

Next we study the correlations between the energy ratios of the other states from set S. These correlations show different degrees of scattering of the data points. Therefore, we have selected for further considerations only the most compact correlations. Fig. 2 shows the correlations between $R(0_3)$, $R(2_3)$, and $R(2_2)$ with $R(0_2)$. The most compact is the $R(2_3)$ versus $R(0_2)$ correlation, which is very close to a straight line. The correlation of $R(2_2)$ with $R(0_2)$ shows the largest scattering of the points, but one can distinguish two distinct branches, namely one with $R(2_2) > R(0_2)$, and the other with $R(2_2) < R(0_2)$, the former representing, very likely, the nuclei in which the two states belong to the quasi-β band.

Figure 3 shows the best correlations observed for $R(3_1)$ and $R(4_2)$, namely those with $R(2_2)$. In both cases, one can see that the nuclei with $R(2_2) > R(0_2)$ and those with $R(2_2) < R(0_2)$ practically follow the same trajectory.

We have not made yet any assumption about the nature of the different states in set S. Nevertheless, the most compact correlations that we have selected indicate certain relationships between different states. For example, the correlations of the 3_1 and 4_2 states with the 2_2 state might indicate that in most of the cases these states belong to the gamma vibrational quasi-band as assumed by Sakai [8]. On the other hand, as shown in [1], and later in [9, 10], the collective nuclei with $R(4/2)$ from about 2.0 (vibrators) to about 3.15 (near rotors) show an yrast structure well described by an anharmonic vibrator (AHV) with an almost constant anharmonicity of the two-phonon 4^+ state (the 4_1^+ state). This finding prompted a checking of the multi-phonon picture for non-yrast states as well: in Ref. [11] this check was made for the states of the quasi-gamma band. In the same idea, we will compare our correlations for the 3_1 and 4_2 states with predictions of the multi-phonon model for the vibrational nuclei [12]. Within this model, the two-phonon multiplet is described by energies $E(I) = 2E(2_1) + \varepsilon_i$, with $I = 0,2,4$ and $i = 0,2,4$ respectively, ε_i being anharmonicities of these states (we assume that the three states correspond to our $0_2,\ 2_2$ and 4_1 states).

For the three-phonon multiplet, we specify the energy expressions of the states with spin 0, 3, and 4, [12] which we will check against our $0_3,\ 3_1$, and 4_2 states:

$$\begin{aligned} E(0_3) &= 3E(2_1) + 3\varepsilon_2 \\ E(3_1) &= 3E(2_1) + \frac{15}{7}\varepsilon_2 + \frac{6}{7}\varepsilon_4 \\ E(4_2) &= 3E(2_1) + \frac{11}{7}\varepsilon_2 + \frac{10}{7}\varepsilon_4 \end{aligned} \tag{2}$$

¿From these equations one can deduce that in the AHV description we must have:

$$\begin{aligned} R(3_1) - \frac{6}{7}R(4/2) - \frac{15}{7}R(2_2) &= -3 \\ R(4_2) - \frac{10}{7}R(4/2) - \frac{11}{7}R(2_2) &= -3 \end{aligned} \tag{3}$$

In Fig. 4 we show how the experimental data compare to these predictions. Although these predictions are not relevant for the pre-collective nuclei ($R(4/2) < 2.0$), they are also included in the graph. These nuclei continue smoothly the trajectory from the collective nuclei, cutting through the AHV value. On the other hand, one can see that only up to $R(4/2) \approx 2.5$ the experimental values are close to the AHV value. At about this $R(4/2)$ value, we observe a change of the slope of the correlation, followed by another one, around $R(4/2) \approx 3.26$. It is conspicuous that this last value corresponds to the x_0 value used in the fits in Fig. 1 (see Table 1).

Until now we have studied energy ratio correlations by considering together all nuclei, without regarding at their structure. If we classify the nuclei according to the three broad classes: precollective ($R(4/2) \leq 2.0$), AHV – anharmonic vibrators ($R(4/2)$ between 2.0 and 3.05), and rotors ($R(4/2)$ larger than 3.05), then some of these correlations become much better defined (smaller scattering of the points around some average trajectory).

