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EVIDENCE FOR OPTIMAL RESONANCES FROM HADRON-NUCLEUS SCATTERING IN ELEMENTARY RESONANCE REGIONS

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Abstract. In this paper, by using the *Principle of Minimum Distance in Space of Quantum States* (PMD-SQS), a new description of the hadron-nucleus scattering in the elementary resonance region, in terms of optimal resonances, is presented. Then, all the essential characteristic features of the hadron-nucleus in the optimal resonance limits are compared with the available experimental data. All optimal resonance predictions are found in a good agreement with the experimental data in the region corresponding to the elementary resonances.

Key word: PMD–SQS, hadron-nucleus scattering, optimal resonances.
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1. INTRODUCTION

The hadron-nucleus interaction is at present one of the most interesting topics in the realm of the conventional hadron physics. Pion-nucleus interactions have been extensively investigated in the past three decades with the meson factories at LAMPF, TRIUMF, and PSI. An excellent review of the advances and discussion of open questions as well as issues related to current research are published in ref. [1]. We recall that, more than twenty years ago, in the papers [2-5] we introduced new exotic collective states of nucleus, called *dual diffractive resonances* (DDR) which can be excited in the hadron-nucleus scattering. Then, the revival of interest was caused by accumulation of more accurate experimental data [6-17] as well as by new theoretical ideas on this subject [18-20]. One of the most actual problems of great experimental and theoretical importance is in what form can the elementary hadron-nucleon resonances $(\Delta, N^*, Y^*, \text{etc.})$ be expected to manifest themselves in the hadron-nucleus total cross sections? Exotic nuclei are those nuclei in which one nucleon is substituted by a heavier baryon. Since the heaviest baryons are the hadron-nucleon resonances $(\Delta, N^*, Y^*, \text{etc.})$, then the most exotic nuclei are expected to exist only as exotic resonances. These exotic nuclei pose completely new problems of nuclear structure such as "exotic giant resonances" or exotic $(\Delta, N^*, Y^*, \text{etc.})$-nuclear doorway stated, etc. Therefore, the hadron-nucleus scattering in the elementary resonance regions yields the true collective exotic states, just as the nucleon-hole and nucleon-nucleon interactions in the usual nuclei give rise to a large variety of collective states. A substantial amount of experimental data on the pion-nucleus scattering was obtained in the region corresponding to $\Delta(3,3)$ – resonance in the elementary πN- interaction. The essential results obtained from the experimental data on pion-nucleus scattering, in the region corresponding to the $\Delta(3,3)$ resonance in the elementary pion-nucleon interaction, are characterized by the following *resonance-diffraction duality*:

I. *A resonant energy behaviour* manifested in the total, integrated elastic and inelastic cross sections (see refs. [6-14]), as well as, in each pion-nucleus partial wave.

II. *A typical diffraction pattern* observed in the pion-nucleus angular distributions ([14-17] see also Refs. [37, 38, 41-76] in ref. [1]).

III. The resonance width Γ_A becomes broader as nuclear mass *A* increases. A behaviour of form: $\Gamma_A = \Gamma_\Delta A^{1/3}$ is verified experimentally with high accuracy ([2, 5]).

IV. The *resonance peak shifts* downward with increasing *A* to lower kinetic energy.

In order to explain consistently all the above essential characteristic features I-IV of the pion-nucleus scattering, a new concept of nuclear collective resonance state was introduced in refs. [2-5]. According to their diffraction pattern observed in the angular distributions these collective nuclear resonant states were called *dual diffractive resonances* (DDR).

On the other hand recent years were marked by a substantial progress in the theoretical description of the strong interacting particles *via* the *Principle of Minimum Distance in Space of Quantum States* (PMD-SQS) [18, 21-22]. Then, the universal optimal atoms as well as the universal optimal resonances are the most fascinating PMD-SQS-optimal states predictions. More concretely, among the most important predictions for the PMD-SQS-optimal resonances there are: (i) the high total intensity proportional with $(L_o+1)^2$, (ii) typical diffraction pattern of the angular distributions, and (iii) very high widths characterized by the relation $\Gamma_{L_o+1} = \Gamma_1(L_o+1)$, where L_o is the optimal angular momentum. Consequently in ref. [5, 19] the dual diffractive resonances (DDR) [2-5] are proved to be genuine PMD-SQS-optimal resonances. Therefore, it is of fundamental interest to understand as much as possible the PMD-SQS-optimal status of the above I-IV characteristic *resonance-diffraction* features manifested in pion-nucleus scattering since they can be signatures of a new kind of resonances called optimal resonances. Therefore, in this paper some new results and detailed description of the pion-nucleus scattering in the $\Delta(3,3)$-resonance region, in terms of PMD-SQS-optimal resonance, are presented.

2. EXPERIMENTAL EVIDENCES FOR OPTIMAL RESONANCES IN PION-NUCLEUS SCATTERING IN $\Delta(3,3)$ – RESONANCE REGION

2.1. OPTIMAL RESONANCE PREDICTIONS IN HADRON-NUCLEUS SCATTERING

We start with the usual decomposition of the hadron-nucleus scattering amplitude in the Coulombian and nuclear parts: $f(E,x) = f_C + f_N(E,x)$, where E is the c.m. hadron-nucleon energy, $x \equiv \cos\theta$, and θ. is the c.m. pion-nucleus scattering angle.

Next, the nuclear amplitude $f_N(E,x)$ of the hadron-nucleus scattering is developed in partial amplitudes in the usual form

$$f_N(E,x) = \sum (2l+1) f_l(E) P_l(x), \tag{1}$$

where $x \in [-1,+1]$ and $P_l(x)$ are Legendre polynomials while l = 0,1,2,…., are orbital angular momenta.

It is well known [18-19] that, in the PMD-SQS approach for the scattering of spinless particles, the partial amplitudes $f_l(E)$ are considered variational variables, while $\sum (2l+1) |f_l|^2$ is distance in the Hilbert space of the $\pi A-$scattering states. Then, according to the PMD-SQS (see for details in Refs. [19]) we must solve the following constrained optimization problem:

$$\min\left[\sum (2l+1)|f_l|^2\right] \text{ with } |\sum (2l+1) f_l|^2 = \text{fixed} \text{ to obtain:}$$

$$\begin{aligned} &f_l(E) = f_N(E,1)/(L_o+1\}^2, \text{ for all } l \in [0, L_o] \\ &f_l(E) = 0, \text{ for all } l > L_o, \end{aligned} \tag{2a}$$

where L_o – is called optimal angular momentum.

