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THEORETICAL PHYSICS

SOLUTIONS OF THE FRACTIONAL DAVEY-STEWARTSON EQUATIONS WITH VARIATIONAL ITERATION METHOD

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Abstract. This paper presents approximate analytical solutions for the fractional Davey-Stewartson equations using the Variational iteration method. The fractional derivatives are described in the Caputo sense. This method is based on the incorporation of the correction functional for the equation. The results obtained by this method have been compared with the exact solutions and show that the introduced approach is a promising tool for solving many linear and nonlinear fractional differential equations.

Key words: The fractional Davey-Stewartson equation, Variational iteration method, Caputo fractional derivative.

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1. INTRODUCTION

In recent years, fractional-order calculus has been studied as an alternative calculus in mathematics and it was applied in various fields of science and engineering [4, 24, 27]. Numerous problems in physics, chemistry, biology and engineering can be modeled with fractional derivatives [2, 10, 18–23, 25, 28–30]. Since most fractional differential equations do not have exact analytic solutions, approximate and numerical techniques, therefore, they are used extensively. Recently, the Adomian decomposition method and Homotopy perturbation method, differential transform method have been used for solving a wide range of problems [5, 12, 13].

Another powerful analytical method, called the variational iteration method (VIM), was first by Ji-Huan He [7]. This technique has successfully been applied to many situations, for example (see [1, 8, 14]). Ji-Huan He [9] was the first to apply the variational iteration method to fractional differential equations. Recently Odibat and Momani [26] implemented the variational iteration method to solve partial differential equations of fractional order.

In this paper, we introduce a new application of the variational iteration method to provide approximate solutions of the fractional Davey-Stewartson equations in the following form:

$$\frac{1}{2}\,\sigma^4 \frac{\partial^\alpha q}{\partial y^\alpha} + \frac{1}{2}\,\sigma^2 \frac{\partial^2 q}{\partial x^2} + i\frac{\partial q}{\partial t} + \lambda\,|q|^2\,q - \frac{\partial \phi}{\partial x} q = 0,$$
$$\frac{\partial^2 \phi}{\partial x^2} - \sigma^2 \frac{\partial^\alpha \phi}{\partial y^\alpha} - 2\,\lambda\,\frac{\partial(|q|^2)}{\partial x} = 0, \qquad 1 < \alpha \le 2$$

where $\frac{\partial^\alpha}{\partial y^\alpha}$ is Caputo fractional derivative.

The special case $\{\alpha = 2\ ,\ \sigma = 1\}$ is called the classical DS-I equation, while $\{\alpha = 2\ ,\ \sigma = i\}$ is the classical DS-II equation. The parameter λ characterizes the focusing or defocussing case. The classical Davey-Stewartson I and II are two well-known examples of integrable equations in two space dimensions, which arise as higher dimensional generalizations the nonlinear Schrödinger (NLS) equation [3]. Although there are a lot of studies for the classical Davey-Stewartson equation and some profound results have been established, it seems that detailed studies of the fractional Davey-Stewartson equation are only beginning. Jafari and Alipour obtained the solutions of the classical Davey-Stewartson by variational iteration method and homotopy analysis method respectively [15, 16]. We intend to take apply the variational iteration method to investigate the approximate solutions of the fractional Davey-Stewartson equations. We will also present numerical results to show the nature of the curves/surfaces as the fractional derivative parameter changed.

2. BASIC DEFINITIONS

A real function $f(y), y > 0$ is said to be in the space $C_\alpha, \alpha \in \Re$ if there exists a real number $p(> \alpha)$, such that $f(y) = y^p f_1(y)$, where $f_1 \in C[0,\infty]$. Clearly $C_\alpha \subset C_\beta$ if $\beta \le \alpha$. A function $f(y), y > 0$ is said to be in the space $C_\alpha^m, m \in N \bigcup \{0\}$, if $f^{(m)} \in C_\alpha$.

The left sided Riemann-Liouville fractional integral of order $\mu \ge 0$, [24, 27] of a function $f \in C_\alpha, \alpha \ge -1$ is defined as:

$$I_y^\mu f(y) = \begin{cases} \frac{1}{\Gamma(\mu)} \int_0^y \frac{f(\tau)}{(y-\tau)^{1-\mu}} d\tau, & \mu > 0,\ y > 0, \\ f(y), & \mu = 0. \end{cases} \tag{1}$$

The (left sided) Caputo fractional derivative of f, $f \in C_{-1}^{m}$, $m \in I\!N \bigcup \{0\}$, is defined as [24]:

$$D_y^{\mu} f(y) = \frac{\partial^{\mu} f(y)}{\partial y^{\mu}} = \begin{cases} I^{m-\mu}\left[\frac{\partial^m f(y)}{\partial y^m}\right], & m-1<\mu<m, \quad m \in I\!N, \\ \frac{\partial^m f(y)}{\partial y^m}, & \mu = m \end{cases} \tag{2}$$

