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Dedicated to Professor Marin Ivaşcu's 80^{th} Anniversary

THE BOUNDARY CONDITION DISPLACEMENT FOR WEAKLY BOUND AND QUASISTATIONARY STATES

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Abstract. The boundary condition displacement (Thomas-Ehrman shift) topic, of interest for spectroscopic study of mirror nuclei, is discussed according to R-Matrix theory and Tombrello's one body potential model. The relative shift for mirror levels is larger when asymptotic behavior of bound neutron and unbound proton wave functions are drastically different. One discusses some aspects of spatial extension of near-threshold states in relation to Thomas-Ehrman shift. The spatial extension outside channel radius of the wave function is described in terms of R-matrix compression factor. This concept is slightly generalized resulting into equation of "single channel resonance"; this equation describes both bound and quasistationary states. The use of natural boundary condition and of Thomas approximation for channel logarithmic derivative results into modification of Thomas-Ehrman shift formula.

Key words: Thomas-Ehrman shift, R-matrix theory.

1. INTRODUCTION

The Thomas-Ehrman level displacement becomes of current interest for nuclear astrophysics studies if it is dealing with excited states in mirror nuclei. The states in the two mirror nuclei belong to an isospin doublet and, due to charge symmetry of nuclear force, their spectroscopic properties (energy, spin and parity, spectroscopic strength) should be identical. The spectroscopic data on states in neutron rich nucleus could be accordingly translated to the analog states in proton rich nucleus. However the isospin symmetry is not an exact one and this results into level displacement between mirror pairs states. For example, at higher excitation energy, the neutron states can be bound while the mirror proton states are unbound. The main problem is to establish one to one correspondence between mirror states at higher excitation energies; for this purpose the relative level displacements should be calculated. The level displacement is due to Coulomb force, to isospin non-conserving interaction and to change in boundary condition. The first two sources of displacement are taken into account in Shell Model calculations, (Brown, unpublished). The "boundary condition displacement" was firstly studied in framework of R-matrix

theory [14], and it is called "Thomas-Ehrman shift" [6, 17]. Later on, Tombrello (1966) has proposed a one body potential model to calculate this level displacement. Both approaches are now used in nuclear astrophysics studies (*e.g.* [9, 11], respectively [1, 5, 10]). In this report both variants are analyzed. One discusses possible connection of Thomas-Ehrman shift to spatial extension of threshold states, as well as to single channel resonances related to bound and quasistationary states.

2. THOMAS-EHRMAN SHIFT ACCORDING TO R-MATRIX THEORY

The Thomas-Ehrman shift describes the level displacement between mirror levels of an isospin doublet [6, 14, 17]. It originates in coupling of (same) internal state to different reaction channels. The reaction channel wave functions play an essential role in study of study of channel induced shifts and threshold effects [14].

The Schroedinger equation for the radial wave function of the relative motion in channel region has two independent solutions, the Coulomb regular F_l and Coulomb irregular G_l (at origin), which define the *in* I_l and *out* O_l channel wave functions, [14]

$$I_l^+ = (G_l - iF_l)\exp(i\omega_l) \tag{1}$$

$$O_l^+ = (G_l + iF_l)\exp(-i\omega_l) \tag{2}$$

For negative energy (closed) channel only *out* type solution occurs in applications and it is just "exponentially decaying" Whittaker function

$$O_l^- = W(-\eta, l+1/2, 2\rho) \simeq \exp(-\rho - \eta ln 2\rho) \tag{3}$$

The quantities used are, as usual, $\rho = kr$, $\eta = Z_1 Z_2 e^2 m/\hbar k$, (coulombian parameter), r- radial distance of reaction fragments, $k = \sqrt{2mE}/\hbar$ - channel wave number, l - angular momentum and coulombian phase shift $\sigma_l = \arg\Gamma(l+1+i\eta)$

$$\omega_l = \sigma_l - \sigma_0, \quad \sigma_l = \arg\Gamma(l+1+i\eta) \tag{4}$$