Consequently, for the nuclear $hA-$scattering amplitude we obtain

$$f_N(E,x) = f_N(E,1)\frac{K_{L_o}(x,1)}{K_{L_o}(1,1)}, \quad L_o = \text{integer}\sqrt{\frac{4\pi}{\sigma_{el}}\frac{d\sigma}{d\Omega}(E,1)} - 1, \tag{2b}$$

where $K_{L_o}(x,y)$, $x,y \in [-1,+1]$, are reproducing kernels of the Hilbert space of states (for more details see also the papers [20-22]). In our particular case for the hadron-nucleus scattering on spinless nuclei we have

$$2K(x,1)=\sum_{l=0}^{L_o}(2l+1)P_l(x)P_l(1)=\dot{P}_{L_o+1}(x)+\dot{P}_{L_o}(x),\ \dot{P}_l(x)\equiv\frac{\mathrm{d}P_l(x)}{\mathrm{d}x}\,, \tag{3}$$

$$2K(1,1)=\Sigma_{l=0}^{L_o}(2l+1)=(L_o+1)^2=\frac{4\pi}{\sigma_{el}}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(E,1)\,. \tag{4}$$

Now, as a direct consequence of the PMD-SQS optimality conditions (2a), we obtain that all πA-resonant states which are derived from a single $\pi N(\Delta)$-resonant state become degenerate. Hence, for the nuclear part of the pion-nucleus scattering, we obtain the following expression:

$$f_N(E,1)=\frac{1}{2k}\frac{\Gamma_{el}(L_o+1)^2}{E_0-E-\frac{i}{2}[\Gamma-\gamma_0(E_0-E)]}\,, \tag{5}$$

where E is the c.m. energy, k is the c.m. momentum, $x\equiv\cos\theta$, θ - c.m. scattering angle, $E_o,\Gamma,\Gamma_{el,}$ and γ_0, are the effective optimal resonance parameters: mass, total width, elastic width and asymmetry parameter, respectively. We note that the resonant amplitude (5) divided by $(L_o+1)^2$ is the *generalized Breit Wigner* (GBW) amplitude obtained by introducing [2] the asymmetry parameter γ_0 in a natural way starting with a Regge pole expression $f_l(E)\propto\beta/[l-\alpha_l(E)]$, for the hadron-nucleus partial amplitudes $f_l(E)$.

Therefore, in the optimal resonance (OR) limit [2, 5], the hadron-nucleus scattering is characterized by the following essential OR-characteristic features:

(i) *The energy behavior of the total hadron-nucleus cross section is of asymmetric Breit-Wigner form [2] given by*

$$\sigma_T=\pi\,\lambda\!\!\!^{-2}(L_o+1)^2\,\frac{\Gamma_{el}[\Gamma-\gamma_0(E_0-E)]}{(E_0-E)^2+\frac{1}{4}[\Gamma-\gamma_0(E_0-E)]^2}\,, \tag{6}$$

$$\sigma_{el}=\pi\,\lambda\!\!\!^{-2}(L_o+1)^2\,\frac{\Gamma_{el}^2}{(E_0-E)^2+\frac{1}{4}[\Gamma-\gamma_0(E_0-E)]^2}\,. \tag{7}$$

(ii) *The real part of the forward hadron-nucleus scattering amplitude has a resonant behavior described by*

$$\mathrm{Re}\,f_{\pi A}(E,0^\circ)=\frac{\lambda\!\!\!^{-}(L_o+1)^2}{2}\frac{\Gamma_{el}(E_0-E)}{(E_0-E)^2+\frac{1}{4}[\Gamma-\gamma_0(E_0-E)]^2}\,. \tag{8}$$

(iii) *The angular distributions of the optimal resonances are typical diffractive patterns, very sensitive to the values of optimal angular momentum* L_o. *They are described by*

$$\frac{d\sigma}{d\Omega}(E,x)=\frac{d\sigma}{d\Omega}(E,1)\left[\frac{K_{L_o}(x,1)}{K_{L_o}(1,1)}\right]^2 \tag{9a}$$

$$\frac{d\sigma}{d\Omega}(E,x)\approx\frac{d\sigma}{d\Omega}(E,1)\left[\frac{2J_1(\tau)}{\tau}\right]^2, \quad L_o>>1 \tag{9b}$$

for $L_o>>1$, where $J_1(\tau)$ is the Bessel function of the first order and $\tau\equiv 2L_o\sin\frac{\theta}{2}$.

(iv) *The logarithmic slope* b_{OR} *of the forward diffraction peak for the optimal resonances is described by the relation* [2]

$$b_{OR}=\frac{ƛ^2}{4}L_o(L_o+2)=\frac{ƛ^2}{4}\left[\frac{4\pi}{\sigma_{el}}\frac{d\sigma}{d\Omega}(E,1)-1\right]. \tag{9c}$$

(v) *The OR-cross sections* $\sigma_{el}(E)$ *and* $\sigma_T(E)$ *and* Γ_{A^8} *saturate the "axiomatic" optimal bounds*:

$$\sigma_T^2(E)\le 4\pi ƛ^2(L_o+1)^2\sigma_{el}(E) \tag{10}$$

$$\Gamma_\Delta\le\Gamma_{A^*}\le\Gamma_\Delta n, \tag{11}$$

where $n\equiv L_o+1$ at the energy $E=E_0$, where $ƛ=1/k$ (we are working in the system $\hbar=c=1$).

2.2. EXPERIMENTAL EVIDENCES FOR OPTIMAL RESONANCE IN PION-NUCLEUS SCATTERING IN $\Delta(3,3)-$ RESONANCE REGION

Now we examine the energy behavior of the experimental data [6-17] on the pion-nucleus scattering in the region corresponding to the $\Delta(1236)$ resonance in the elementary pion-nucleon scattering. Γ_1 from Eq. (11) is identified with the $\Delta-$*total width*.

In the next part of the paper, the optimal angular momentum is taken as $L_o\approx kR$, where R ($R\approx r_0A^{1/3}$) is the equivalent spherical radius of a nucleus with A-nucleons. Then in all Eqs. (1–11) we can write

$$ƛ^2(L_o+1)^2\cong(R+ƛ)^2. \tag{12}$$

Consequently, at optimal hadron-nucleus resonance, from (10)-(11) we have the saturation of the following important bounds

$$\sigma_T^2(E_o) \le 4\pi(R+\lambda\!\!\!^{-})^2 \sigma_{el}(E_o), \quad (13)$$

$$\Gamma_1 \le \Gamma_{A^8} \le \Gamma_\Delta(A^{1/3} + \lambda\!\!\!^{-}/r_0). \quad (14)$$

Now, the experimental data on the pion-nucleus total cross sections [6-14] are presented in Figs. 1 and 2. We see that the cross sections for all nuclei exhibit a maximum clearly related to $\Delta(1\,236)$ resonance, which shifts downward and broadens with increasing A and has an asymmetry which also increases with increasing A. It is easy to see that all these characteristic features are fairly reproduced by the optimal resonance predictions.

In fitting Eq. (6) to the experimental data we have considered the E_0 fixed by the relation

$$E_o = M_A + 1\,236\ \text{MeV} - m_N\,,\ \ \Gamma_{el} = k\gamma_1,\ \ L_o \approx kR \quad (15)$$

and the geometric radius R fixed as in the Tables 1 and 2 of Ref. [2], for each nucleus. The other parameters γ_0, γ_1 and Γ from Eq. (6) are allowed to vary for each nucleus in order to obtain the best χ^2-fit of the total cross sections. The optimal resonance parameters are presented in Fig. 1b.

Therefore, from the results presented in Figs. 1a,b we see that all experimental data on the pion-nucleus total cross sections are well described by Eq. (6). Moreover, by using the fitted parameters of the total cross sections we obtain absolute numerical predictions (see an example in Fig. 2) for σ_{el}, σ_{in} as well as for the upper bound $\left[4\pi(R+\lambda\!\!\!^{-})^2\sigma_{el}\right]^{1/2}$ which is saturated at the optimal resonance energy $E = E_0$. Then, we see that all optimal resonance predictions (see Figs. 1–2) are in excellent agreement with the experimental data.

