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MAY BE THE ELECTRON TAKEN OFF THE CHARGE?

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Abstract. By using a Hamiltonian based on the coupling through flux lines we have calculated the electron-electron interaction energy via bosons. We have found that the electron-electron interaction via mass less bosons occurs according to the law $\alpha\hbar c/R$, which is an equivalent expression for the Coulomb's law. α is the fine structure constant equals with 1/137 in the three dimensional space and with 0.053 in the two dimensional space, respectively. In our treatment we do not taken into account the concept of the electronic charge.

Key words: electron-electron interaction, Coulomb's law, magnetic moment.

1. INTRODUCTION

In nature all forces can be divided into two classes: the forces that are induced by an exchange of massless particles and fall as a power of distance, and the rest [1]. In the first class there are the forces which follow the inverse square decay law, $F\sim 1/r^2$ (the Newton and the Coulomb forces), and forces that fall faster or slower than that. Among them are such extremes as the van der Waals forces between electrically neutral atoms which behave as $1/r^7$. Long range forces, behaving as $1/r^2$, can be created through an exchange of mass les bosons with different spins s = 0, 1, 2. In this paper we use a Hamiltonian based on the elastic coupling through flux lines and evaluate the electron-electron interaction energy *via* bosons; particularly we find an equivalent expression for the Coulomb's law. We have evaluated the fine structure constant in 3D and 2D spaces, respectively. In our treatment we do not taken into account the concept of the electronic charge. Neither the charge nor the mass of the electron or any other charged particle can actually calculated in QED- they have to be assumed.

2. THE ELECTRON – ELECTRON INTERACTION *VIA* BOSONS

In order to write the Hamiltonian for the electron-boson interaction consider a string (rope) on which electrons may walk. We have therefore the "tight-rope walking electrons". The Hamiltonian density of the isotropic elastic continuum is defined by [2, 3]

$$H_d == \frac{1}{2\rho}\Pi_\mu\Pi_\mu + \frac{DR_l}{2}\frac{\partial u_\mu}{\partial z_\mu}\frac{\partial u_l}{\partial z_l} + \frac{\beta R_l}{2}\frac{\partial u_\mu}{\partial z_l}\frac{\partial u_\mu}{\partial z_l} - \Psi^{"}(\mathbf{z})\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z_\mu^2}\right]\Psi(\mathbf{z}) + \\ + \frac{D}{2}s_l\frac{\partial u_\mu}{\partial z_\mu}\frac{\partial u_l}{\partial z_l}\Psi^*(\mathbf{z})\Psi(\mathbf{z}) + \frac{\beta}{2}s_l\frac{\partial u_\mu}{\partial z_l}\frac{\partial u_\mu}{\partial z_l}\Psi^+(\mathbf{z})\Psi(\mathbf{z}). \tag{1}$$

We are to sum over repeated indices. The coordinate axes z_μ are assumed to be orthogonal. The term in D is the square of the trace of the strain tensor, the term in β is the sum of the squares of the tensor components. R_l is the distance between the two electrons, s_l is the electron displacement near its equilibrium position,

$$\rho = \rho_o + \frac{(D\delta_{\mu l} + \beta)R_l}{c^2}, \tag{2}$$

ρ_o is "massive the density" of the interacting field, if this is a massive field, c is the velocity of the boson waves, and $(D\delta_{\mu l} + \beta)R_l / c^2$ is the "massless density" of the interacting field. Π_μ are the components of the momentum density, $u_\mu(\mathbf{z})$ is the displacement of the string (that is of the coupling field) at the position $\mathbf{z}$. The Hamiltonian of interaction is

$$H_I = \frac{D}{2}\int\sum_l R_l s_l \frac{\partial u_\mu}{\partial z_\mu}\frac{\partial u_l}{\partial z_l}\Psi^+(\mathbf{z})\Psi(\mathbf{z})\mathrm{d}\mathbf{z} + \frac{\beta}{2}\int\sum_l R_l s_l \frac{\partial u_\mu}{\partial z_l}\frac{\partial u_\mu}{\partial z_l}\Psi^+(\mathbf{z})\Psi(\mathbf{z})\mathrm{d}\mathbf{z} \tag{3}$$

where we have introduces the sum over neighbours, and [3]