Fig. 5 shows the impact of this classification on the correlation between the ratios $R(2_2)$ and $R(0_2)$. It is further useful in these correlations to split the nuclei according to the relative position of the two states ($E(2_2) < E(0_2)$ and $E(2_2) > E(0_2)$), the later situation being likely to correspond to the beta vibration band in deformed (rotor) nuclei. One can see that, at least in the AHV and ROT (rotor) nuclei, the correlation splits into two patterns, that are relatively well described, separately, by a straight line. For the rotational nuclei (ROT) with $E(2_2) > E(0_2)$ one can see that the very well defined correlation is $R(2_2) \approx R(0_2) + 1$, or, in terms of energy, $E(2_2) = E(0_2) + E(2_1)$, which is expected if the 0_2 state is the head of the $\beta-$band and this band has a moment of inertia equal to that of the ground band.

Figures 6 and 7 show the same data as Fig. 2, but with nuclei classified in the three categories. In each case, to show the general trend, we have fitted a straight line to the data, with the resulting parameters shown in the graph. The 0_3 state shows again the largest scattering of the points (Fig. 6). Nevertheless, for the rotational case, one gets a relatively compact trajectory described by the straight line $E(0_3) \approx E(0_2) + 4E(2_1)$. For the 2_3 state (Fig. 7), one gets, for the rotational case, $E(2_3) \approx E(0_2) + 2E(2_1)$. Figures 8 and 9 show the same data as in Fig. 3 (for the 3_1 and 4_2 states) for the three categories of nuclei, and also the comparison with the AHV prediction. The correlations for AHV and rotational nuclei are rather compact and can be well described by straight lines. In the rotational case we have the remarkable relations $E(3_1) \approx E(2_2) + E(2_1)$, and $E(4_2) \approx E(2_2) + 2E(2_1)$.

4. CONCLUSIONS

In this work we presented correlations between energy ratios for the states 4_1^+, 0_2^+, 0_3^+, 2_2^+, 2_3^+, 3_1^+ and 4_2^+, in all even-even nuclei with Z between 8 and 100. The general image of the correlations between the energy ratio of these states (their energy normalized to that of the 2_1^+ state) and the $R(4/2)$ ratio, for all nuclei, is that of a simple, relatively compact trajectory, as found in the previous study of the 0_2 states [6]. For the correlations between the energy ratios of different states we have selected the most compact ones and represented them by very simple functions. Further simplicity of these correlations is obtained if the nuclei are split according to their $R(4/2)$ in the categories of precollective, AHV, and rotational nuclei. A comparison with the predictions of the vibrational spherical nuclei model (anharmonic vibrator) shows only limited agreement for certain classes of nuclei.

These results are preliminary and only an empirical discussion was made at this stage. We have not tried to assess neither the structure of the intrinsic excitations (such as the 0_2, 0_3, 2_2, and 2_3 states, etc.), nor to discuss the situation of the particular nuclei that deviate significantly from the average trajectories (as made, for a few cases, in ref. [6]). A discussion of such details will be made in a forthcoming publication.

Some of the observed more compact correlations may be useful to predict the energy of excited states in nuclei where they are not known.

