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BRIGHT AND DARK SOLITONS OF THE MODIFIED COMPLEX GINZBURG LANDAU EQUATION WITH PARABOLIC AND DUAL-POWER LAW NONLINEARITY

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Abstract. This paper obtains the exact bright and dark soliton solutions of the complex Ginzburg-Landau equation with parabolic and dual-power law nonlinearities, governing the propagation of solitons through nonlinear optical fibers. The solitary wave ansatz is used to carry out the integration of the considered models. Parametric conditions for the existence of the exact solutions are given.

Key words: exact bright and dark soliton solutions, complex Ginzburg-Landau equation.

1. INTRODUCTION

The study of the modified complex Ginzburg-Landau (mCGL) equation has been going on for the past few years [1–20]. There has been quite a bit of focus on the study of this equation with Kerr and power laws of nonlinearity. One of the most important aspects of this equation is in its integrability. Previously some authors have addressed the integrability of this mCGL equation in Kerr and power law nonlinearity. However, in those papers the semi-inverse variational principle was applied to carry out the integration of this equation [1].

Several methods of integrating such types of equations have been developed in the past few years. They are the G'/G method, exp-function method, Riccati equation method, simplest equation method, Fan's F-expansion method, Lie symmetry analysis, collective variable approach, and many others. In this paper, there will be one such method that will be addressed to solve the mCGL equation, that is known as the *ansatz method*. It is only the parabolic law as well as the dual-power laws of nonlinearity that will be addressed.

2. GOVERNING EQUATION

The dimensionless form of the mCGL equation that will be studied in this paper is given by [1]

$$\mathrm{i}q_t + aq_{xx} + bF\left(|q|^2\right)q = \alpha\frac{|q_x|^2}{q^*} + \frac{\beta}{4|q|^2 q^*}\left[2|q|^2\left(|q|^2\right)_{xx} - \left\{\left(|q|^2\right)_x\right\}^2\right]. \quad (1)$$

Equation (1) is the mCGL equation with nonlinearity given by the function F. This equation for Kerr law nonlinearity was studied using the Hirota bilinear method. The modulational stability of this equation was also addressed [11].

In (1) the dependent variable $q(x,t)$ represents the wave profile that arises in various physical systems, including nonlinear optics, plasma physics and others. The independent variables are x and t that represents the spatial and temporal variables. The real valued constants are a, b, α and β where a and b represents the coefficients of dispersion and nonlinearity. The function F is a real-valued algebraic function and it is necessary to have the smoothness of the complex function $F\left(|q|^2\right)q : C \mapsto C$. Considering the complex plane C as a two-dimensional linear space R^2, the function $F\left(|q|^2\right)q$ is k times continuously differentiable, so that [1]

$$F\left(|q|^2\right)q \in \bigcup_{m,n=1}^{\infty} C^k\left((-n,n)\times(-m,m);R^2\right). \quad (2)$$

In this paper the two forms of nonlinearity of the function F that will be considered are the parabolic law and the dual-power law. The next two subsections are going to focus on the ansatz method to extract the 1-soliton solutions for these two types of nonlinear media.

3. PARABOLIC LAW

For parabolic law medium, $F(s) = bs + cs^2$ so that equation (1) can be rewritten as [4]

$$\mathrm{i}q_t + aq_{xx} + \left(b|q|^2 + c|q|^4\right)q = \alpha \frac{|q_x|^2}{q^*} + \frac{\beta}{4|q|^2 q^*}\left[2|q|^2\left(|q|^2\right)_{xx} - \left\{\left(|q|^2\right)_x\right\}^2\right], \quad (3)$$

where c is another real-valued constant. Parabolic law nonlinearity arises in the context of plasma physics to study the nonlinear interaction between Langmuir waves and electrons. This equation (2) will be studied in this section and the bright and dark 1-soliton solution will be obtained in this context. This study will now be split into the following two subsections where the bright and dark solitons will be studied separately.

3.1. BRIGHT SOLITONS

In order to obtain the bright soliton solution to (3), the starting hypothesis is taken to be [4]

$$q(x,t) = \frac{A}{\left[D_1 + \cosh\tau\right]^p}\mathrm{e}^{\mathrm{i}\varphi}, \quad (4)$$

where

$$\tau = B\left(x - vt\right) \quad (5)$$

and the phase is given by

$$\varphi(x,t) = -\kappa x + \omega t + \theta. \quad (6)$$

Here, in (4–6), A is the amplitude of the soliton, while v is the velocity and B is the inverse width of the soliton. Also, κ is the frequency of the soliton, while ω is the wave number of the soliton and θ is the phase constant. The exponent p is unknown at this point and its value will fall out in the process of deriving the solution of this equation. Also D_1 is a constant whose value in terms of the known parameters will be determined during the course of derivation of the soliton solution. Thus from ansatz (4),

$$q_t = \left\{\frac{pBvA\sinh\tau}{\left[D_1 + \cosh\tau\right]^{p+1}} + \frac{\mathrm{i}\omega A}{\left[D_1 + \cosh\tau\right]^p}\right\}\mathrm{e}^{\mathrm{i}\varphi}, \quad (7)$$