Pion-nucleus interactions have been extensively investigated in the past three decades with the meson factories at LAMPF, TRIUMF, and PSI. The article [1] gives a pedagogical review of the advances and discusses some open questions as well as issues related to current research. Extensive data on pion-nucleus elastic scattering from a wide range of nuclei, obtained at meson factories (see Refs. [37, 38, 41–78] from Ref.[1]), all show the characteristics of diffractive scattering. That is, minima in the elastic scattering angular distributions correspond to the diffractive minima produced by strong optical absorption. These experimental results are also important signatures of the PMD-SQS-optimal resonance dominance since as can be seen in Eq. (9) theory of optimal resonances includes such absorption in a more general and exact form and consequently with the correct nuclear size will reproduce with high accuracy all the diffractive features of the angular distributions.

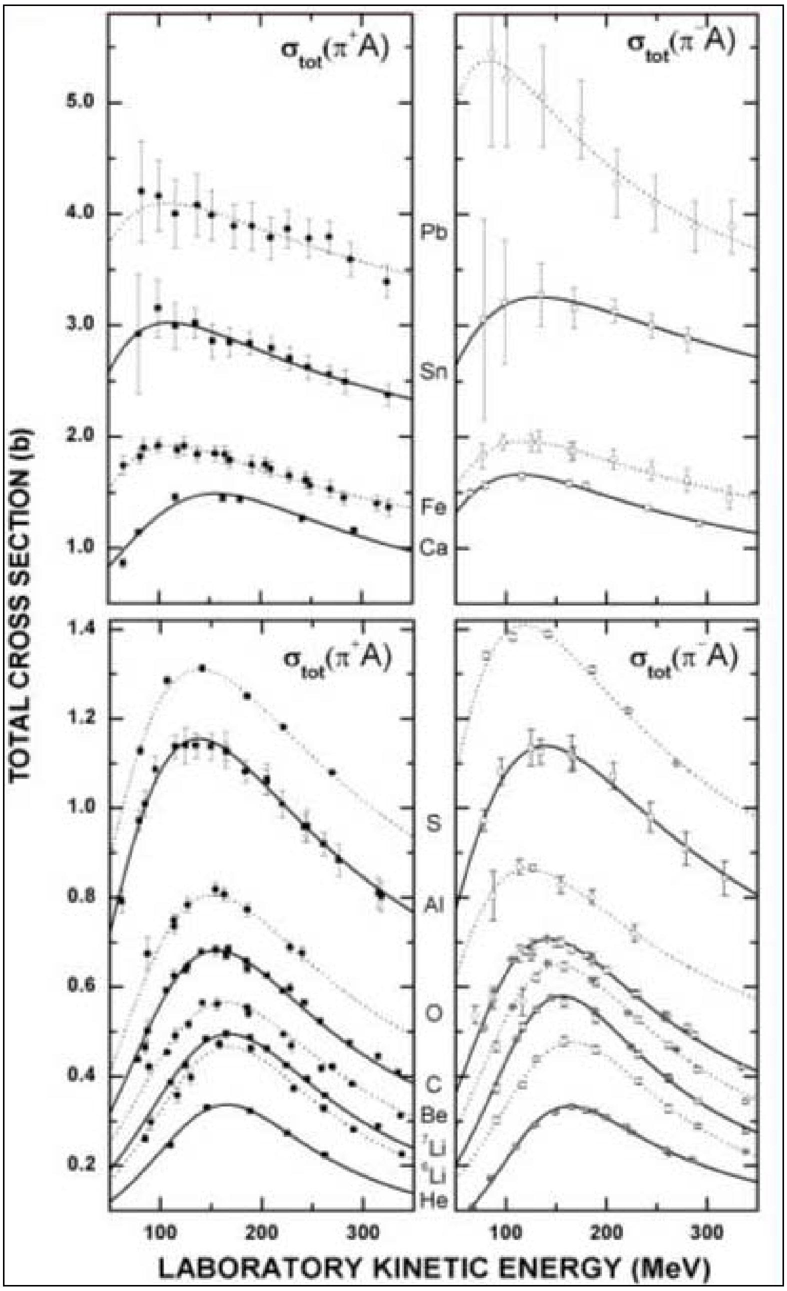


Fig. 1 – The pion-nucleus total cross section in the $\Delta(1\,236)$ - energy region. The experimental data are from Refs. [3–11]. The curves are results of the best fit to the data with the optimal resonance predictions, Eq. (6) with $L_o = kR$ and $m_\Delta = 1236$ MeV fixed.