$$u_\mu(\mathbf{z}) = \frac{1}{\sqrt{NR}}\sum_{\mathbf{q}}\left(\frac{\hbar}{2\rho\omega_{\mathbf{q}\mu}}\right)^{1/2}\left(a_{\mathbf{q}\mu}\mathrm{e}^{i\mathbf{qz}} + a^+_{\mathbf{q}\mu}\mathrm{e}^{-i\mathbf{qz}}\right) \tag{4}$$

$$s_l = \frac{1}{NR_l}\sum_{\mathrm{q_o}}\frac{\hbar}{2m\omega_{q_o l}}\left(b_{\mathrm{q_o}l} + b^+_{-\mathrm{q_o}l}\right)\mathrm{e}^{i\mathrm{q_o}(\mathbf{z}-\mathbf{z}_1)}$$

$\omega_{\mathbf{q}\mu}$ is the classical oscillation frequency

$$\omega_{\mathrm{q}\,\mu} = \left(\frac{D\delta_{\mu l} + \beta}{\rho}R\right)^{1/2} q, \tag{5}$$

where l denotes the longitudinal boson, a_q^+, a_q are the boson creation and annihilation operators, b_{qo}^+, b_{qo} are the creation and annihilation operators associated with the electron oscillations, m is the electron mass and

$$\Psi(\mathbf{z}) = \frac{1}{\sqrt{NR}} \sum_{\mathbf{k},\sigma} c_{\mathbf{k},\sigma} \mathrm{e}^{\mathrm{i}\mathbf{kz}}, \qquad \Psi^+(\mathbf{z}) = \frac{1}{\sqrt{NR}} \sum_{\mathbf{k},\sigma} c^+_{\mathbf{k},\sigma} \mathrm{e}^{-\mathrm{i}\mathbf{kz}}, \tag{6}$$

where N is the number of the links in the case of the linear lattice, $c_{k,\sigma}^+$, $c_{k,\sigma}$ are the electron creation and annihilation operators, **k** is the wave vector of an electron and σ is the spin quantum number. Finally, the interaction Hamiltonian may be written

$$H_I = -\hbar \sum_{\mathbf{k},\mathbf{k}',\mathbf{q},\mathbf{q}',\mathbf{q}_o,\sigma,\sigma'} \{ g_{\mathbf{q}o} b_{\mathbf{q}o} (a_{\mathbf{q}} + a^+_{-\mathbf{q}})(a_{-\mathbf{q}'} + a^+_{\mathbf{q}'}) c^+_{\mathbf{k}',\sigma'} c_{\mathbf{k},\sigma} \Delta(\mathbf{q} - \mathbf{q}' + \mathbf{k} - \mathbf{k}' + \mathbf{q}_o) + \\ + g^*_{\mathbf{q}o} b^+_{\mathbf{q}o} (a_{-\mathbf{q}} + a^+_{\mathbf{q}})(a_{\mathbf{q}'} + a^+_{-\mathbf{q}'}) c^+_{\mathbf{k},\sigma} c_{\mathbf{k}',\sigma'} \Delta(\mathbf{q}' - \mathbf{q} + \mathbf{k}' - \mathbf{k} - \mathbf{q}_o) \}, \tag{7}$$

where

$$g_{\mathbf{q}_o} = \frac{\hbar D}{8N^2 mR\left(\rho_o + \frac{DR}{c^2}\right)} \frac{\mathbf{qq}'}{\omega_{\mathbf{q}} \omega_{\mathbf{q}_o}} \mathrm{e}^{\sum_l \mathrm{i}\mathbf{q}_o \mathbf{z}_l}, \tag{8}$$

where shear term and also the indices μ were omitted. We use an approximation of nearest neighbours. In the interaction picture the effective Hamiltonian is given by [3, 4]