The logarithmic derivative of *out* type of wave function O_l

$$L_l = \rho(O_l^{'}/O_l) = S_l + iP_l \tag{5}$$

defines the shift S_l and penetration P_l factors of open $(+)$ and closed $(-)$ channels in terms of Coulomb regular and irregular functions

$$S_l^+ = \rho A^{'}/A \tag{6}$$

$$S_l^- = \rho W^{'}/W \tag{7}$$

$$P_l^+ = \rho/A \tag{8}$$

$$P_l^- = 0 \tag{9}$$

$$A = (F^2 + G^2)^{1/2} \tag{10}$$

where $(')$ is differentiation with respect to ρ. Of peculiar interest are penetration P_0 and shift S_0 factors for neutron s-wave channel, $P_0 = \rho$, $S_0^+ = 0$, $S_0^- = -\rho$, [14]. The shift factors for charged particles are monotone energy dependent near thresholds [16].

The energy dependence of neutron and proton s-wave shift factor is presented in Fig.1 for the scattering on ^{26}Al ($A = 26$, $Z = 13$) at radius $r = 1.4A^{1/3}$.

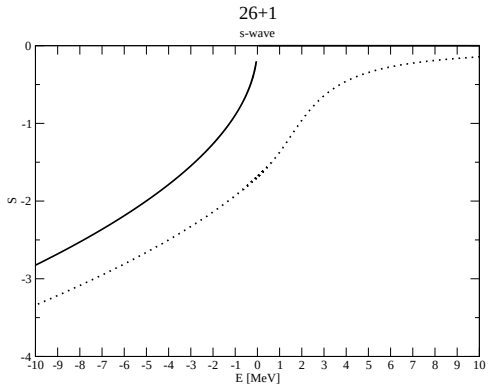

Fig. 1 – Neutron s-wave shift factor (solid line) and proton s-wave shift factor (dotted line) according to R-matrix.

The resonance energy E_r is defined in R-matrix theory by condition

$$E_r - E_\lambda - \Delta_\lambda = 0 \tag{11}$$

where E_λ is the eigenenergy of the R-matrix state λ defined in the internal region. The level shift Δ_λ = - $\gamma_\lambda^2(S(E_r) - b)$ is defined in terms of the reduced width γ_λ, of the channel shift function $S(E)$ and of the boundary condition b at channel radius a. Let us consider the two states of an isospin doublet belong to nuclei n and p. Assuming that the internal wave functions of the doublet are identical, both reduced widths $\gamma_{\lambda n}$ and $\gamma_{\lambda p}$ are identical, $\gamma_{\lambda n}$ = $\gamma_{\lambda p}$ = γ_λ ; same for the boundary conditions b_p = b_n = b. The difference between the resonant energies of the two states of the doublet is

$$E_{rn} - E_{rp} = [E_{\lambda n} - E_{\lambda p}] + [\Delta_{\lambda n} - \Delta_{\lambda p}] \tag{12}$$

$$\Delta_{\lambda p} - \Delta_{\lambda n} = [S_n(E_{rn}) - S_p(E_{rp})]\gamma_\lambda^2. \tag{13}$$

The quantity $[E_{\lambda p} - E_{\lambda n}]$ is the level displacement in the doublet due to different structure of the state λ in the two nuclei n and p. It is related to Coulomb internal energy and to isospin non-conserving (INC) nuclear effective interactions. One assumes that this component of the level displacement is same for pairs of excited (λ) mirror states as for the ground states of the two nuclei. The net level displacement

$\Delta_{\lambda p} - \Delta_{\lambda n}$ is Thomas-Ehrman shift; it is related to different couplings, via shifts functions, to p and n reaction channels and accounts esp. for the channel influence on level positions. Of peculiar interest is the case when a mirror state is bound (below neutron channel threshold) while its counterpart is unbound (above proton channel threshold). The neutron and proton shift functions have different energy dependences esp at corresponding thresholds. This results into threshold energy dependence of the Thomas-Ehrman shift.