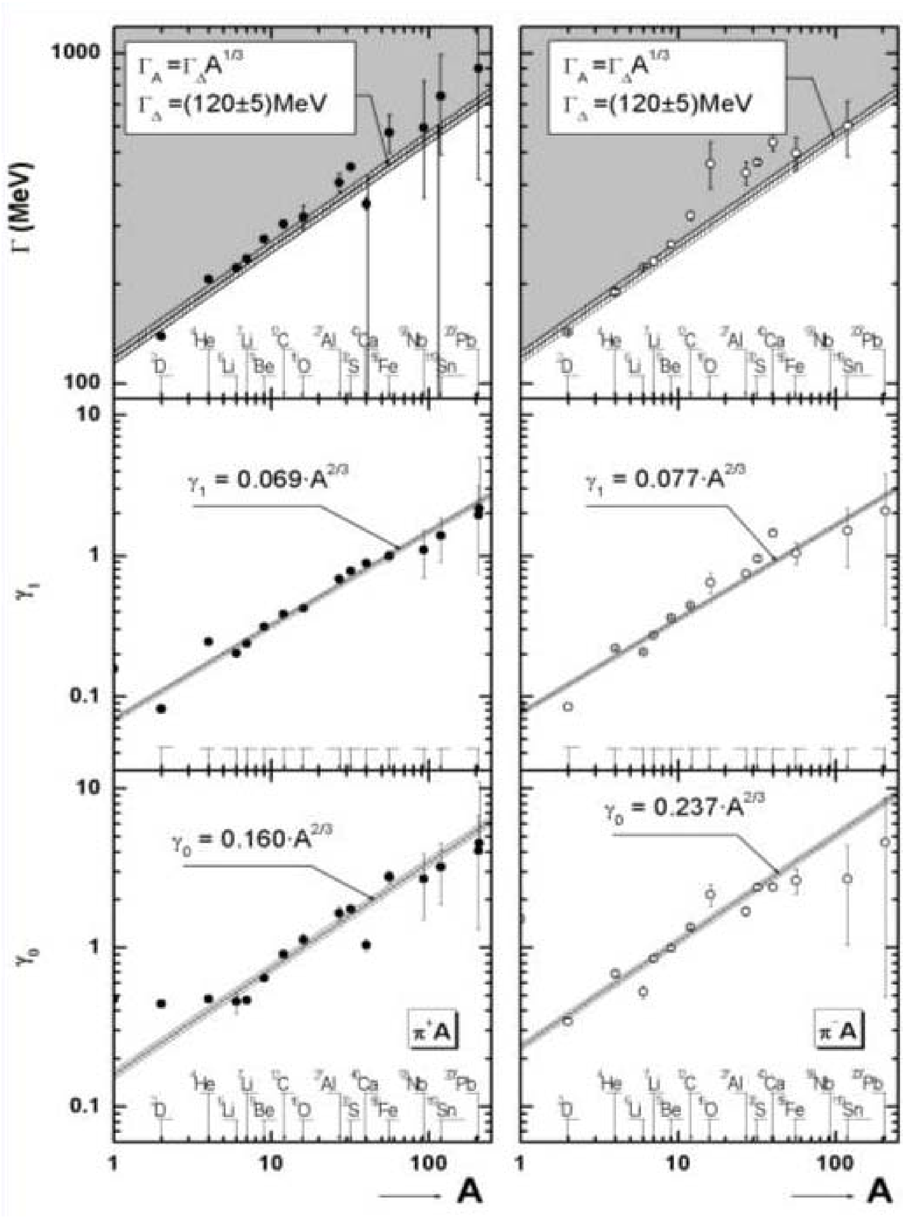
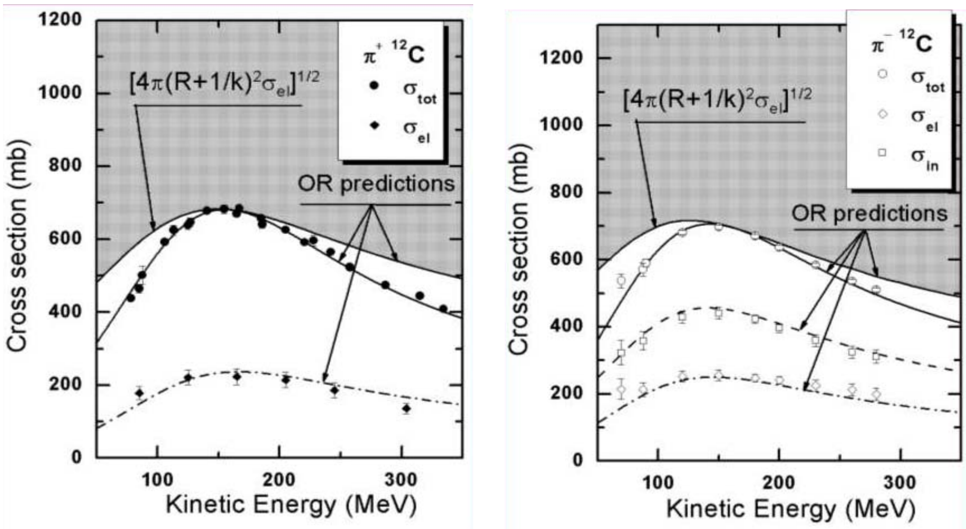
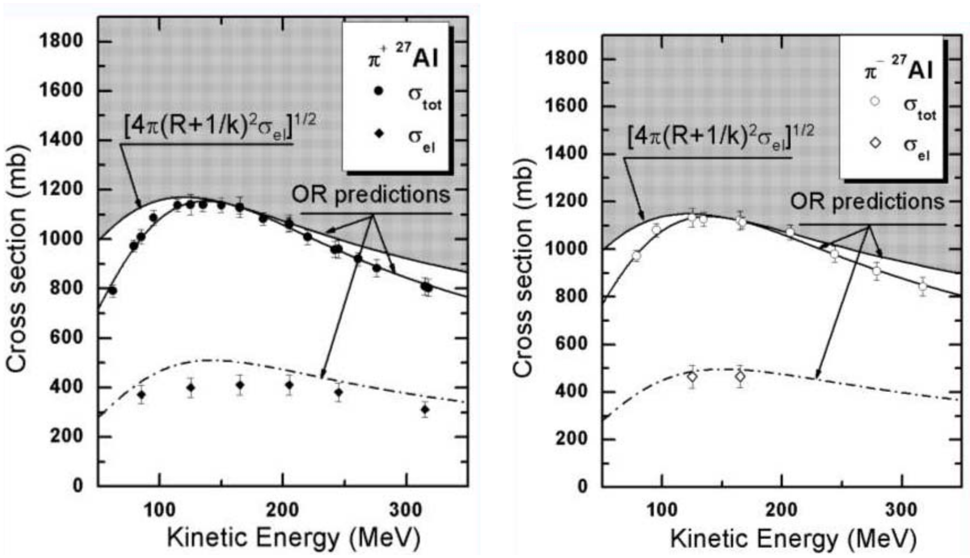


Fig. 2 – The OR-parameters Γ_A, γ_0, γ_1 obtained by minimum χ^2 -fits to the total pion-nucleus cross sections (6). The solid circles are obtained from the fits to $\pi^+ -$ nucleus data, while the open circles are from fits to $\pi^- - nucleus$ data. The solid lines represent the smooth A-dependence of the corresponding OR-parameters. The saturation of axiomatic upper limit: $\Gamma_A \le \Gamma_\Delta(L_o + 1\} \approx \Gamma_\Delta A^{1/3}$ is evidentiated with high accuracy.



a



b

Fig. 3 – The saturation of the axiomatic bounds (13) on $[\pi^{\pm}C$ (Fig.3a) and $\pi^{\pm}$Al (Fig.3b)] total cross sections in the $\Delta(1\,236)$ -energy region. The experimental data are from Ref. [6-14]. The solid curves are results of the best fit to the data with the optimal resonance predictions Eq. (6), while the elastic and inelastic dashed curves are absolute predictions using Eqs. (7) with the fitted parameters of Eq. (6). The saturation of the axiomatic optimal bound (10) is experimentally evidentiated with high accuracy.

2.3. EXPERIMENTAL ENTROPIC TESTS OF OPTIMAL RESONANCE

The angle and angular momenta information entropies as well as the entropic angle-angular momentum uncertainty relations in hadron-hadron scattering are introduced in the paper [45, 50]. Here, the experimental information entropies for the pion-nucleus scattering were calculated by using the available phase shifts analyses [17] and definitions

$$S_L = -\sum_{l=0}^{L_0} (2l+1) p_l \ln p_l \le S_L^{01}, \tag{15}$$

where $p_l = |f_l(E)|^2 / \sum_0^{L_o} (2l+1)|f_l(E)|^2$.

Now, it easy to see that, as a direct consequence of the optimal solution (2a), *at optimal resonance the entropic limit* $S_L = S_L^{01} = \ln\left[(L_o+1)^2\right]$ *must be saturated.*

Indeed, the results obtained for experimental angular momentum entropies are presented in Figs. 6–7, in comparison with the optimal entropies. Then, the saturation of the OR-entropic in pion-nucleus scattering in the optimal resonance region is also evidentiated with high accuracy by the OR-test.