Note that [24, 27]

$$(i) I_y^{\mu} f(x,y) = \frac{1}{\Gamma(\mu)} \int_0^y \frac{f(x,s)}{(y-s)^{1-\mu}} ds, \quad \mu>0,\ y>0,$$

$$(ii) D_y^{\mu} f(x,y) = I_y^{m-\mu} \frac{\partial^m f(x,y)}{\partial y^m}, \qquad m-1<\mu \le m. \tag{3}$$

$$(iii) I_y^{\mu} y^{\gamma} = \frac{\Gamma(\gamma+1)}{\Gamma(\gamma+\mu+1)} y^{\gamma+\mu}, \qquad \mu>0,\ \gamma>-1,\ y>0, \tag{4}$$

$$(iv) I_y^{\mu} D_y^{\mu} f(x,y) = f(x,y) - \sum_{k=0}^{m-1} \frac{\partial^k f(x,0^+)}{\partial y^k} \frac{y^k}{k!}, \quad m-1<\mu \le m. \tag{5}$$

3. VARIATIONAL ITERATION METHOD

The principles of the variational iteration method and its applicability for various kinds of differential equations are given in [17]. In [9], Ji-Huan He showed that the variational iteration method is also valid for fractional differential equations. In this section, following the discussion presented in [9], we extend the application of the variational iteration method to solve the fractional Davey-Stewartson equations:

$$\begin{aligned} &\frac{1}{2}\,\sigma^4 \frac{\partial^{\alpha} q}{\partial y^{\alpha}} + \frac{1}{2}\,\sigma^2 \frac{\partial^2 q}{\partial x^2} + i \frac{\partial q}{\partial t} + \lambda |q|^2 q - \frac{\partial \phi}{\partial x}\, q = 0, \\ &\frac{\partial^2 \phi}{\partial x^2} - \sigma^2 \frac{\partial^{\alpha} \phi}{\partial y^{\alpha}} - 2\,\lambda\, \frac{\partial(|q|^2)}{\partial x} = 0. \end{aligned} \tag{6}$$

Without loss of generality, first we separate the amplitude of a surface wave packet q into real part and imaginary part, i.e., $q = u + iv$. Then, we rewrite the fractional Davey-Stewartson (DS) equations Eq.(6) in the following form :

$$\begin{aligned} &\frac{\partial^{\alpha} u}{\partial y^{\alpha}} + \frac{1}{\sigma^2}\,\frac{\partial^2 u}{\partial x^2} - \frac{2}{\sigma^4}\,\frac{\partial v}{\partial t} + \frac{2\lambda}{\sigma^4}(u^3 + v^2 u) - \frac{2}{\sigma^4}(\frac{\partial \phi}{\partial x}\, u) = 0, \\ &\frac{\partial^{\alpha} v}{\partial y^{\alpha}} + \frac{1}{\sigma^2}\,\frac{\partial^2 v}{\partial x^2} + \frac{2}{\sigma^4}\,\frac{\partial u}{\partial t} + \frac{2\lambda}{\sigma^4}(v^3 + u^2 v) - \frac{2}{\sigma^4}(\frac{\partial \phi}{\partial x}\, v) = 0, \\ &\frac{\partial^{\alpha} \phi}{\partial y^{\alpha}} - \frac{1}{\sigma^2}\,\frac{\partial^2 \phi}{\partial x^2} + \frac{2\,\lambda}{\sigma^2}\,\frac{\partial(u^2+v^2)}{\partial x} = 0. \end{aligned} \tag{7}$$

According to the variational iteration method, we can construct the correction functional for Eq. (7) as:

$$
\begin{aligned}
u_{n+1}(x,y,t) &= u_n(x,y,t) + I_y^{\beta}\Bigg[\lambda_1\Bigg(\frac{\partial^{\alpha}u_n(x,y,t)}{\partial y^{\alpha}} + \frac{1}{\sigma^2}\,\frac{\partial^2 u_n(x,y,t)}{\partial x^2} \\
&- \frac{2}{\sigma^4}\,\frac{\partial v_n(x,y,t)}{\partial t} + \frac{2\lambda}{\sigma^4}\big(u_n(x,y,t)^3 + v_n(x,y,t)^2 u_n(x,y,t)\big) \\
&- \frac{2}{\sigma^4}\frac{\partial \phi_n(x,y,t)}{\partial x}u_n(x,y,t)\Bigg)\Bigg] = u_n(x,y,t) + \frac{1}{\Gamma(\beta)}\int_0^y (y-\zeta)^{(\beta-1)}\lambda_1(\zeta) \\
&\times\Bigg[\frac{\partial^{\alpha}u_n(x,\zeta,t)}{\partial \zeta^{\alpha}} + \frac{1}{\sigma^2}\frac{\partial^2 u_n(x,\zeta,t)}{\partial x^2} - \frac{2}{\sigma^4}\frac{\partial v_n(x,\zeta,t)}{\partial t} \\
&+ \frac{2\lambda}{\sigma^4}\Big(u_n(x,\zeta,t)^3 + v_n(x,\zeta,t)^2 u_n(x,\zeta,t)\Big) - \frac{2}{\sigma^4}\frac{\partial \phi_n(x,\zeta,t)}{\partial x}u_n(x,\zeta,t)\Bigg].
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
v_{n+1}(x,y,t) &= v_n(x,y,t) + I_y^{\beta}\Bigg[\lambda_2\Bigg(\frac{\partial^{\alpha}v_n(x,y,t)}{\partial y^{\alpha}} + \frac{1}{\sigma^2}\,\frac{\partial^2 v_n(x,y,t)}{\partial x^2} \\
&+ \frac{2}{\sigma^4}\,\frac{\partial u_n(x,y,t)}{\partial t} + \frac{2\lambda}{\sigma^4}\big(v_n(x,y,t)^3 + u_n(x,y,t)^2 v_n(x,y,t)\big) \\
&- \frac{2}{\sigma^4}\frac{\partial \phi_n(x,y,t)}{\partial x}v_n(x,y,t)\Bigg)\Bigg] = u_n(x,y,t) + \frac{1}{\Gamma(\beta)}\int_0^y (y-\zeta)^{(\beta-1)}\lambda_2(\zeta) \\
&\times\Bigg[\frac{\partial^{\alpha}v_n(x,\zeta,t)}{\partial \zeta^{\alpha}} + \frac{1}{\sigma^2}\frac{\partial^2 v_n(x,\zeta,t)}{\partial x^2} + \frac{2}{\sigma^4}\frac{\partial v_n(x,\zeta,t)}{\partial t} \\
&+ \frac{2\lambda}{\sigma^4}\big(v_n(x,\zeta,t)^3 + u_n(x,\zeta,t)^2 v_n(x,\zeta,t)\big) - \frac{2}{\sigma^4}\frac{\partial \phi_n(x,\zeta,t)}{\partial x}v_n(x,\zeta,t)\Bigg].
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
\phi_{n+1}(x,y,t) &= \phi_n(x,y,t) + I_y^{\beta}\Bigg[\lambda_3\Bigg(\frac{\partial^{\alpha}\phi_n(x,y,t)}{\partial y^{\alpha}} - \frac{1}{\sigma^2}\frac{\partial^2 \phi_n(x,y,t)}{\partial x^2} \\
&+ \frac{2\lambda}{\sigma^2}\frac{\partial}{\partial x}\left(v_n(x,y,t)^2 + u_n(x,y,t)^2\right)\Bigg)\Bigg] = \phi_n(x,y,t) + \frac{1}{\Gamma(\beta)}\int_0^y (y-\zeta)^{\beta-1}\lambda_3(\zeta) \\
&\times\left[\frac{\partial^{\alpha}\phi_n(x,\zeta,t)}{\partial \zeta^{\alpha}} - \frac{1}{\sigma^2}\frac{\partial^2 \phi_n(x,\zeta,t)}{\partial x^2} + \frac{2\lambda}{\sigma^2}\frac{\partial}{\partial x}\left(v_n(x,\zeta,t)^2 + u_n(x,\zeta,t)^2\right)\right].
\end{aligned}
\tag{10}
$$

Here I_y^{β} is the Riemann-Liouville fractional integral operator of order $\beta = \alpha - \mathrm{floor}(\alpha)$, that is $\beta = \alpha - 1$ (see for more details [26]) with respect to the variable y and $\lambda_i,\ (i=1,2,3)$ are the general Lagrange multipliers, which can be identified optimally via variational theory [11].

To identify approximately Lagrange multipliers, some approximation must be made. The correction functional for (8), (9) and (10) can be approximately expressed as follows:

$$
\begin{aligned}
&u_{n+1}(x,y,t)=u_n(x,y,t)\\
&+\int_0^y \lambda_1(\zeta)\left[\frac{\partial^2 u_n(x,\zeta,t)}{\partial\zeta^2}+\frac{1}{\sigma^2}\frac{\partial^2\widetilde{u_n}(x,\zeta,t)}{\partial x^2}-\frac{2}{\sigma^4}\frac{\partial\widetilde{v_n}(x,\zeta,t)}{\partial t}\right.\\
&\left.+\frac{2\lambda}{\sigma^4}(\widetilde{u_n}(x,\zeta,t)^3+\widetilde{v_n}(x,\zeta,t)^2\widetilde{u_n}(x,\zeta,t))-\frac{2}{\sigma^4}(\frac{\partial\widetilde{\phi_n}(x,\zeta,t)}{\partial x}\widetilde{u_n}(x,\zeta,t))\right],\\
&v_{n+1}(x,y,t)=v_n(x,y,t)\\
&+\int_0^y \lambda_2(\zeta)\left[\frac{\partial^2 v_n(x,\zeta,t)}{\partial\zeta^2}+\frac{1}{\sigma^2}\ \frac{\partial^2\widetilde{v_n}(x,\zeta,t)}{\partial x^2}+\frac{2}{\sigma^4}\frac{\partial\widetilde{u_n}(x,\zeta,t)}{\partial t}\right.\\
&\left.+\frac{2\lambda}{\sigma^4}(\widetilde{v_n}(x,\zeta,t)^3+\widetilde{u_n}(x,\zeta,t)^2\widetilde{v_n}(x,\zeta,t))-\frac{2}{\sigma^4}(\frac{\partial\widetilde{\phi_n}(x,\zeta,t)}{\partial x}\ \widetilde{v_n}(x,\zeta,t))\right],\\
&\phi_{n+1}(x,y,t)=\phi_n(x,y,t)+\int_0^y \lambda_3(\zeta)\left[\frac{\partial^2\phi_n(x,\zeta,t)}{\partial\zeta^2}-\frac{1}{\sigma^2}\ \frac{\partial^2\widetilde{\phi_n}(x,\zeta,t)}{\partial x^2}+\right.\\
&\left.\frac{2\lambda}{\sigma^2}\frac{\partial}{\partial x}(\widetilde{v_n}(x,\zeta,t)^2+\widetilde{u_n}(x,\zeta,t)^2))\right],
\end{aligned}
$$

where $\widetilde{u_n},\widetilde{v_n}$ and $\widetilde{\phi_n}$ are considered as restricted variations, which $\delta\widetilde{u_n}=\delta\widetilde{v_n}=\delta\widetilde{\phi_n}=0$. To find the optimal λ_1,λ_2 and λ_3 we proceed as follows:

$$
\begin{cases}
\delta u_{n+1}(x,y,t)=\delta u_n(x,y,t)+\delta\int_0^y \lambda_1(\zeta)\Big[\frac{\partial^2 u_n(x,\zeta,t)}{\partial\zeta^2}+\frac{1}{\sigma^2}\frac{\partial^2\widetilde{u_n}(x,\zeta,t)}{\partial x^2}\\
-\frac{2}{\sigma^4}\frac{\partial\widetilde{v_n}(x,\zeta,t)}{\partial t}+\frac{2\lambda}{\sigma^4}(\widetilde{u_n}(x,\zeta,t)^3+\widetilde{v_n}(x,\zeta,t)^2\widetilde{u_n}(x,\zeta,t))\\
-\frac{2}{\sigma^4}\frac{\partial\widetilde{\phi_n}(x,\zeta,t)}{\partial x}\widetilde{u_n}(x,\zeta,t)\Big]=0,\\
\delta v_{n+1}(x,y,t)=\delta v_n(x,y,t)+\delta\int_0^y \lambda_2(\zeta)\Big[\frac{\partial^2 v_n(x,\zeta,t)}{\partial\zeta^2}+\frac{1}{\sigma^2}\ \frac{\partial^2\widetilde{v_n}(x,\zeta,t)}{\partial x^2}\\
+\frac{2}{\sigma^4}\frac{\partial\widetilde{u_n}(x,\zeta,t)}{\partial t}+\frac{2\lambda}{\sigma^4}(\widetilde{v_n}(x,\zeta,t)^3+\widetilde{u_n}(x,\zeta,t)^2\widetilde{v_n}(x,\zeta,t))\\
-\frac{2}{\sigma^4}\frac{\partial\widetilde{\phi_n}(x,\zeta,t)}{\partial x}\ \widetilde{v_n}(x,\zeta,t)\Big]=0,\\
\delta\phi_{n+1}(x,y,t)=\delta\phi_n(x,y,t)+\delta\int_0^y \lambda_3(\zeta)\Big[\frac{\partial^2\phi_n(x,\zeta,t)}{\partial\zeta^2}-\frac{1}{\sigma^2}\ \frac{\partial^2\widetilde{\phi_n}(x,\zeta,t)}{\partial x^2}\\
+\frac{2\lambda}{\sigma^2}\frac{\partial}{\partial x}(\widetilde{v_n}(x,\zeta,t)^2+\widetilde{u_n}(x,\zeta,t)^2)\Big]=0.
\end{cases}
$$