$$
q_{xx}=\left\{\frac{p(p+1)B^2A\left(D_1^2-1\right)}{\left[D_1+\cosh\tau\right]^{p+2}}+\frac{Ap^2B^2}{\left[D_1+\cosh\tau\right]^{p}}-\frac{p(2p+1)D_1B^2A}{\left[D_1+\cosh\tau\right]^{p+1}}\right.
$$
$$
\left.+\frac{2i\kappa pBA\sinh\tau}{\left[D_1+\cosh\tau\right]^{p+1}}-\frac{A\kappa^2}{\left[D_1+\cosh\tau\right]^{p}}\right\}e^{i\varphi}, \tag{8}
$$

$$
\frac{|q_x|^2}{q^*}=\left\{\frac{A\left(p^2B^2+\kappa^2\right)}{\left[D_1+\cosh\tau\right]^{p}}-\frac{2Ap^2B^2D_1}{\left[D_1+\cosh\tau\right]^{p+1}}+\frac{Ap^2B^2\left(D_1^2-1\right)}{\left[D_1+\cosh\tau\right]^{p+2}}\right\}e^{i\varphi}, \tag{9}
$$

$$
\frac{\left(|q|^2\right)_{xx}}{q^*}=\left\{\frac{4Ap^2B^2}{\left[D_1+\cosh\tau\right]^{p}}-\frac{2p(4p+1)AB^2D_1}{\left[D_1+\cosh\tau\right]^{p+1}}+\frac{2p(2p+1)AB^2\left(D_1^2-1\right)}{\left[D_1+\cosh\tau\right]^{p+2}}\right\}e^{i\varphi} \tag{10}
$$

$$
\frac{\left\{\left(|q|^2\right)_x\right\}^2}{|q|^2q^*}=\left\{\frac{4Ap^2B^2}{\left[D_1+\cosh\tau\right]^{p}}-\frac{8p^2AB^2D_1}{\left[D_1+\cosh\tau\right]^{p+1}}+\frac{4p^2AB^2\left(D_1^2-1\right)}{\left[D_1+\cosh\tau\right]^{p+2}}\right\}e^{i\varphi}. \tag{11}
$$

Substituting (7–11) into (4) yields

$$
\begin{aligned}
&-\frac{\omega A+aA\kappa^2-aAp^2B^2+\alpha A\left(p^2B^2+\kappa^2\right)+\beta Ap^2B^2}{\left[D_1+\cosh\tau\right]^{p}}\\
&+\frac{2\alpha Ap^2B^2D_1+(\beta-a)p(2p+1)AB^2D_1+ipBvA\sinh\tau+2ai\kappa pBA\sinh\tau}{\left[D_1+\cosh\tau\right]^{p+1}}\\
&+\frac{ap(p+1)B^2A\left(D_1^2-1\right)-\alpha Ap^2B^2\left(D_1^2-1\right)-\beta p(p+1)AB^2\left(D_1^2-1\right)}{\left[D_1+\cosh\tau\right]^{p+2}}\\
&+\frac{bA^3}{\left[D_1+\cosh\tau\right]^{3p}}+\frac{cA^5}{\left[D_1+\cosh\tau\right]^{5p}}=0.
\end{aligned} \tag{12}
$$

Now, separating the real and imaginary parts in Eq. (12), we get the following pair of relations:

$$
\frac{pBA(v+2a\kappa)\sinh\tau}{\left[D_1+\cosh\tau\right]^{p+1}}=0 \tag{13}
$$

and

$$
\begin{aligned}
&-\frac{\omega A + aA\kappa^2 - aAp^2B^2 + \alpha A\left(p^2B^2 + \kappa^2\right) + \beta Ap^2B^2}{\left[D_1 + \cosh\tau\right]^p} \\
&+\frac{2\alpha Ap^2B^2D_1 + (\beta - a)\,p(2p+1)AB^2D_1}{\left[D_1 + \cosh\tau\right]^{p+1}} \\
&+\frac{ap(p+1)B^2A\left(D_1^2 - 1\right) - \alpha Ap^2B^2\left(D_1^2 - 1\right) - \beta p(p+1)AB^2\left(D_1^2 - 1\right)}{\left[D_1 + \cosh\tau\right]^{p+2}} \\
&+\frac{bA^3}{\left[D_1 + \cosh\tau\right]^{3p}} + \frac{cA^5}{\left[D_1 + \cosh\tau\right]^{5p}} = 0.
\end{aligned}
\tag{14}
$$

From (13), the velocity of the soliton is given by

$$v = -2\kappa a. \tag{15}$$

From the real part equation (14), equating the exponents $p+2$ and $5p$ gives

$$p + 2 = 5p \tag{16}$$

which implies

$$p = \frac{1}{2}. \tag{17}$$

The same value of p is also obtained when the exponents $p+1$ and $3p$ are equated against each other. Now, from (14), the linearly independent functions are $1/\left[D + \cosh\tau\right]^{p+j}$ for $j = 0,1,2$. Therefore, setting their respective coefficients to zero yields

$$\omega A + aA\kappa^2 - aAp^2B^2 + \alpha A\left(p^2B^2 + \kappa^2\right) + \beta Ap^2B^2 = 0, \tag{18}$$

$$2\alpha Ap^2B^2D_1 + (\beta - a)\,p(2p+1)AB^2D_1 + bA^3 = 0, \tag{19}$$

$$(a - \beta)\,p(p+1)B^2A\left(D_1^2 - 1\right) - \alpha Ap^2B^2\left(D_1^2 - 1\right) + cA^5 = 0. \tag{20}$$