This threshold concept was later on applied to analysis of other types of broken symmetries [7].

3. TOMBRELLO MODEL FOR SINGLE PARTICLE MOTION

Tombrello (1966) has developed the one-particle potential model in order to describe the energy shifts between components of an isospin doublet, [19]. This model is able to describe the most important two sources of the level shift, change in Coulomb energy and Thomas-Ehrman shift, due to proximity of threshold. The Tombrello model is presently used for calculus of Thomas-Ehrman shifts for light and light medium nuclei [1, 5, 10].

One consider two mirror nuclei both consisting of same core and an additional nucleon, either neutron or proton. Both neutron and proton single-particle states are calculated with same nuclear (Woods-Saxon) potential, proton being in addition subject of Coulomb potential. The calculated energies correspond to neutron bound state and to proton unbound state. The known experimental energy of neutron single-particle bound state results into establishing depth of nuclear potential. The proton resonant state is calculated with same nuclear potential as for neutron plus the Coulomb interaction. The radial wave functions of the proton in analogue state is not identical to that of corresponding neutron state. The Coulomb potential results into increase of radius of the proton over that of neutron. The energy of proton resonance is defined, thereafter, as energy at which the scattering phase-shift becomes $\pi/2$.

Apparently the Tombrello model is parallel to R-matrix Thomas-Ehrman model. This assertion could be verified in an numerical experiment. It is expected that for a neutron bound state the logarithmic derivative in Tombrello model is identical with that of Thomas-Ehrman model (8); both are related to same Whittaker function. The neutron reduced width should be the Wigner unit because this neutron state is a single particle one. In Tombrello model one calculates the proton resonant state parameters as resonance width. The proton reduced width is obtained from its resonant width, provided the single particle width is known. The proton penetration factor is proportional to proton single particle width $P_p \sim \Gamma_p$. Thereafter one should compare the proton reduced width with neutron reduced width (Wigner unit) and Tombrello pro-

ton shift factor with R-Matrix proton shift factor. The possible differences between $S_p\gamma_p^2$(Tombrello) and $S_p\gamma_p^2$ (R-matrix), will result into different boundary conditions displacements.

The Tombrello model is valid for those states which have large spectroscopic factors, *i.e.* single-particle character. Assuming the single-particle state is spread, by residual interaction, into actual nuclear states one has to modify the Tombrello procedure (*e.g.* [10]). One calculates the Thomas-Ehrman shift for every actual state with Tombrello model, in same way as for a single-particle one. This Tombrello shift is thereafter multiplied by corresponding spectroscopic factor, resulting into Thomas-Ehrman shift for each actual state. The final Thomas-Ehrman shift for a particular proton state is obtained by summing the Thomas-Ehrman shifts of those actual states which originate in same single-particle state.d, by residual interaction, into actual nuclear states one has to modify the Tombrello procedure (*e.g.* [10]). One calculates the Thomas-Ehrman shift for every actual state with Tombrello model, in same way as for a single-particle one. This Tombrello shift is thereafter multiplied by corresponding spectroscopic factor, resulting into Thomas-Ehrman shift for each actual state. The final Thomas-Ehrman shift for a particular proton state is obtained by summing the Thomas-Ehrman shifts of those actual states which originate in same single-particle state.

The 3 mechanism of a single-particle state consists, according to Lane (1969) "Line Broadening" theory, both from residual interaction and by change in boundary conditions. The single particle state is fragmented into actual nuclear states, whose reduced widths becomes renormalized. The broadening of single particle state due to residual interactions is described by the Shell Model, resulting in spectroscopic factors of actual nuclear states. For light-medium mass nuclei, $s-d$ configuration space, which include nuclei involved in Ne-Na and Mg-Al stellar cycles [5] the calculus of nuclear states (energies, spins, spectroscopic factors) is done with OXBASH code [4].

Figure 2 shows the energy dependence of the Tombrello single particle shifts, $\Delta\epsilon_{TE}$, in respect with the $2s1/2$ neutron energy. The $\Delta\epsilon_{TE}$ shifts were derived from hypothetical single particle mirror levels in ^{27}Al and ^{27}Si nuclei obtained within a one body potential model describing neutron/proton scatterings on a ^{26}Al core.