$$H_{\mathrm{I}}^{\mathrm{eff}} = H_{\mathrm{I1}} + H_{\mathrm{I2}},$$
$$H_{\mathrm{I1}} = \hbar \sum_{\mathbf{q},\mathbf{q}_o,\mathbf{k}} 2|g_{\mathbf{q}o}|^2 \frac{\omega_{\mathbf{q}}}{(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}-\mathbf{q}})^2 - \omega^2_{\mathbf{q}}} a_{\mathbf{q}} a_{\mathbf{q}o} a^+_{\mathbf{q}o} a^+_{\mathbf{q}} c^+_{\mathbf{k}-\mathbf{q},\sigma'} c^+_{\mathbf{k},\sigma} c_{\mathbf{k}-\mathbf{q},\sigma'} c_{\mathbf{k},\sigma}, \tag{9}$$
$$H_{\mathrm{I2}} = 2\hbar \sum_{\mathbf{q},\mathbf{k}} 2|g_{\mathbf{q}}|^2 \frac{1}{(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}-\mathbf{q}}) - \omega_{\mathbf{q}}} a_{\mathbf{q}} a^+_{\mathbf{q}} c^+_{\mathbf{k}-\mathbf{q},\sigma} c_{\mathbf{k}-\mathbf{q},\sigma},$$

where in the Kronecker Δ we have used $\mathbf{q}' = \mathbf{q}_o$, $\mathbf{k}' = \mathbf{k} - \mathbf{q}$. The expectation value of the energy of H_{I1} is the electron-electron interaction energy *via* bosons, while the expectation value of H_{I2} is the electron-boson interaction energy [3]. The expectation value of the energy of interaction is

$$E_I = \frac{\hbar^3 D^2}{32N^3 m^2 R^2 \left(\rho_o + \frac{DR}{c^2}\right)^2} \sum_{\mathbf{q},\mathbf{q_o},\mathbf{k}} \frac{(\mathbf{q}.\mathbf{q_o})^2}{\omega_{\mathbf{q}}^2 \omega_{\mathbf{q_o}}^2} \left| \frac{1}{\sqrt{N}} \sum_l e^{i\mathbf{q_o}.\mathbf{z}_l} \right|^2 \frac{\omega_{\mathbf{q}}}{\left(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k-q}}\right)^2 - \omega_{\mathbf{q}}^2} \times \tag{10}$$
$$\times \left(n_{\mathbf{q}} + 1\right)\left(n_{\mathbf{q_o}} + 1\right) n_{\mathbf{k}} n_{\mathbf{k-q}}.$$

In the case the electron-electron repulsion energy E_i' is given by Eq. (10) where we substitute the Fröhlich fraction [4]

$$\frac{\omega_{\mathbf{q}}}{\left(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k-q}}\right)^2 - \omega_{\mathbf{q}}^2} \tag{11a}$$

which give an attractive interaction when $\left(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k-q}}\right)^2 << \omega_{\mathbf{q}}^2$, by the fraction

$$\frac{-1}{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k-q}} - \omega_{\mathbf{q}}}, \tag{11b}$$

which give, in the same approximation, a repulsive interaction. The explanation consists in the fact that the fraction (11a) assumes that one particle delivers a boson and the other absorbs it, while the fraction (11b) assume that both electrons absorb or emit bosons.

3. AN EQUIVALENT EXPRESSION FOR THE COULOMB'S LAW

The expectation value of the energy of the interaction Hamiltonian H_{I1} (10) with (11b) is

$$E_i = 2\hbar \sum_{\mathbf{q},\mathbf{q_o},\mathbf{k}} \left|g_{q_o}\right|^2 \frac{-1}{\left(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k-q}}\right) - \omega_{\mathbf{q}}} \left(n_{\mathbf{q}} + 1\right)\left(n_{\mathbf{q_o}} + 1\right) n_{\mathbf{k}} n_{\mathbf{k-q}}, \tag{12}$$

where $g_{\mathbf{q}_o}$ is given by Eq. (8). For $n_{\mathbf{q}}, n_{\mathbf{q_o}} = 0, n_{\mathbf{k}}, n_{\mathbf{k-q}} = 1$, $N = 1$, from Eqs. (12) and (8) one gets

$$E_i = \sum_{\mathbf{q},\mathbf{q_o},\mathbf{k}} \frac{D^2}{32} \frac{\hbar^2}{\mu^2 R^2 \omega_{\mathbf{q}_o}^2} \frac{(\mathbf{q}.\mathbf{q_o})^2}{\left(\rho_o + \frac{DR}{c^2}\right)^2 \omega_{\mathbf{q}}^2} \left| \sum_l \mathrm{e}^{i\mathbf{q}_o.\mathbf{R}_l} \right|^2 \frac{-1}{\left(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k-q}}\right) - \omega_{\mathbf{q}}}. \tag{13a}$$