REFERENCES

1. R.F. Casten, N.V. Zamfir, and D.A. Brenner, Phys. Rev. Lett., **71**, 227 (1993).
2. R.F. Casten, D. Kuznezov, and N.V. Zamfir, Phys. Rev. Lett., **82**, 501 (1999).
3. P.E. Garrett, J. Phys. G: Nucl. Part. Phys., **27**, R1 (2001).
4. C.A. Mallmann, Phys. Rev. Lett. **2**, 507 (1959).
5. D. Bucurescu, N.V.Zamfir, G.Căta-Danil, M.Ivaşcu, and N. Mărginean, Phys. Rev. **C78**, 044322 (2008); Acta Phys. Polonica **40**, 503 (2009).
6. W.-T. Chou, G.Căta-Danil, N.V. Zamfir, and N. Pietralla, Phys. Rev. **C64**, 057301 (2001).
7. Evaluated Nuclear Structure Data File (ENSDF), maintained by the National Nuclear Data Center, Brookhaven National Laboratory.
8. At. Data Nucl. Data Rables **31**, 399 (1984).
9. N.V. Zamfir and R.F. Casten, Phys. Rev. Lett. **75**, 1280 (1995).
10. D. Bucurescu and N. Mărginean, Phys. Rev. Lett. **79**, 31 (1997).
11. W.-T. Chou, R.F. Casten, N.V. Zamfir, D.S. Brenner, and D. Bucurescu, Nucl. Phys. **A580**, 33 (1994).
12. D.M. Brink, A.F.R. De Toledo Piza, and A.K. Kerman, Phys. Lett. **19**, 413 (1965).

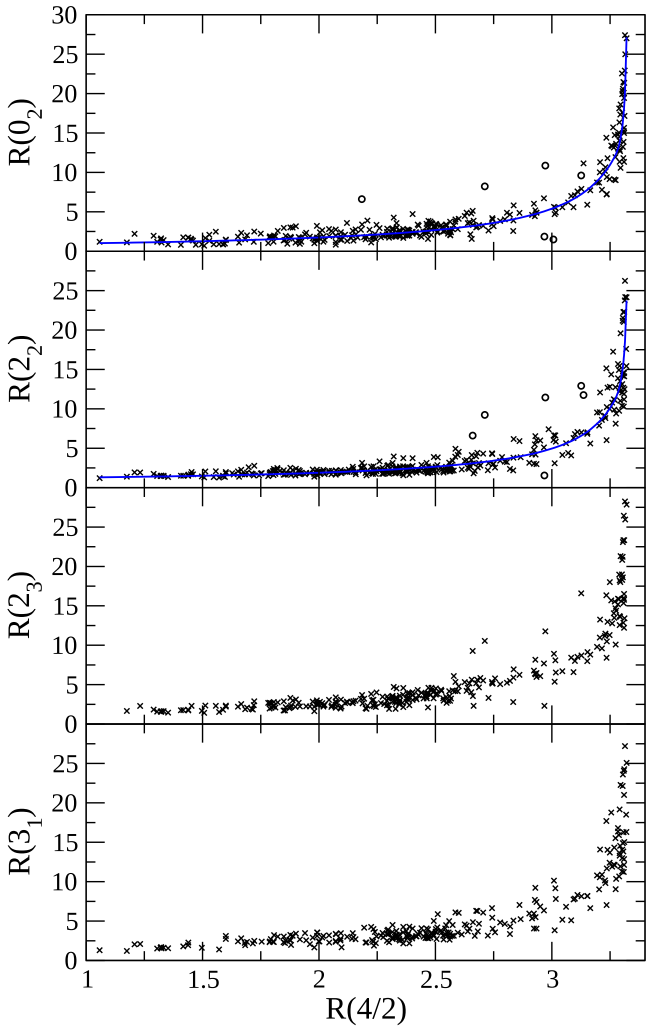

Fig. 1 – Examples of correlations of some energy ratios with $R(4/2)$. In the case of $R(0_2)$ and $R(2_2)$, the curves represent fits to the data with formula (1) (two smoothly joined $\cosh$ functions), excluding the data points with large deviations from the average behaviour (represented by circles). There are 323 data points for $R(0_2)$, 362 for $R(2_2)$, 286 for $R(2_3)$, and 265 for $R(3_1)$, respectively.