An open problem is how 'generalized' Tombrello model, (taking explicitly in account the residual interaction via spectroscopic factors of actual states,) could account for effect of boundary conditions. In this report we will pay attention to the change in boundary conditions effect on nuclear states; it is more visible for near-threshold states. The neutron logarithmic derivative near threshold is strongly energy dependent, *i.e.* in this energy range the boundary condition is significantly modified. An illustrative example for role of boundary conditions for a neutron near-threshold state was provided by Lane in study of single particle $E1$ matrix elements (giant

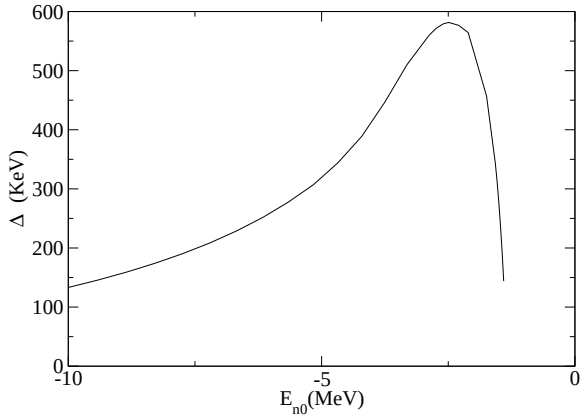

Fig. 2 – The $\Delta\epsilon_{TE}$ Tombrello shifts assuming pure $2s1/2$ and $1d5/2$ single particle states from a potential model for $A = 26$ as a function of the s-wave neutron energy.

dipole resonance). The authors [8] found that modification of boundary conditions for near-threshold neutron state is responsible for 'pigmy resonance' ie decoupling of $E1$ strength from GDR. Some aspects of threshold states, related to threshold dependence of logarithmic derivative and to "Line-Broadening" mechanism, are discussed in following chapter.

4. THRESHOLD STATES: SPATIAL EXTENSION AND THRESHOLD COMPRESSION

The Thomas-Ehrman shift is related to those states in an isomultiplet which have different asymptotic behavior; one state is bound while the other is unbound. The shift results from matching of same internal wave function to different channel logarithmic derivatives. This is why it is called also "boundary condition displacement". The shift for mirror levels is larger when asymptotic behavior of wave functions is drastically different; *e.g.* for bound neutron and unbound proton. The spatial extension outside channel radius plays an essential role in producing Thomas-Ehrman shift. On the other hand the spatial extension is a definitory characteristic of "threshold states". This characteristic is determined by channel reduced width and by channel logarithmic derivative which has a strong energy dependence near threshold, esp. near neutron threshold.

The threshold states, with energies near zero, display a large spatial extension as $\exp(-kr) \to 1$, because the wave number is vanishingly small, $(k \to 0)$. The concept of "threshold state" was introduced in nuclear physics by Baz (1959) [2], also [3]. According to Baz the "threshold states" have some specific properties: (*i*) they are connected with threshold of nucleus decay in two particles, (*ii*) they have a

cluster structure and the threshold particles are in a definite state of relative motion, and (*iii*) they have an anomalously large radius. The underlying physical picture of a threshold state is that of a state extended spatially out of channel radius. One defines in R-Matrix theory [12, 14] a "spatial extension factor" for renormalizing the wave function out of channel radius; it can be used here for description of a "threshold state". For negative energy, just below threshold, the "spatial extension factor" is, ($u(r)$- radial wave function),

$$\beta(E) = \frac{\int_0^a u^2(r)\,\mathrm{d}r}{\int_0^a u^2(r)\,\mathrm{d}r + u^2(a)\int_a^\infty [O^-(r)/O^-(a)]^2\,\mathrm{d}r} \tag{14}$$

On continuity reasons it is expected this result is extended for positive energy and large channel barrier; the integral is extended to the classical turning point a_t,

$$\beta(E) = \frac{\int_0^a |u|^2(r)\,\mathrm{d}r}{\int_0^a |u|^2(r)\,\mathrm{d}r + |u|^2(a)\int_a^{a_t} |O^+(r)/O^+(a)|^2\,\mathrm{d}r} \tag{15}$$

One proves that the wave function "spatial extension factor" can be related to R-Matrix "compression factor", defined below in terms of threshold channel logarithmic derivative and level reduced width.