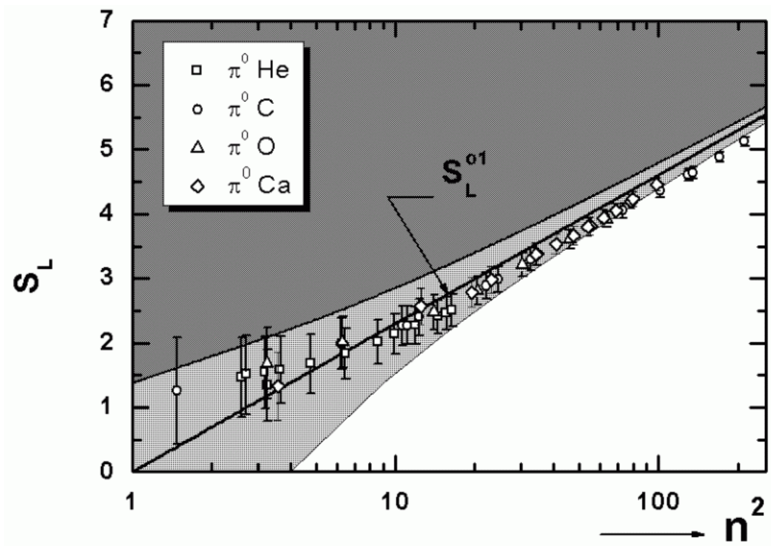


Fig. 4a – The experimental values of the pion-nucleus scattering entropies S_L, are compared with their PMD-SQS optimal values $S_L^{01} = \ln\left[(L_o+1)^2\right]$ (solid curve). The numerical values for S_L are obtained by using the available phase shifts analyses [17]. The grey region around the optimal entropies is obtained by assuming an error of $\Delta L_o = \pm 1$ in the estimation of optimal angular momentum while $n^2 = (L_o+1)^2$.

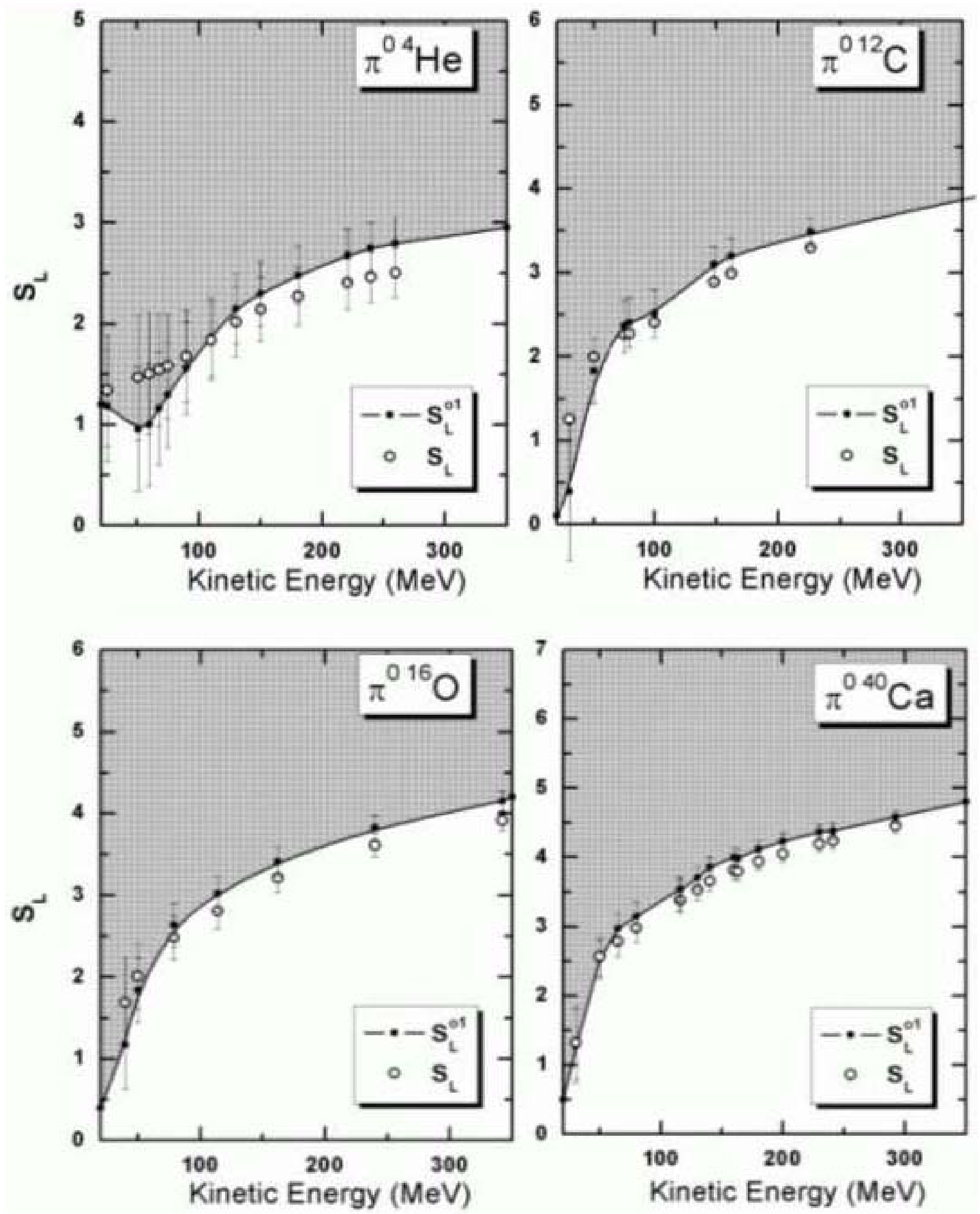


Fig. 4b – The experimental values of the π^0- nucleus scattering entropies S_L (blake circles) are compared with their PMD-SQS optimal values $S_L^{01} = \ln\left[(L_o + 1)^2\right]$ (white circles). The numerical values for S_L are obtained by using the available π^0- nucleus phase shifts analyses [17].

3. EXPERIMENTAL EVIDENCES FOR OPTIMAL RESONANCES IN NUCLEON-NUCLEUS SCATTERING

In this section the experimental data on the nucleon-nucleus scattering, for the laboratory momentum between 0.9 and 10 GeV/c are analyzed in terms of optimal resonance predictions of form (6–11).