Further,

$\left|\sum_{l} e^{i\mathbf{q}_o.\mathbf{R}_l}\right|^2 = 2\left[1+\cos\left(\mathbf{q}_o.\mathbf{R}\right)\right]$, $\omega_{\mathbf{q}} = cq, \omega_{\mathbf{q_o}} = \hbar q_o^2/2m, \varepsilon_{\mathbf{k}} = \hbar\mathbf{k}^2/2m$, where c is the light velocity. If the boson field is a mass less field, then $\rho_o = 0$ and Eq. (13a) becomes

$$E_I = \frac{\hbar^3 c^4}{32\mu^2 R^4}\sum_{\mathbf{q},\mathbf{q_o},\mathbf{k}}\frac{\left(\mathbf{q}.\mathbf{q_o}\right)^2}{\omega_{\mathbf{q}}^2\omega_{\mathbf{qo}}^2}\left|\sum_n e^{i\mathbf{q_o}\mathbf{R_n}}\right|^2 \frac{-1}{\left(\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k-q}}\right)-\omega_{\mathbf{q}}}. \tag{13b}$$

Now we apply this equation to a system of two electrons at $\mathbf{R}_1$ and $\mathbf{R}_2$ acting on the vacuum of the less mass boson field. In this case $\sum_n \left|e^{i\mathbf{q_o}\mathbf{R_n}}\right|^2 = 2\left(1+\cos(\mathbf{q_o}\mathbf{R})\right), \varepsilon_{\mathbf{k}} = \hbar k^2/2m, \omega_{\mathbf{q}} = cq, \omega_{\mathbf{q_o}} = \hbar q_o^2/2m,$ $n_{\mathbf{q}}$ is the occupation number of bosons, $n_{\mathbf{k}}$ is the occupation number of fermions and $\mathbf{R} = \mathbf{R}_1 - \mathbf{R}_2$. Further, if we assume $\left(\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k-q}}\right) << \omega_{\mathbf{q}}$, we write

$$\sum_{\mathbf{q}}\frac{\left(\mathbf{q}.\mathbf{q_o}\right)^2}{\omega_{\mathbf{q}}^3\omega_{\mathbf{q_o}}^2} = \left(\frac{2m}{\hbar}\right)^2 \frac{1}{q_o^2 c^3}\frac{\Omega}{\left(2\pi\right)^2}\int_0^{\pi}\cos^2\alpha\sin\alpha\,\mathrm{d}\alpha\int_0^{q_o} q\,\mathrm{d}q = \left(\frac{2m}{\hbar}\right)^2\frac{R^3}{9\pi c^3}$$

and

$$\sum_{\mathbf{q_o}}\left[1+\cos\left(\mathbf{q}_o\mathbf{R}\right)\right] = \sum_{\mathbf{q_o}}1 + \sum_{\mathbf{q_o}}\cos\left(\mathbf{q_o}\mathbf{R}\right) =$$
$$= 1 + \frac{\Omega}{\left(2\pi\right)^2}\int_0^{\pi}\mathrm{d}\theta\int_0^{0.76\pi/R}\mathrm{d}q_o q_o^2\cos\left(q_o R\cos\theta\right)\sin\theta = 1.514.$$

We have considered $\sum_{\mathbf{k}} 1 = 1$ and $\Omega = 4\pi R^3/3$. The upper limit of the integrals over q, q_o appear from the requirement

$$\frac{4\pi R^3/3}{\left(2\pi\right)^3}\times 4\pi\frac{q_m^3}{3} = 1.$$

The interaction energy becomes [5]

$$E_I = 2\times 7.24\times 10^{-3}\frac{\hbar c}{R} \approx 2\times\frac{1}{137}\frac{\hbar c}{R} = 2\times\frac{\alpha\hbar c}{R}, \tag{14}$$

where α is the fine structure constant. The factor 2 appears because we have considered the two nearest neighbours of an electron. This is an equivalent expression of the Coulomb's law, because $\alpha\hbar c = e^2/4\pi\varepsilon_0$. In the case of the electron-proton interaction, as for example in the hydrogen atom, in front of the interaction energy (14) is the sign *minus*. When a particle has a charge Q, that is Q/e electronic charges, the term from the right-hand side of Eq. (14) is multiplied by Q_1Q_2/e^2, because in this case we must define $|\Psi|^2 \propto Q/e$ (the number of electronic charges per particle). In the case of the hydrogen like atoms $Q_1Q_2/e^2 = Z\cdot 1 = Z$, where Z is the nucleus charge.