The energy derivative of the shift function $\mathrm{d}S/\mathrm{d}E$ has a special physical meaning [14]. Let consider firstly the negative energy channels (bound states). The amplitude of external part (out of channel radius a) of the radial wave function is, up to a normalization constant (ANC), just O^- type of wave function, $u(r) \sim O^-(r)/O^-(a)$. The integral $\int u^2(r)\,\mathrm{d}r$ can be transformed into, [14]

$$\int_a^\infty u^2(r)\,\mathrm{d}r = \gamma^2 \mathrm{d}S^-/\mathrm{d}E \tag{16}$$

where $\gamma^2 = (\hbar^2/2ma)u^2(a)$ is Wigner reduced width. This quantity is used to define the threshold compression factor or the spatial extension of the radial wave function out of channel radius (16) or

$$\beta(E) = \frac{1}{1+\gamma^2 \mathrm{d}S^-/\mathrm{d}E} \tag{17}$$

For s- and p- wave neutrons, above $(+)$ and below $(-)$ threshold,

$$\gamma^2 \mathrm{d}S_0^+/\mathrm{d}E = 0 \tag{18}$$

$$\gamma^2 \mathrm{d}S_0^-/\mathrm{d}E = 1/\rho \tag{19}$$

$$\gamma^2 \mathrm{d}S_1^+/\mathrm{d}E = 2/(1+\rho^2)^2 \tag{20}$$

$$\gamma^2 \mathrm{d}S_1^-/\mathrm{d}E = (2+\rho)/(1+\rho^2)^2 \tag{21}$$

It is proved numerically the shift functions of protons for different partial waves are monotone increasing with energy, both below and above threshold, $\gamma^2 \mathrm{d}S_l/\mathrm{d}E > 0$.

One can conclude that, at least in these cases, the R-Matrix β factor (19) is subunitary; from here the denomination of "compression factor".

The threshold compression factor has a direct impact on neutron spectroscopic factor (reduced width).The problem of fragmentation of single particle state over actual nuclear states and of corresponding spectroscopic factors is subject of "Line-Broadening", (Lane 1969). One defines a special state, *e.g.* a single particle state, which has a large overlap on one reaction channel. The special state, bound or unbound, is fragmented over many actual states. The mechanisms responsible for line-broadening are both residual interactions (which act at level of actual states) as well as change in boundary conditions (acting esp on single particle states near threshold). The last mechanism results in threshold dependence of spectroscopic factor. It follows that the spectroscopic factor of near-threshold state follows the energy dependence of threshold compression factor.

The physical interpretation of this result is related to the alternative meaning of the compression factor, [12]. The R-Matrix compression factor is identical to wave function renormalization out of channel radius; it is the ratio of the usual normalization of the state over internal region to the normalization over all space including reaction channel. A large spatial extension of wave function in reaction channel corresponds to a small compression factor. The fragmentation of single particle state into actual states is determined by their overlap inside nucleus which, at its turn, depends on single particle state localization in the internal region. Small compression factor, *i.e.* small localization of single particle state inside nucleus, results in small overlap on actual states, *i.e.* small spectroscopic factor.

5. SINGLE CHANNEL RESONANCES

The channel equation $R_{nn}^{-1} = S_n^{(-)}$ defines the bound state in R-matrix theory [14]. Extended to positive energy channel, the corresponding state should be the quasistationary one. A pole in Collision Matrix is obtained by a condition which is analog to the bound state one, $R_{nn}^{-1} = L_n$; the logarithmic derivative L_n is the corresponding, at positive energy, of the shift function $S_n^{(-)}$ defined for negative energy.