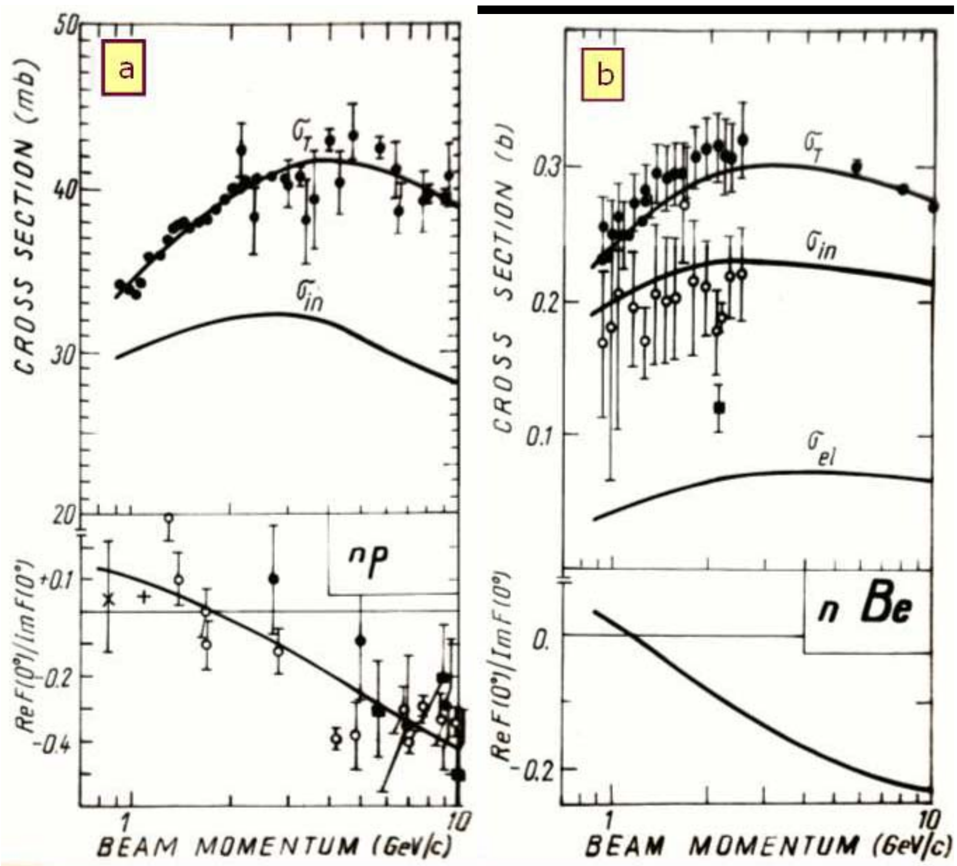


Fig. 5a – The experimental data [23–27] on neutron-proton scattering for p_{LAB}=0.9–10GeV/c, are compared with the optimal resonance predictions eqs. (6–8) with the parameters obtained by fit of the total cross sections; b) the experimental data [24–32] on n^9Be scattering for p_{LAB}=0.9-10GeV/c, are compared with the optimal resonance predictions eqs. (6–8) with the parameters obtained by fit of the total cross sections.

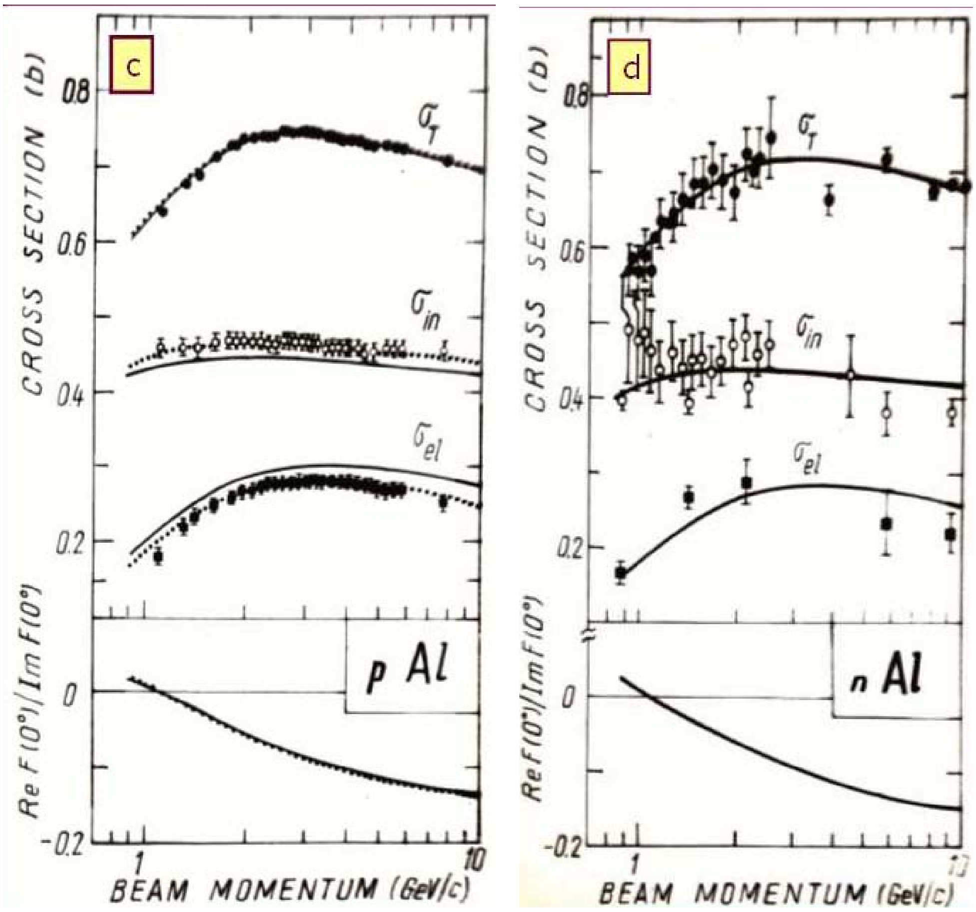


Fig. 5c – The experimental data [39] on p^{27}Al scattering for p_{LAB}=0.9–10GeV/c, are compared with the optimal resonance predictions eqs. (6–8) with the parameters obtained by fit of the total cross section and with R=3.76fm (full lines) and R=3.90Fm (dashed lines); d) the experimental data [25–38] on n^{27}Al scattering for p_{LAB}=0.9–10GeV/c, are compared with the optimal resonance predictions eqs.(6–8) with the parameters obtained by fit of the total cross sections.

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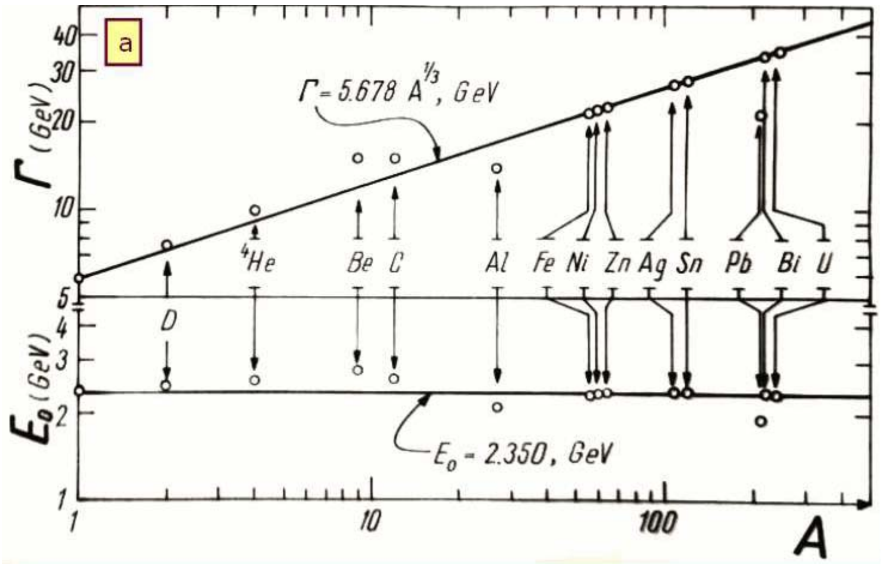

Fig. 6a – The nucleon-nucleus total cross sections for laboratory momenta between 0.9 Gev and 10 GeV are consistent with the optimal resonance predictions given in Eq. (6) and (14). The optimal resonance parameters are as follows: E_o=2.35 GeV, $\Gamma = \Gamma_{np} A^{1/3} = 5.678 A^{1/3}$ while the values of γ_0 and γ_1 are presented in Fig. 6b (see the paper [4] for references and details).