4. THE COULOMB'S LAW IN THE TWO DIMENSIONAL SPACE

How does show the Coulomb interaction energy (14) in a two dimensional space? Let us consider Eq. (13*b*) in a two dimensional space. In a two dimensional space we have

$$\left|\sum_l \mathrm{e}^{i\mathbf{q}_o\cdot\mathbf{R}_l}\right|^2 \delta_{R_{12},R_n} = 2\left(1+\frac{1}{2\pi}\int_0^{2\pi}\mathrm{e}^{iq_oR_n\cos\theta}\mathrm{d}\theta\right) = 2(1+J_o(q_oR_n)), \tag{15}$$

where $J_o(x)$ is the Bessel function of the first kind, $\mathbf{R}_{12} = \mathbf{R}_1 - \mathbf{R}_2$, and the index n is the principal quantum number. Next,

$$\begin{aligned}\sum_{\mathbf{q}}\frac{(\mathbf{q}.\mathbf{q}_o)^2}{\omega_{\mathbf{q}}^3\omega_{\mathbf{q}_o}^2} &= \left(\frac{2\mu}{\hbar}\right)^2\sum_{\mathbf{q}}\frac{q^2q_o^2\cos^2\alpha}{q_o^4q^3c^3} = \left(\frac{2\mu}{\hbar}\right)^2\sum_{\mathbf{q}}\frac{\cos^2\alpha}{q_o^2qc^3} = \\ &= \frac{S}{(2\pi)^2}\left(\frac{2\mu}{\hbar}\right)^2\frac{1}{c^3q_o^2}\int_0^{2\pi}\cos^2\alpha\,\mathrm{d}\alpha\int_0^{q_o}\mathrm{d}q = \frac{\pi R^2}{(2\pi)^2}\left(\frac{2\mu}{\hbar}\right)^2\frac{\pi}{c^3q_o} = \left(\frac{2\mu}{\hbar}\right)^2\frac{R^2}{4c^3q_o}\end{aligned} \tag{16a}$$

and therefore,

$$\begin{aligned}\sum_{\mathbf{q},\mathbf{q_o}}\frac{(\mathbf{q}.\mathbf{q_o})^2}{\omega_{\mathbf{q}}^3\omega_{\mathbf{q}_o}^2}\{J_o(q_oR_n)+1\} &= \left(\frac{2\mu}{\hbar}\right)^2\frac{R^2}{4c^3}\frac{S}{(2\pi)^2}2\pi\int_0^{2/R}\frac{1+J_o(q_oR_n)}{q_o}q_o\mathrm{d}q_o = \\ &= \left(\frac{2\mu}{\hbar}\right)^2\frac{R^2}{4c^3}\frac{\pi R^2}{2\pi}\frac{1}{R}\left\{2+\int_0^2 J_o(q_oR_n)\,\mathrm{d}(q_oR_n)\right\} = \left(\frac{2\mu}{\hbar}\right)^2\frac{R^3}{8c^3}\{2+1.426\}.\end{aligned} \tag{16b}$$

The interaction energy in the two dimensional space becomes

$$E_i^{2D} = \frac{\hbar^3c^4}{32\mu^2R^4}\frac{4\mu^2}{\hbar^2}\frac{R^3}{8c^3}\times 3.426 = 2\times 0.053\frac{\hbar c}{R} = 2\times\frac{\alpha_{2D}\hbar c}{R}, \tag{17}$$

where the fine structure constant in the two dimensional space is

$$\alpha_{2D} = 0.053 \approx 7.26\alpha, \tag{18}$$

where α is the fine structure constant in the three dimensional space.