The resonance condition, $1 - L_n R_{nn} = 0$, for nuclear reactions can be approached in different ways. If the energy dependence of logarithmic derivative L_n is a parametric one then the root of resonance equation becomes energy dependent; it is Kapur-Peierls approach to resonance. Another approach is in terms of complex energy pole H_π which is root of implicit equation $1 - L_n R_{nn}(H_\pi) = 0$. The boundary conditions are those of out waves at the state energy H_π (not at prescribed energy E). In nuclear studies it is convenient to relate the quasistationary state parameters

to the R-Matrix parametrization. For only one level, one obtains by means of Level Matrix A, [14]

$$\begin{aligned}(1-RL)^{-1} &= 1+L\gamma_\pi^2 A_{\pi\pi}\\ H_\pi &= E_\pi - i\Gamma_\pi - L\gamma_\pi^2\\ A_{\pi\pi}^{-1} &= H_\pi - E\end{aligned} \tag{22}$$

with γ_π- channel reduced width and E_π- resonance energy including channels couplings level shift and width Γ_π originating in complementary channels; for a single channel case $\Gamma_\pi = 0$.

It results in description of the resonance π in the n-channel

$$\frac{\gamma_{\pi n}^2}{E_\pi - E - L_n\gamma_{\pi n}^2 - i\Gamma_\pi} \tag{23}$$

The resonance denominator has a peculiar property due to energy dependence near threshold of the n-channel logarithmic derivative $L_n = S_n + iP_n$ (S_n- Shift Factor, P_n- Penetration Factor). In Thomas approximation [14], the resonance parameters are renormalized in terms of R-Matrix 'compression factor' $\beta_{\pi n}$, (Lane 1970),

$$\beta_{\pi n} = \frac{1}{1+\gamma_{\pi n}^2(\mathrm{d}L_n/\mathrm{d}E)_{E=E_\pi}} \tag{24}$$

$$\tilde{\gamma}_{\pi n}^2 = \beta_{\pi n}\gamma_{\pi n}^2; \quad \tilde{\Gamma}_\pi = \beta_{\pi n}\Gamma_\pi \tag{25}$$

The compression factor is significantly smaller than unity provided the reduced width is large and shift factor has large positive slope. A large reduced width, of order of Wigner unit, is vital in obtaining small value of compression factor. The neutron shift factor is constant far-away from threshold; near threshold it is monotone increasing with energy, $\mathrm{d}S_n/\mathrm{d}E > 0$. Accordingly the compression factor is essential non-unity only near neutron channel threshold and for large reduced width.

6. THOMAS-EHRMAN SHIFT FOR BOUND AND QUASISTATIONARY STATES

The bound states are defined in R-matrix Theory [14] by equation

$$1 - R(E_b)S(E_b) = 0 \tag{26}$$

A similar equation is valid for quasistationary states (*e.g.* [14])

$$1 - R(E_q)L(E_q) = 0 \tag{27}$$

In above equations $S = L^-$ and $L = L^+$ are logarithmic derivatives at negative and positive energies, respectively. For resonance scattering the phase shift is defined by

$$\delta = \arctan PR/(1-RS) \tag{28}$$

with penetration and shift factors as imaginary and real parts of logarithmic derivative $L = S + iP$. The resonance phenomenon occurs for scattering phase shift equating $\pi/2$ (or odd multiples of $\pi/2$), *i.e.*

$$1 - R(E_r)S(E_r) = 0. \tag{29}$$

Let's consider an energy E in vicinity of the resonance energy E_r

$$1 - R(E)S(E) = 1 - R(E_r)S(E_r) - (E - E_r)(RS)'_{E_r} = (E_r - E)(RS)'_{E_r} \tag{30}$$

with $(')$ as differentiation with respect to energy. The equation for the resonance scattering phase shift, defined in terms of resonance energy E_r and partial decay width $\Gamma = P\gamma^2_{obs}$,