The stationary conditions for $i=1,2,3$ can be obtained as follows:

$$\lambda_i''(\zeta)|_{\zeta=y}=0, \quad 1-\lambda_i'(\zeta)|_{\zeta=y}=0, \quad \lambda_i(\zeta)|_{\zeta=y}=0 \quad \Leftrightarrow \quad \lambda_i(\zeta)=\zeta-y.$$

Substituting $\lambda_i(\zeta)=\zeta-y$, $(i=1,2,3)$ into the functional (8), (9) and (10), we obtain the following iteration formula:

$$\begin{aligned} u_{n+1}(x,y,t) = u_n(x,y,t) - (\alpha-1)I_y^{\alpha}\Bigg[&\frac{\partial^{\alpha}u_n(x,y,t)}{\partial y^{\alpha}} + \frac{1}{\sigma^2}\,\frac{\partial^2 u_n(x,y,t)}{\partial x^2} \\ &- \frac{2}{\sigma^4}\,\frac{\partial v_n(x,y,t)}{\partial t} + \frac{2\lambda}{\sigma^4}\left(u_n(x,y,t)^3 + v_n(x,y,t)^2\,u_n(x,y,t)\right) \\ &- \frac{2}{\sigma^4}\left(\frac{\partial \phi_n(x,y,t)}{\partial x}u_n(x,y,t)\right)\Bigg], \end{aligned} \tag{11}$$

$$\begin{aligned} v_{n+1}(x,y,t) = v_n(x,y,t) - (\alpha-1)I_y^{\alpha}\Bigg[&\frac{\partial^{\alpha}v_n(x,y,t)}{\partial y^{\alpha}} + \frac{1}{\sigma^2}\frac{\partial^2 v_n(x,y,t)}{\partial x^2} \\ &+ \frac{2}{\sigma^4}\frac{\partial u_n(x,y,t)}{\partial t} + \frac{2\lambda}{\sigma^4}\left(v_n(x,y,t)^3 + u_n(x,y,t)^2 v_n(x,y,t)\right) \\ &- \frac{2}{\sigma^4}\left(\frac{\partial \phi_n(x,y,t)}{\partial x}v_n(x,y,t)\right)\Bigg], \end{aligned} \tag{12}$$

$$\begin{aligned} \phi_{n+1}(x,y,t) = \phi_n(x,y,t) - (\alpha-1)I_y^{\alpha}\Bigg[&\frac{\partial^{\alpha}\phi_n(x,y,t)}{\partial y^{\alpha}} - \frac{1}{\sigma^2}\,\frac{\partial^2 \phi_n(x,y,t)}{\partial x^2} \\ &+ \frac{2\lambda}{\sigma^2}\frac{\partial}{\partial x}\left(v_n(x,y,t)^2 + \, u_n(x,y,t)^2\right)\Bigg]. \end{aligned} \tag{13}$$

The initial approximations $u_0(x,y,t)$, $v_0(x,y,t)$ and $\phi_0(x,y,t)$ can be freely chosen if they satisfies the initial conditions of the problem. Finally, we approximate the solutions $u(x,y,t)=\lim\limits_{n\longrightarrow\infty}u_n(x,y,t)$, $v(t)=\lim\limits_{n\longrightarrow\infty}v_n(x,y,t)$ and $\phi(x,y,t)=\lim\limits_{n\longrightarrow\infty}\phi_n(x,y,t)$ by the N^{th} term $x_N(x,y,t)$, $y_N(x,y,t)$ and $\phi_N(x,y,t)$.

3.1. NUMERICAL RESULTS

In this section, some numerical results are presented to support our theoretical analysis. We consider the following initial conditions :

$$\begin{aligned} u(x,0,t) &= r\,\mathrm{sech}\,[s(x-ct)]\cos[(k_1x+k_3t)], \\ v(x,0,t) &= r\,\mathrm{sech}\,[s(x-ct)]\sin[(k_1x+k_3t)], \\ \phi(x,0,t) &= f\tanh\,[s(x-ct)]\,, \end{aligned} \tag{14}$$

where $c=k_2+\sigma^2k_1$, $r=\sqrt{-(2k_3+k_1^2\sigma^2+k_2^2)/\lambda}$, $s=\sqrt{(2k_3+k_1^2\sigma^2+k_2^2)/\sigma^2}$, $f=(2\sigma\sqrt{-\lambda})/(1-\sigma_2)$ and $k_i(i=1;2;3)$ are arbitrary constants.