It appears that in the three dimensional space the Coulomb interaction (17) is approximately 7 times stronger than in the three dimensional space. The Bohr radius of the Hydrogen atom becomes

$$a_o^{2D} \equiv R_1^{2D} = \frac{\hbar}{4\alpha_{2D}\mu\, c},$$

that is 29 times smaller than in the three dimensional space, and the ground state energy of the hydrogen atom

$$E_{10}^{2D} = -\frac{4\alpha_D^2 m_r\, c^2}{2}, \tag{19}$$

that is 200 times larger than in the three dimensional space. Here m_r is the reduces mass of the proton and the electron. The factor 4 in the two last relations appears because in the two dimensional space [6] the quantum number n is substituted by $(n - ½)$.

5. A RELATION BETWEEN THE ELECTROSTATIC ENERGY OF THE ELECTRON IN THE HYDROGEN ATOM AND THE MAGNETIC MOMENT

Till now we can not found a definition of the magnetic filed or moment, without taking into account the electronic charge concept, as we have found for the electrostatic energy of interaction. If magnetism is the oldest great mystery of matter, it has also remained on of the most difficult to explain. Magnetic fields are produced by electric currents, which can be macroscopic currents in wires, or microscopic currents associated with electrons in atomic orbits. The magnetic field **B** is defined in terms of force on moving charge in the Lorentz force law. The magnetic field in the electron frame of reference arising from the orbital motion is

$$\mathbf{B} = \frac{\mu_o Ze\mathbf{L}}{4\pi m a_o^2}, \tag{20}$$

where **L** is the orbital magnetic momentum. For a hydrogen electron in a 2*p* state at a radius 4×Bohr radius, this translates to a magnetic filed of about 0.3 Tesla. This is fairly consistent with the splitting of levels observed in the hydrogen fine structure.

Now, if we can see the electron in the hydrogen atom as a quantum ring (if we neglect the dimensions of the proton with respect to the Bohr radius) the magnetic flux threading the ring, in z direction, is $\Phi_o = -(h/e)m_l$ and the corresponding magnetic filed is

$$B_o = -\frac{1}{\pi n^4 a_o^2}\frac{h}{e}m_l. \tag{21}$$

For $n = 2$, one obtains $B_o = 2.9\times10^4$ Tesla. Here n is the principal quantum number and m_l is the magnetic quantum number. On the other hand, the orbital magnetic moment of an electron in the hydrogen atom is

$$\vec{\mu} = -\frac{e}{2m}\mathbf{L}.$$

If the magnetic field is in the z direction we only care about the z-component of μ

$$\mu_z = -\frac{e}{2m}L_z = -\frac{e}{2m}\left(m_l\hbar\right). \tag{22}$$

The magnetic potential "selfenergy" of the electron may be written

$$E_i = -\mu_z B_o = -\frac{\hbar^2}{ma_o^2}\frac{m_l^2}{n^4} = -\frac{\alpha^2 mc^2 m_l^2}{n^4} = \frac{m_l^2}{n^2}E_c. \tag{23}$$

where E_c is just the Coulomb energy of electron in the hydrogen atom. But the existence of a field B_o (21) is not evidenced. In the fine structure of the hydrogen atom is evidenced the field $\boldsymbol{B}$ (20) only.

6. CONCLUSIONS

We have used a Hamiltonian based on the coupling through flux lines, in order to evaluate the electron-electron interaction energy *via* bosons, particularly the Coulomb's law. Relation (10) is a general formula, which is valid for any boson types, when $\rho_o = 0$ or $\rho_o << DR/c^2$ that is at large separation between particles and small velocity of the bosons.

To summarize, a testable prediction of our theory is the finding of a relation of interaction between two electrons *via* mass less bosons on the form $\alpha\hbar c/R$, which is equivalent to the Coulomb's law, $e^2/4\pi\varepsilon_o R$. In the two dimensional space we have found that the fine structure constant $\alpha_{2D} \approx 0.053$, which is by 7 times larger than that of three dimensional space. Also, we have established a relation between the electron magnetic moment of the electron in the hydrogen atom and its Coulomb energy of interaction. In another paper we have calculated the Lamb shift without taken into account the electronic charge [7]. The question is: the electron may be considered as taken off the charge or is an indestructible charge? What is this the charge? We must answer to these questions in the future.

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