$$\delta = \arctan\Gamma/(E_r - E) = \arctan PR/(E_r - E)(RS)'_{E_r} \tag{31}$$

results into definition of observed reduced width γ_{obs}

$$\Gamma = PR/(RS)'_{E_r} = P\gamma^2_{obs} \tag{32}$$

$$\gamma^2_{obs}(E) = R(E)/(RS)'_{E_r} \tag{33}$$

The observed reduced width evaluated at resonance energy is

$$\gamma^2_{obs} = \gamma^2_{obs}(E_r) = R(E_r)/(RS)'_{E_r} \tag{34}$$

For case of single resonance $R = \gamma^2/(E_\lambda - E)$, $R' = R^2/\gamma^2$, with γ as R-matrix reduced width, one obtains

$$\gamma^2_{obs}(E) = (R(E)/R(E_r))/(S'_{E_r} + 1/\gamma^2) = (R(E)/R(E_r))\beta\gamma^2 \tag{35}$$

with β as R-matrix compression factor

$$\beta = \beta(E_r) = 1/(1 + \gamma^2(\mathrm{d}S/\mathrm{d}E)_{E_r}) \tag{36}$$

$$\gamma^2_{obs} = \beta\gamma^2 \tag{37}$$

This way the bound- or quasistationary- or resonance-state equation results in natural way into renormalization of R-matrix reduced width γ in terms of R-matrix compression factor β. The β factor is effective for large R-matrix reduced width, *i.e.* for single-particle reduced width, and for large shift factor derivative, ie near threshold.

Observe that extended definition of β from previous chapter

$$\beta_{\pi n} = (1 - R^\pi_{nn}S_n)^{-1} \tag{38}$$

is compatible, in limit E_π = E_r, $(r = \pi)$, with above result. Indeed $1 - RS$ = $(E_r - E)(RS)'_{E_r}$ provided $(RS)_{E_r}$ = 1 is taken into account.

$$(1 - R^\pi_{nn}S_n) \to (E_r - E)(RS)'_{E_r} = \gamma^2/\gamma^2_{obs}(E) \to_{(E=E_r)} 1/\beta \tag{39}$$

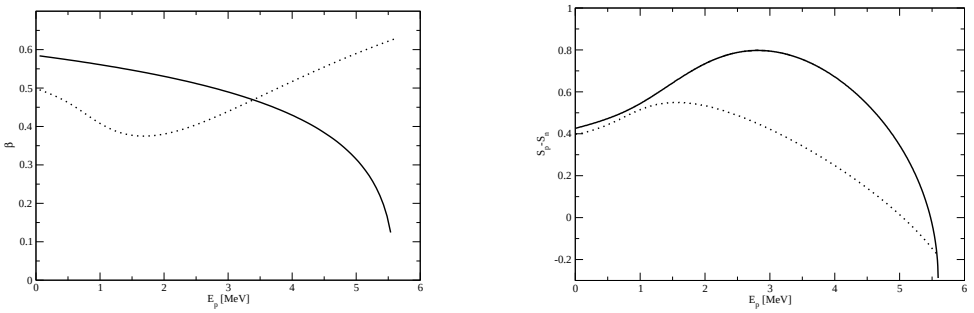

Fig. 3 – Left: compression factor for proton level (system ^{26}Al⊗p=^{27}Si) (dotted line) and for corresponding mirror neutron level (^{26}Al⊗n=^{27}Al nucleus)(solid line). Right: difference between the s-wave shift factor for proton level at energy E_p and the shift factor for isobaric analogue neutron state (solid line) and the same quantity renormalized (dotted line).

Another approach to problem of renormalization of reduced widths is in terms of Kapur-Peierls type of boundary conditions, $B = L$, or even natural boundary conditions, $B = S$, (Lane and Thomas 1958),

$$R(B) = (R^{-1} - B)^{-1} = \gamma^2/(E_\pi - B\gamma^2 - E) \tag{40}$$

$$\gamma^2(B) = R^2(B)/(\mathrm{d}R(B)/\mathrm{d}E) = \gamma^2/(1+\gamma^2\mathrm{d}B/\mathrm{d}E) = \gamma^2\beta \tag{41}$$

with either $B = L$ or $B = S$.