The exact solutions, for the special case $\alpha = 2$ is given by [16]:

$$\begin{aligned} u(x,y,t) &= r\,\mathrm{sech}\,[s(x+y-ct)]\cos[(k_1x+k_2y+k_3t)], \\ v(x,y,t) &= r\,\mathrm{sech}\,[s(x+y-ct)]\sin[(k_1x+k_2y+k_3t)], \\ \phi(x,y,t) &= f\tanh[s(x+y-ct)]. \end{aligned} \tag{15}$$

According to the variational iteration formulas Eqs.(11), (12) and (13), if we begin with:

$$\begin{aligned} u_0(x,y,t) &= r\,\mathrm{sech}\,[s(x-ct)]\cos[(k_1x+k_3t)], \\ v_0(x,y,t) &= r\,\mathrm{sech}\,[s(x-ct)]\sin[(k_1x+k_3t)], \\ \phi_0(x,y,t) &= f\tanh[s(x-ct)]. \end{aligned}$$

We can obtain the following approximations:

$$\begin{aligned} u_1 &= r\cos[xk_1+tk_3]\,\mathrm{sech}[s(-ct+x)] + \frac{2frsy^\alpha\,\mathrm{sech}[s(-ct+x)]^3\cos[xk_1+tk_3]}{\sigma^4\Gamma(\alpha)} \\ &- \frac{2frsy^\alpha\,\mathrm{sech}[s(-ct+x)]^3\cos[xk_1+tk_3]}{\sigma^4\Gamma(\alpha+1)} + \frac{rs^2y^\alpha\cos[k_1x+k_3t]\,\mathrm{sech}[s(-ct+x)]^3}{\sigma^2\Gamma(\alpha)} \\ &- \frac{rs^2y^\alpha\cos[xk_1+tk_3]\,\mathrm{sech}[s(-ct+x)]^3}{\sigma^2\Gamma(\alpha+1)} - \frac{2r^3y^\alpha\lambda\cos[xk_1+tk_3]^3\,\mathrm{sech}[s(-ct+x)]^3}{\sigma^4\Gamma(\alpha)} \\ &+ \ldots], \end{aligned}$$

$$\begin{aligned} v_1 &= r\sin[xk_1+tk_3]\,\mathrm{sech}[s(-ct+x)] + \frac{2frsy^\alpha\,\mathrm{sech}[s(-ct+x)]^3\sin[xk_1+tk_3]}{\sigma^4\Gamma(\alpha)} \\ &- \frac{2frsy^\alpha\,\mathrm{sech}[s(-ct+x)]^3\sin[xk_1+tk_3]}{\sigma^4\Gamma(\alpha+1)} + \frac{rs^2y^\alpha\sin[k_1x+k_3t]\,\mathrm{sech}[s(-ct+x)]^3}{\sigma^2\Gamma(\alpha)} \\ &- \frac{rs^2y^\alpha\sin[xk_1+tk_3]\,\mathrm{sech}[s(-ct+x)]^3}{\sigma^2\Gamma(\alpha+1)} - \frac{2r^3y^\alpha\lambda\sin[xk_1+tk_3]^3\,\mathrm{sech}[s(-ct+x)]^3}{\sigma^4\Gamma(\alpha)} \\ &+ \ldots], \end{aligned}$$

$$\begin{aligned} \phi_1 &= f\tanh[s(-ct+x)] - \frac{2fs^2y^\alpha\,\mathrm{sech}[s(-ct+x)]^2\tanh[s(-ct+x)]}{\sigma^2\Gamma(\alpha)} \\ &\quad + \frac{2fs^2y^\alpha\tanh[s(-ct+x)]\,\mathrm{sech}[s(-ct+x)]^2}{\sigma^2\Gamma(\alpha+1)}] \\ &\qquad + \frac{4r^2sy^\alpha\lambda\,\mathrm{sech}[s(-ct+x)]^2\tanh[s(-ct+x)]}{\sigma^2\Gamma(\alpha)} + \ldots], \end{aligned}$$

$$\vdots$$

and so on, in the same manner the rest of components of the iteration formula can be obtained using the Mathematica package.

Tables 1-3 show the absolute errors between the approximate solutions obtained for value of $\alpha = 1.98$ by the variational iteration method and the exact solutions.

Table 1.

Absolute errors of $u(x, y, t)$

x/t	0.1	0.2	0.3	0.4	0.5
20	1.28402×10^{-8}	1.17181×10^{-8}	1.05973×10^{-8}	9.47895×10^{-9}	8.36397×10^{-9}
17	1.97597×10^{-8}	8.07466×10^{-9}	3.49088×10^{-9}	1.49275×10^{-8}	2.6226×10^{-8}
14	9.95054×10^{-7}	1.10588×10^{-6}	1.21449×10^{-6}	1.32079×10^{-6}	1.42471×10^{-6}
11	2.18989×10^{-5}	2.28375×10^{-5}	2.37448×10^{-5}	2.46202×10^{-5}	2.54634×10^{-5}
7	7.73714×10^{-4}	7.8569×10^{-4}	7.96811×10^{-4}	8.07073×10^{-4}	8.16478×10^{-4}

Table 2.