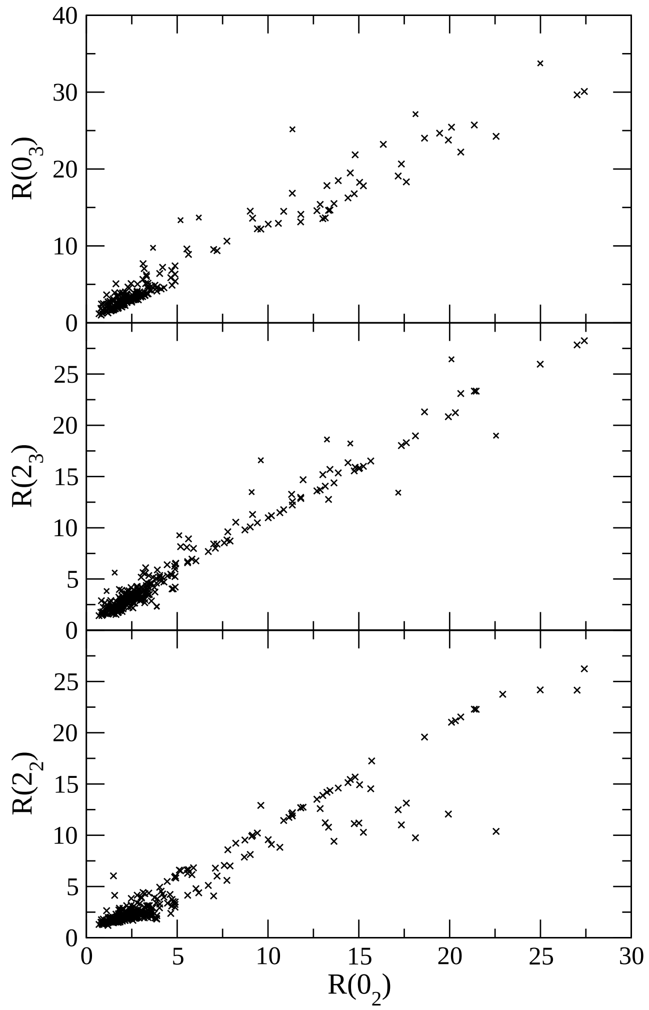

Fig. 2 – The correlations $R(0_3)$ versus $R(0_2)$ (204 points), $R(2_3)$ versus $R(0_2)$ (259 points), and $R(2_2)$ versus $R(0_2)$ (310 points).

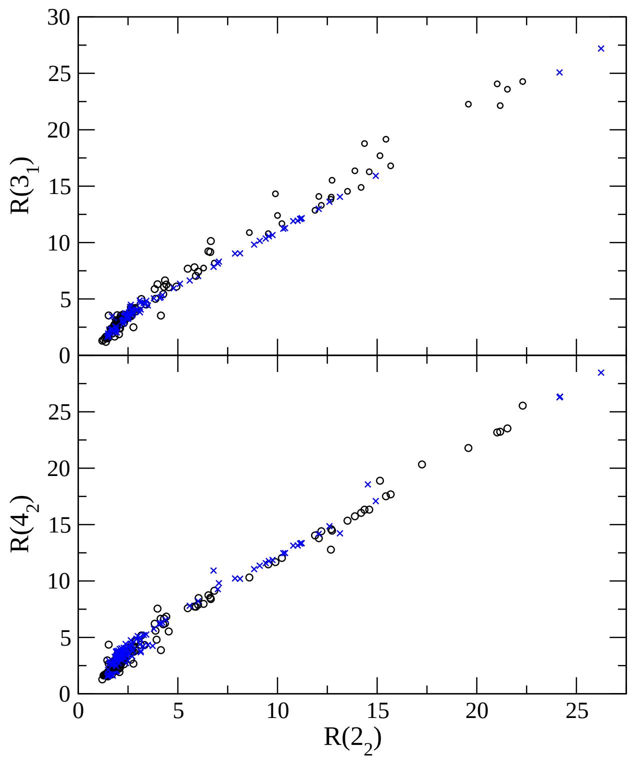

Fig. 3 – The correlations $R(3_1)$ versus $R(2_2)$ (249 points), and $R(4_2)$ versus $R(2_2)$ (284 points). The circles represent nuclei with $R(2_2) > R(0_2)$, and the crosses the nuclei with $R(2_2) < R(0_2)$.