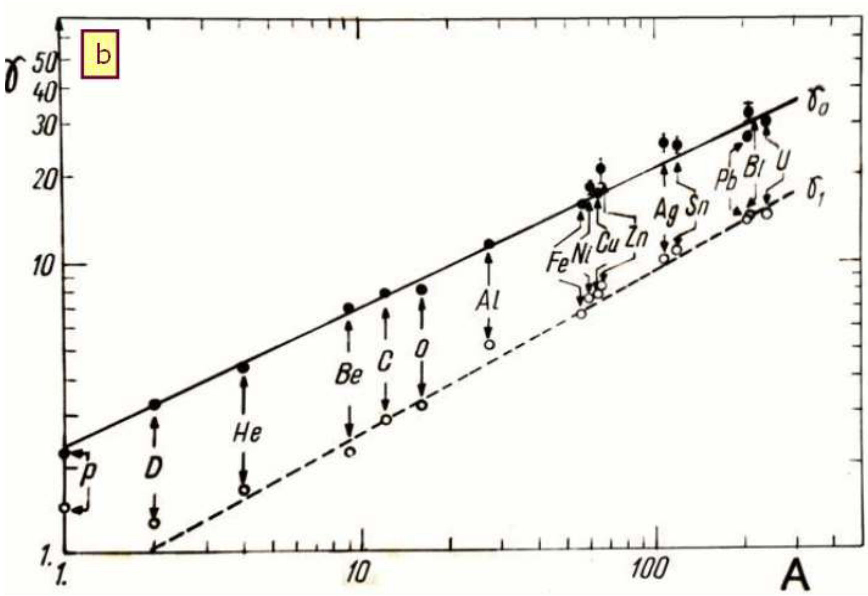

Fig. 6b – The values of the optimal parameters γ_0 and γ_1 obtained by fit are presented in Fig. 6b (see the paper [4] for references and details).

Next, we must underline that the experimental nucleon-nucleus angular differential cross sections in the laboratory momentum display diffraction patterns: periodic side maxima and minima with a pronounced forward diffraction peak. All these optimal resonance characteristic features are well described by the optimal predictions (9a,b,c). Here a quantitative analysis of the experimental data [43] on the logarithmic slope of the forward diffraction peak in terms of OR- predictions (9c) is presented in Fig. 7 (full circles). Then, we see that b_{OR} calculated using Eq. 9c (open circles) are in good agreement with the experimental result from ref. [43].

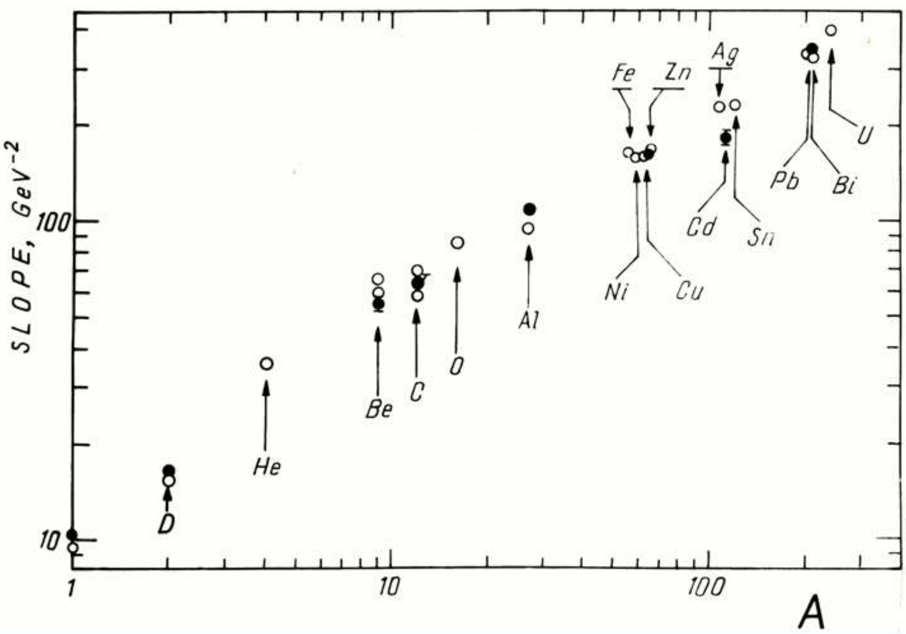

Fig. 7 – The experimental data (Rigia *et al.* Phys Lett. B, **28**, 185 (1972)) with full circles on the logarithmic slope of the neutron-nucleus forward peak at p_{LAB} = 4.2 GeV/c compared with the *optimal resonance* predictions (o) calculated using Eq. 9c.

4. CONCLUSIONS AND OUTLOOK

Now, it is important to recall the philosophical optimism of Leonhard Euler (1707–1783) having written: "*Since the fabric of universe is the most perfect and is the work of the wisest Creator, nothing whatsoever takes place in this universe in which some relation of maximum or minimum does not appear*. *Wherefore, there is absolutely no doubt that every effect in universe can be explained as satisfactory from final causes themselves with the aid of the method of Maxima and Minima, as can from the effective causes*".

Therefore, having in mind such kind of optimism, in the papers [18–21, 44–55] we investigated the possibility to construct a predictive analytic theory of the elementary particle interaction based on *the principle of minimum distance in the space of quantum states* (PMD-SQS). So, choosing the partial transition amplitudes as the system variational variables and the *distance in the space of the quantum states* as a measure of the system effectiveness, we obtained the results [1–16] from Ref. [22]. These results proved that the principle of minimum distance in space of quantum states (PMDSQS) can be chosen as variational principle by which we can find the analytic expressions of the partial transition amplitudes.

In conclusion, by using the Principle of Minimum Distance in Space of Quantum States (PMD-SQS) in this paper a new description of the hadron-nucleus scattering in the elementary-resonances regions, in terms of optimal resonances predictions, is presented. Then, all the essential characteristic features (i)-(v) of the hadron-nucleus in the optimal resonance limit are given in Eqs. (6–11). The results and conclusions of this analysis may be summarized as follows:

1. The hadron-nucleus total cross sections, in the energy region corresponding to resonance in the elementary resonance, are well described by optimal resonance predictions (see Figs. 1–7).

2. The total widths of the optimal resonances obtained by fit of the total cross sections are consistent with $\Gamma_{\pi A} = \Gamma_1 A^{1/3}$ with $\Gamma_1 = \Gamma_\Delta = (120 \pm 5)$ MeV (see Figs. 3) in pion-nucleus and $\Gamma_1 = 5.678$ GeV (see Fig. 6a) in nucleon-nucleus scattering, respectively.

3. The available experimental data on σ_{el}, σ_{in} are also in good agreement with the predictions of the PMD-SQS-resonance mechanism (see Figs. 3 and 5).

4. The saturations of the axiomatic bound (10–11), at the optimal resonance energy $E = E_0$, are verified experimentally with high accuracy (see Figs. 2–3, 6a).

5. As a direct consequence of the optimal solution (2a), *at optimal resonance the entropic limit* $S_L \leq S_L^{01} = \ln\left[(L_o + 1)^2\right]$ *must be saturated*. The saturation of these entropic limits are also verified experimentally with high accuracy in Figs. 4a,b.

Finally, we note that further investigations and extension of these results are needed since one of the immediate consequences of the optimal resonance concept is that reactions other than pion-nucleus or nucleon-nucleus scattering can excite the same exotic collective (optimal resonance) states.

REFERENCES

1 T.-S.H. Lee and R.P. Redwine, Annu. Rev. Nucl. Part. Sci., **52**, 23–63 (2002), and there in references.
2. D.B. Ion, *Dual Diffractive Resonances, New Exotic States in Hadron-Nucleus Scattering,* Rev.Roum.Phys. **26,** 15 (1981); Rev.Roum.Phys., **26**, 25 (1981).