Absolute errors of $v(x, y, t)$

x/t	0.1	0.2	0.3	0.4	0.5
20	4.73679×10^{-8}	4.75531×10^{-8}	4.76923×10^{-8}	4.77859×10^{-8}	4.78342×10^{-8}
17	5.18236×10^{-7}	5.15453×10^{-7}	5.12222×10^{-7}	5.08554×10^{-7}	5.04455×10^{-7}
14	5.17454×10^{-6}	5.09924×10^{-6}	5.02004×10^{-6}	4.93706×10^{-6}	4.85041×10^{-6}
11	4.67751×10^{-5}	4.55803×10^{-5}	4.43562×10^{-5}	4.31045×10^{-5}	4.18265×10^{-5}
7	7.05161×10^{-4}	6.69475×10^{-4}	6.33556×10^{-4}	5.97442×10^{-4}	5.61168×10^{-4}

Table 3.

Absolute errors of $\phi(x, y, t)$

x/t	0.1	0.2	0.3	0.4	0.5
20	9.992×10^{-16}	1.1102×10^{-15}	1.1102×10^{-15}	9.992×10^{-16}	9.992×10^{-16}
17	1.169×10^{-13}	1.157×10^{-13}	1.1457×10^{-13}	1.1324×10^{-13}	1.1224×10^{-13}
14	1.2696×10^{-11}	1.256×10^{-11}	1.2439×10^{-11}	1.231×10^{-11}	1.219×10^{-11}
11	1.3837×10^{-9}	1.370×10^{-9}	1.3565×10^{-9}	1.3431×10^{-9}	1.3299×10^{-9}
7	7.2190×10^{-7}	6.6947×10^{-4}	7.07465×10^{-7}	7.0031×10^{-7}	6.9321×10^{-7}

Figures 1-3 show the approximate solutions (7) obtained for values of $\alpha = 1.98$ and $\alpha = 1.8$ using the variational iteration method and exact solutions. It can be seen from Figures 1-3 that the solutions obtained by the present method is nearly identical with the exact solutions. Clearly, including more higher terms leads to highly accurate results. It is to be noted that only the two-order term of the variational iteration solution for the special case $y = 0.2$, $k_1 = 0.1$, $k_2 = 0.03$, $k_3 = -0.3$, $\sigma = I$, $\lambda = 1$ used in evaluating the approximate solutions for Tables 1-3 and Figures 1-3.

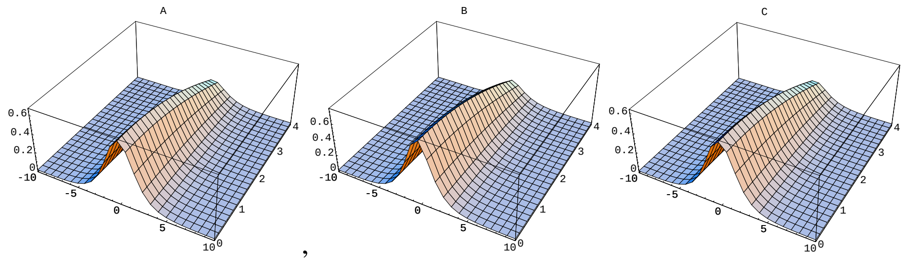

Fig. 1 – The surface shows the solution $u(x,y,t)$ for (7): (A) approximate solution for $\alpha = 1.98$; (B) exact solution ;(C) approximate solution for $\alpha = 1.8$.

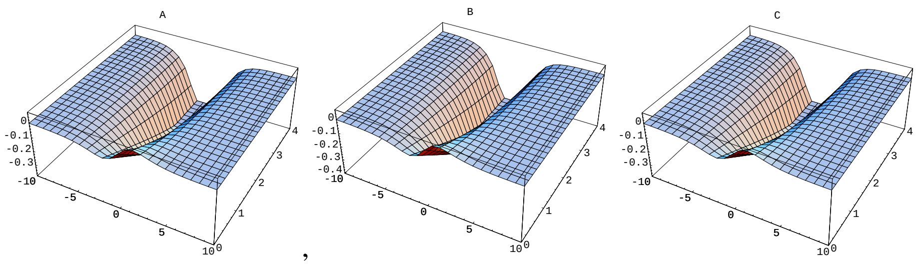

Fig. 2 – The surface shows the solution $v(x,y,t)$ for (7): (A) approximate solution for $\alpha = 1.98$; (B) exact solution ;(C) approximate solution for $\alpha = 1.8$.