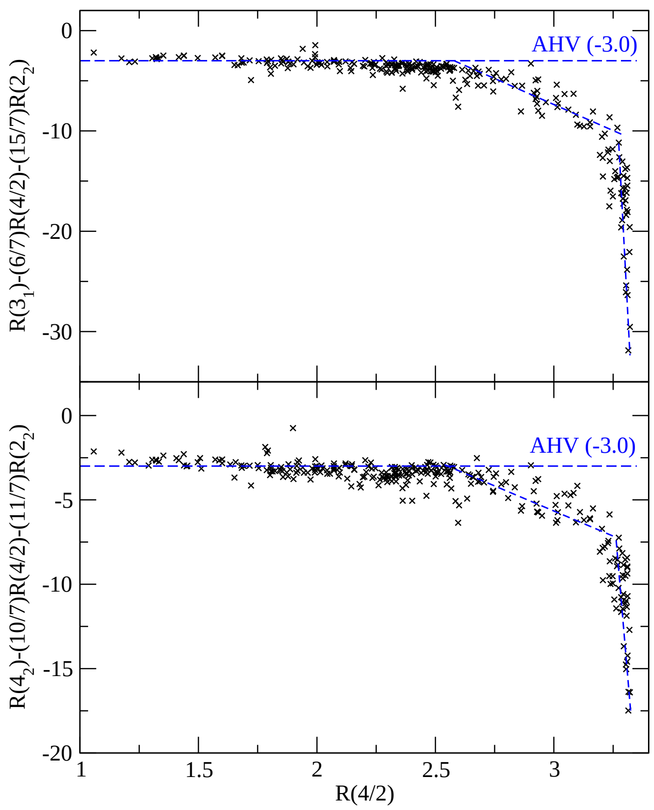

Fig. 4 – Comparison of the experimental data related to the 2_2, 3_1 and 4_2 states assumed to belong to the three-phonon multiplet, to the expressions (3) for the anharmonic vibrators. The observed changes of slope at $R(4/2)$ values of about 2.5 and 3.26 are highlighted by hand-drawn dashed lines.