3. D.B. Ion and R. Ion-Mihai, *Experimental Evidences for Dual Diffractive Resonances in Pion-Nucleus Scattering,* Nucl. Phys., **A 360,** 400 (1981) and the quoted references.
4. D.B. Ion and R. Ion-Mihai, *Experimental Evidences for Dual Diffractive Resonances in Nucleon-Nucleus Scattering,* Rev.Roum.Phys., **36,** 15 (1991).
5. D.B. Ion *et al.*, *Experimental Evidences for Optimal Resonances in Pion-Nucleus Scattering,* Rom. Journ. Phys., **54**, 601 (2009); *Saturation of Optimal Resonance Limits in Pion-Nucleus Scattering,* Rom. Journ. Phys., **54**, 985 (2009); Rom. Journ. Phys., **55**, 296 (2010).
6. M.L. Scott *et al.*, Phys. Rev. Lett., **28**, 1209 (1972).
7. A.S. Clough *et al.*, Nucl. Phys., **B 76,** 15 (1974).
8. C. Wilkin *et al.*, Nucl. Phys., **B 62,** 61 (1973).
9. D. Ashery *et al.*, Phys. Rev., **C 23**, 2173 (1981).
10. F. Binon *et al.*, Nucl. Phys., **B17**, 168 (1970); Nucl. Phys., **B33,** 42 (1971); Nucl. Phys., **B40**, 608 (1972).
11. A.S. Carroll *et al.*, Phys. Rev., **C 14,** 635 (1976).
12. B.W. Allardice *et al.*, Nucl. Phys., **A209,** 1 (1973).
13. E. Pedroni, *et al.*, Nucl. Phys., **A300,** 321 (1978).
14. F. Binon *et al.*, Nucl. Phys., **A298**, 499 (1978).
15. J. Piffaretti *et al.*, Phys. Lett., **71B,** 324 (1977).
16. L.E. Antonuk et al. Nucl. Phys., **A420**, 43 (1984).
17. J. Frolich *et al.*, Z. Phys., **A 302**, 89 (1981);
O. Dumbrais *et al.*, Phys. Rev., **C 29**, 581 (1984);
J. Frolich *et al.*, Nucl. Phys., **A415**, 399 (1984);
B. Brinkmoller and H.G. Schlaile, Phys. Rev., C **48**, 1973 (1993); H. G. Schlaile, Phys. Rev., **C 55**, 2584 (1997).
18. D.B. Ion, *et al.*, *Reproducing kernel Hilbert space and optimal state description of hadron-hadron scattering*, Int. J. Theor. Phys., **24**, 355 (1985).
19. D.B. Ion, *Reproducing kernel Hilbert spaces and extremal problems for scattering of particles with arbitrary spins*, Int. J. Theor. Phys., **24**, 1217 (1985).
20. D.B. Ion, *Scaling and S-channel helicity conservation via optimal state description of hadron-hadron scattering*, Int. J. Theor. Phys., **25**, 1257 (1986).
21. D.B. Ion, *Description of quantum scattering via principle of minimum distance in space of states*, Phys. Lett., **B376**, 282 (1996).
22. D.B. Ion and M.L.D.Ion, *Principle of minimum distance in space of states as new principle of quantum physics,* Rom. Rep. Phys., **59**, 1058 (2007).
23. J. Bistricky *et al.*, Group I,7, 15-21, 83 (see Ref.[42] in [5]).
24. M.H. Mac Gregor *et al.*, Phys.Rev., **139 B**, 362 (1965);
25. L.M.C. Dutton *et al.*, Phys.Rev.Lett., **21**, 1416 (1968).
26. N. Dalhajav *et al.*, Yad. Phys., **8** (1974).
27. H. Hofstadter, in J. Bistricky et al., Landolt-Bornstein, Group I, 9, p.225, 289,16 (see Ref.[41] in [5]).
28. V.A. Nezdel, Phys. Rev., **94**, 174 (1954); Phys. Rev., **91**, 440 (1953).
29. V.P. Dzelepov *et al.*, Dokl. Acad. Nauk. SSSR, **104**, 717 (1955).
30. T. Coor *et al.*, Phys. Rev., **98**, 1369 (1955).
31. W. Schimmerling *et al.*, Phys. Rev., **C 7**, 248 (1973).
32. W. Schimmerling *et al.*, Phys. Lett., **37 B**, 177 (1971).
33. F. Parker *et al.*, Phys.Lett., **31 B**, 246, 250 (1970).
34. J. Engler *et al.*, Phys.Lett., **32 B**, 716 (1970).
35. J. Engler *et al.*, Phys.Lett., **28 B**, 64 (1968).
36. N.E. Booth *et al.*, Proc. Phys. Soc., **A71**, 293 (1958)
37. W.L. Larkin *et al.*, Phys.Lett., **31 B**, 677 (1970).
38. W. Pantuev *et al.*, JETP **42**, 909 (1962).
39. J.H. Atninson *et al.*, Phys. Rev., **123**, 1850 (1961).
40. A. Ashmore, *et al.*, Proc. Phys. Soc., **A71**, 552 (1958).

41. H.P. Barett, Phys. Rev., **114**, 1374 (1959).
42. D.V. Bugg *et al.*, Phys. Rev., **146**, 980 (1966).
43. F.E. Rigia *et al.*, Phys Lett., B **28**, 185 (1972).
44. D.B. Ion and M.L.D. Ion, *Isospin quantum distances in hadron-hadron scatterings*, Phys. Lett., **B 379,** 225 (1996).
45. D.B. Ion and M.L. Ion, *Information entropies in pion-nucleon scattering and optimal state analysis*, Phys. Lett., **B 352**, 155 (1995).
46. D.B. Ion and M.L.D. Ion, *Entropic lower bound for quantum scattering of spinless particles*, Phys. Rev. Lett., **81**, 5714 (1998).
47. M.L.D. Ion and D.B. Ion, *Entropic uncertainty relations for nonextensive quantum scattering*, Phys. Lett., **B 466**, 27 (1999).
48. M.L.D. Ion and D.B. Ion, *Optimal bounds for Tsallis-like entropies in quantum scattering of spinless particles*, Phys. Rev. Lett., **83**, 463 (1999).
49. M.L.D. Ion and D.B. Ion, *Angle-angular-momentum entropic bounds and optimal entropies for quantum scattering of spinless particles*, Phys. Rev., **E 60**, 5261 (1999).
50. D. B. Ion and M. L.D. Ion, *Limited entropic uncertainty as a new principle in quantum physics*, Phys. Lett., **B 474**, 395 (2000).
51. M.L.D. Ion and D.B. Ion, *Strong evidences for correlated nonextensive statistics in hadronic scatterings*, Phys. Lett., **B 482**, 57 (2000).
52. D.B. Ion and M. L.D. Ion, *Optimality entropy and complexity in quantum scattering*, Chaos Solitons&Fractals, **13**, 547 (2002).
53. D.B. Ion and M.L.D. Ion, *Evidences for nonextensive statistics conjugation in hadronic scatterings systems*, Phys. Lett., **B 503**, 263 (2001).
54. D.B. Ion and M.D. Ion, *New nonextensive quantum entropy and strong evidences for the equilibrium of quantum hadronic states*, Phys. Lett., **B 519**, 63 (2001).
55. D.B. Ion and M.D. Ion, *Nonextensive statistics and saturation of PMD-SQS-optimality limits in hadronic scattering*, Physica, **A 340**, 501 (2004).