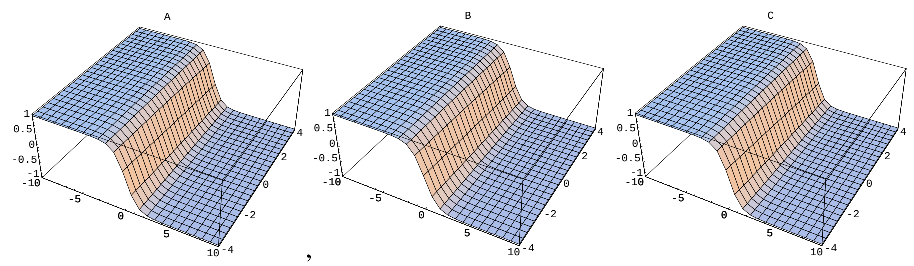

Fig. 3 – The surface shows the solution $\phi(x,y,t)$ for (7): (A) approximate solution for $\alpha = 1.98$; (B) exact solution; (C) approximate solution for $\alpha = 1.8$.

4. CONCLUSIONS

The variational iteration method is a powerful method which is able of handling linear/nonlinear fractional differential equations. The method has been applied to fractional Davey-Stewartson differential equations for their approximate solutions. The results show that the applied method is suitable and inexpensive for obtaining approximate solutions.

REFERENCES

1. M.A. Abdou, A.A. Soliman, Phys. D, **211**, 1 (2005).
2. D. Baleanu, S.I. Muslih, Phys. Scripta **72**(2-3), 119 (2005).
3. A. Davey, K. Stewartson, Proc. R. Soc. Lond.A. **338**, 101 (1974).
4. L. Debnath, J. Math. Educ. Sci. Tech. **35**(4), 487 (2004).
5. V. Erturk, S. Momani, Z. Odibat, Comm. Non. Sci. Numer. Simul. **13**, 1642 (2008).
6. V. Gafiychuk, B. Datsko, Phys. Lett. A **372**, 4902 (2008).
7. J. H. He, Comm. Sci. Num. Sim. **2**, 230 (1997).
8. J. H. He, J. Chaos. Solit. Fract. **19**, 847 (2004).
9. J. H. He, J. Comput. Meth. Appl. Mech. Eng. **167**, 57 (1998).
10. R. Hilfer, *Applications of Fractional Calculus in Physics* (Academic Press, Orlando, 1999).
11. M. Inokuti, H. Sekine, T. Mura, in *Variational Method in the Mechanics of Solids*, p. 156, S.Nemat-Nasser (ed.) (Pergamon Press, Oxford, 1978).
12. H. Jafari, S. Momani, Phys. Lett. A **370**, 388 (2007).
13. H. Jafari, V. Daftardar-Gejji, J. Appl. Math. Comput **180**, 488 (2006).
14. H. Jafari, A. Alipoor, Far East J. Math. Sci. **31**, 9 (2008).
15. H. Jafari, M. Alipour, World Appl. Sci. J. **8**(7), 814 (2010).
16. H. Jafari, M. Alipour, Nonlin. Anal.: Modell. Contr. **15**(4), 423 (2010).
17. S. Momani, S. Abuasad, J. Chaos. Solit. Fract. **12**, 1119 (2006).
18. A. Kadem, Appl. Sci. **11**, 78 (2009).
19. A. Kadem, D.Baleanu, Commun. Nonlin. Sci. Num. Simul. **15**(3), 491 (2010).
20. A. Kadem,D. Baleanu, Com. Math. Appl. **59**(5), 1865 (2010).
21. D. Baleanu, S. I. Muslih, E.M. Rabei, A.K. Golmankhaneh, Ali K.Golmankhaneh, Rom. Rep. Phys. **63**(1), 3 (2011).
22. S. I. Muslih, D. Baleanu, Rom. Rep. Phys. **62**, 689 (2010).
23. M. Klimek, Czech. J. Phys. **52**, 1247 (2002).
24. A. A. Kilbas, H. H. Srivastava, J.J. Trujillo, *Theory and Application of Fractional Differential Equations* (Elsevier, Amsterdam, 2006).
25. R. L. Magin, X. Feng, D. Baleanu, Concept. Magn. Reson. **34A**(1), 16 (2009).
26. Z. Odibat, S. Momani, J. Comm. Math. Appl. **58**, 2199 (2009).
27. I. Podlubny, *Fractional Differential Equations* (Academic Press, San Diego, 1999).
28. E. M. Rabei, K. I. Nawafleh, R. S. Hijjawi, S. I. Muslih, D. Baleanu, J. Math. Anal. Appl. **327**, 891 (2007).
29. A. M. Spasic, M. P. Lazarevic, J. Coll. Int. Sci. **282**, 223 (2005).
30. M. F. Silva, J. A. Tenreiro Machado, A. M. Lopes, Robotica **23**, 595 (2005).
31. Y. Wang, C. Li, Phys. Lett. A **372**, 414 (2007).