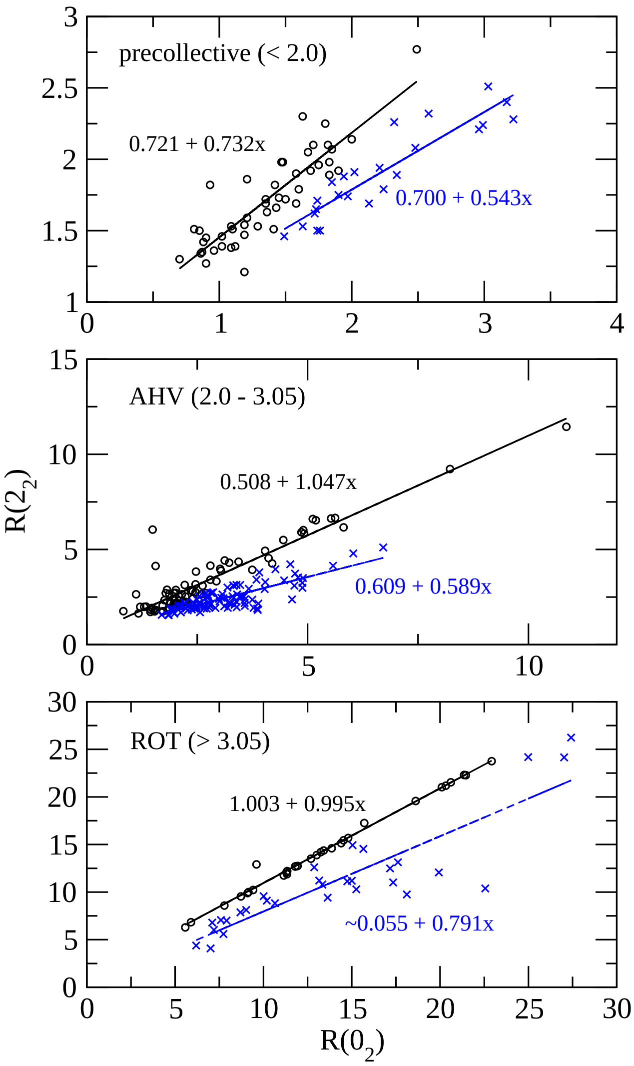

Fig. 5 – Correlation between the energy ratios of the 2_2 and 0_2 states. Nuclei with $R(2_2) < R(0_2)$ and $R(2_2) > R(0_2)$ are represented separately by crosses and circles, respectively. Also, the correlation is shown separately for the three large classes of nuclei, precollective, AHV, and rotational. The data are fitted with straight lines, as indicated.

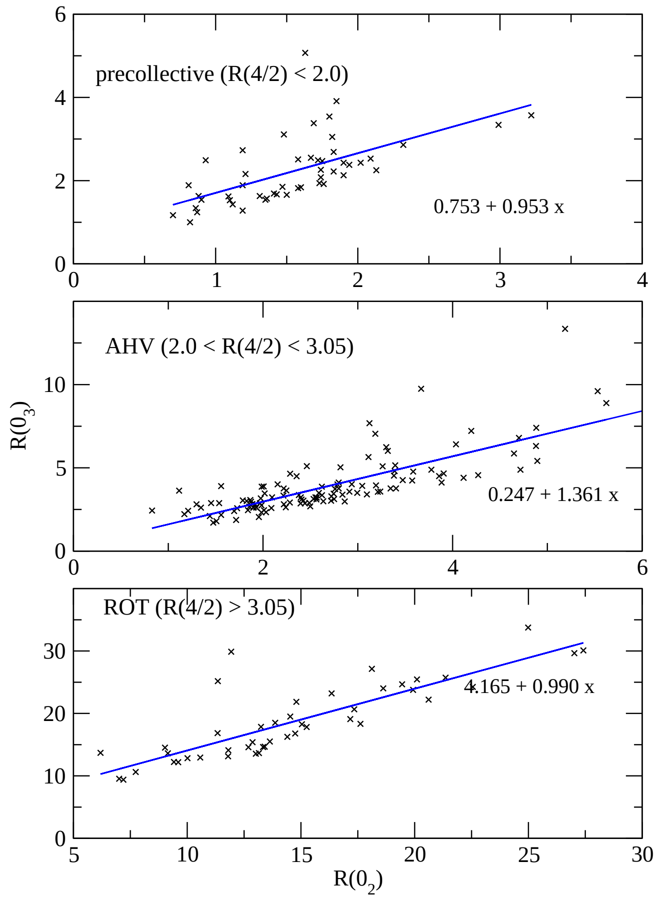

Fig. 6 – Same data for the 0_3 state as in Fig. 2, but with nuclei split according to the three indicated categories.