The conclusion of this chapter is that neutron and proton reduced widths, $\gamma_{\pi n}$ and $\gamma_{\pi p}$, are not identical esp near thresholds, even for mirror states. The next point is to transfer these conclusions to description of the Thomas-Ehrman effect. One obtains a modification of Thomas-Ehrman eq. (14), provided the threshold renormalization of reduced withs is taken into account,

$$\Delta_{\pi p} - \Delta_{\pi n} = [S_n(E_{rn})\beta_n(E_{rn}) - S_p(E_{rp})\beta_p(E_{rp})]\gamma_\pi^2 \tag{42}$$

This result is congruent with Lane's (1970) R-matrix approach to threshold effects. Recently [18] have approached mirror states decays by taking into account threshold effects as those studied in Gamow Shell Model [15].

There are significant differences between Thomas-Ehrman shift according to R-matrix prescription and those calculated with renormalized reduced widths. Fig. 3 displays Thomas-Ehrman shift for s-wave mirror levels involved in ^{26}Al(p,p) and ^{26}Al(p,n) reactions taking into account the corresponding proton and neutron compression factors. The proton energy E_p is measured with respect to proton threshold for p+^{26}Al reaction; the corresponding neutron threshold energy for ^{26}Al(p,n) reaction is E_n=E_p-5.594 MeV.

The compression factors, below neutron threshold, relevant for the system ^{26}Al + proton or neutron are presented in Fig. 3, left; proton and neutron compression fac-

tors are evaluated below neutron threshold. The renormalized factor multiplying the reduced width is presented in the right part of Fig. 3 together with its nonrenormalized counterpart.

7. CONCLUSIONS

The Thomas-Ehrman shift is a subject of prime interest in non-explosive nucleosynthesis, *i.e.* studies of proton threshold states of mirror nuclei. The calculus of astrophysical factor presumes knowledge of spectroscopic properties of proton threshold states involved in (p,γ) capture reaction. The experimental study of proton threshold states is difficult due to very small cross-section at low (near-threshold) energy. The experimental study is then complemented by a theoretical spectroscopic study of proton-rich nucleus. But proton threshold states are located at 'high' energy in spectrum of proton-rich nucleus and the theoretical analysis is not unambiguous. This shortcoming is, at least in part, overcome by a parallel study of the mirror proton and neutron states of the isospin doublet. One assumes that the internal states of neutron-rich and proton-rich nucleus are identical but they are coupled to different reaction channels, the proton and neutron ones, which result into different boundary condition displacements. The neutron-rich and proton-rich levels displacements originate in different matching of same internal state to the two reaction channels, the neutron one which is closed and the proton one which is open. The formal correspondence between the mirror states should take into account the relative energy displacements of proton and neutron states, including the boundary condition one. The first approach to the problem is due to Thomas and Ehrman in framework of R-matrix theory. Later one same problem was approached by Tombrello in 'one-body potential model'. The two approaches result in same qualitative descriptions of the boundary condition displacements; this work is based on a quantitative description of threshold states, spatially extended in reaction channels. The spatial extension of threshold state, out of channel radius, is described in terms of R-matrix compression factor which, at its turn, is related to energy variation of channel logarithmic derivative and to particle (proton or neutron) reduced widths. The R-matrix single particle equation is related also to channel logarithmic derivative. This equation allows, by using natural boundary conditions, to define the 'observed' reduced width, which has same energy dependence as threshold compression factor. (The use of two different boundary conditions, for neutron and proton channels, does not conflict R-matrix. The boundary condition is a channel characteristic rather than a level one.) This study results into conclusion that the proton and neutron reduced widths are not identical as in R-matrix Thomas-Ehrman shift. A modified formula for boundary condition displacement, taking into account the neutron and proton threshold compression factors, is proposed.

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