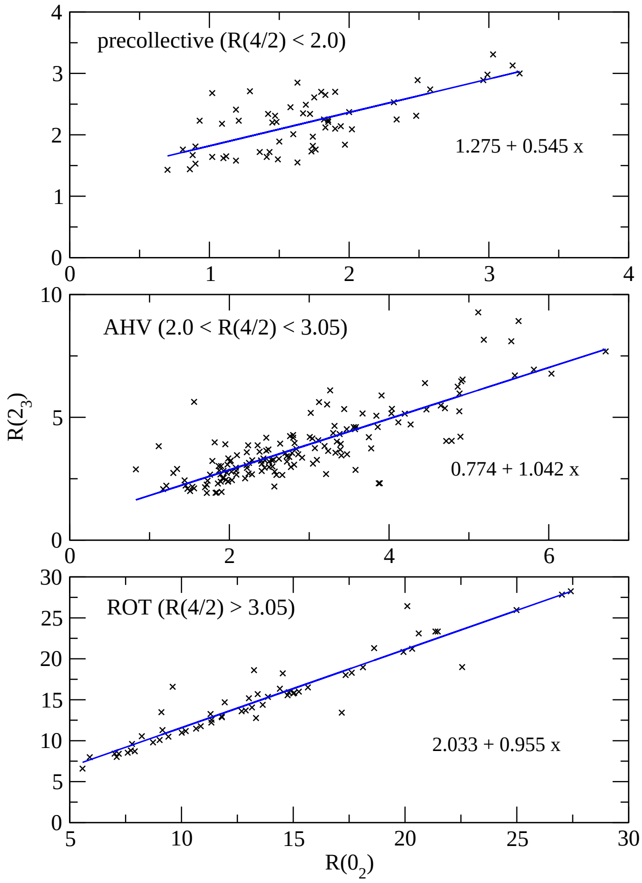

Fig. 7 – Same as Fig. 6 but for the 2_3 state.

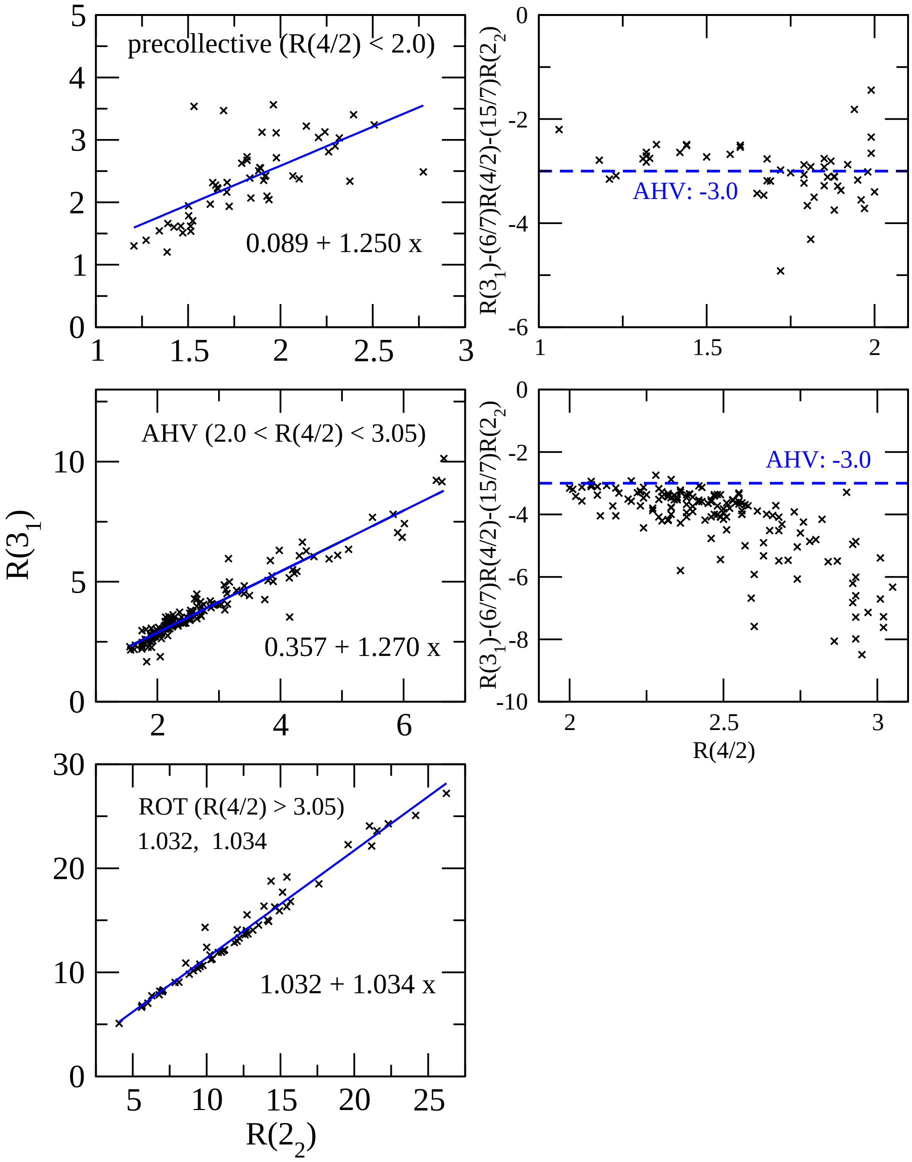

Fig. 8 – The same data for the 3_1 state as in Fig. 3, but with nuclei split according to the three indicated categories. A comparison with the AHV predictions, as in Fig. 4, is again shown for the precollective and AHV nuclei.

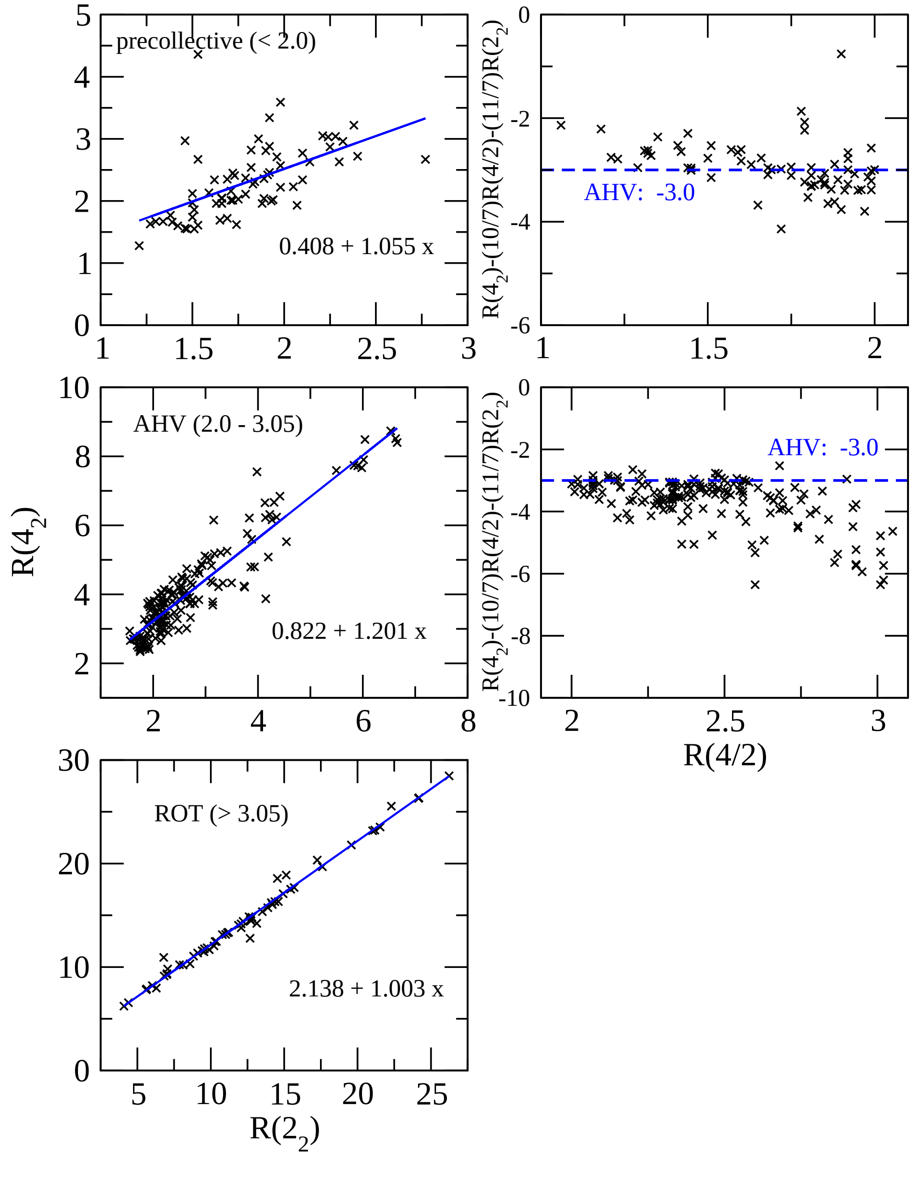

Fig. 9 – Same as Fig. 8, but for the 4_2 state.