Solving the above equations gives

$$\omega = \frac{(a - \alpha - \beta)B^2 - 4(a + \alpha)\kappa^2}{4}, \tag{21}$$

$$A = \sqrt{B}\left[\frac{4b^2}{\left[\alpha + 2(\beta - a)\right]^2 B^2} + \frac{4c}{3(a - \beta) - \alpha}\right]^{-\frac{1}{4}}, \tag{22}$$

$$D_1 = -\frac{2b}{(\alpha + 2\beta - 2a)B}\left[\frac{4b^2}{[\alpha + 2\beta - 2a]^2 B^2} + \frac{4c}{3(a-\beta) - \alpha}\right]^{-\frac{1}{2}}. \tag{23}$$

As seen from (22) and (23), the soliton pulse will exist for

$$\alpha + 2(\beta - a) \neq 0;\ 3(a - \beta) - \alpha \neq 0. \tag{24}$$

Thus, the bright soliton solution to the mCGL equation with parabolic law nonlinearity (3) is given by

$$q(x,t) = \frac{A}{\left[D_1 + \cosh\left[B(x - vt)\right]\right]^{\frac{1}{2}}} e^{i(-\kappa x + \omega t + \gamma t + \theta)}, \tag{25}$$

where the velocity v is given by (15), the wave number ω is shown in (21), the amplitude A of the soliton is given by (22) while the parameter D_1 is given by (23). Note that this solution exists provided that the constraint equations between the model coefficients α, β and a given in (24) are satisfied.

3.2. DARK SOLITONS

Now we are interested by finding the exact dark soliton solution for the considered mCGL equation with parabolic law nonlinearity (3). Dark solitons are also known as topological optical solitons in the context of Theoretical Physics.

The starting *ansatz* is

$$q(x,t) = (A + \lambda \tanh \tau)^p e^{i\varphi}, \tag{26}$$

where τ and φ are the same as in (5) and (6).

Here in (26), A, λ and B are unknown free parameters and v is the velocity of the wave. In this case, the unknown exponent p will also fall out during the course of the derivation of the soliton solution to (3). From the ansatz (26), we get

$$\begin{aligned} q_t = \Bigg[& \frac{Bpv}{\lambda^2}\left\{[A + \lambda \tanh \tau]^{p+1} - 2A[A + \lambda \tanh \tau]^p - (\lambda^2 - A^2)[A + \lambda \tanh \tau]^{p-1}\right\} + \\ & + i\omega[A + \lambda \tanh \tau]^p\Bigg] e^{i\varphi}, \end{aligned} \tag{27}$$

$$
\begin{aligned}
q_{xx} = \Bigg[& \frac{pB^2}{\lambda^4}\Big\{ (p-1)(\lambda^2 - A^2)^2 [A+\lambda\tanh\tau]^{p-2} - (p+1)[A+\lambda\tanh\tau]^{p+2} \\
& - 2A(2p+1)[A+\lambda\tanh\tau]^{p+1} + 2A(2p-1)(\lambda^2 - A^2)[A+\lambda\tanh\tau]^{p-1} \\
& + 2p(3A^2 - \lambda^2)[A+\lambda\tanh\tau]^{p} \Big\} - \kappa^2 [A+\lambda\tanh\tau]^{p} \\
& - \frac{2i\kappa Bp}{\lambda^2}\Big\{ (\lambda^2 - A^2)[A+\lambda\tanh\tau]^{p-1} + 2A[A+\lambda\tanh\tau]^{p} - [A+\lambda\tanh\tau]^{p+1} \Big\} \Bigg] \mathrm{e}^{\mathrm{i}\varphi},
\end{aligned} \tag{28}
$$

$$
\begin{aligned}
\frac{|q_x|^2}{q^*} = \Bigg[& \frac{p^2B^2}{\lambda^4}\Big\{ (\lambda^2 - A^2)^2 [A+\lambda\tanh\tau]^{p-2} + [A+\lambda\tanh\tau]^{p+2} \\
& +2(3A^2 - \lambda^2)[A+\lambda\tanh\tau]^{p} + 4A(\lambda^2 - A^2)[A+\lambda\tanh\tau]^{p-1} - 4A[A+\lambda\tanh\tau]^{p+1} \Big\} \\
& +\kappa^2 [A+\lambda\tanh\tau]^{p} \Bigg] \mathrm{e}^{\mathrm{i}\varphi},
\end{aligned} \tag{29}
$$

$$
\begin{aligned}
\frac{\left(|q|^2\right)_{xx}}{q^*} = \Bigg[& \frac{2pB^2}{\lambda^4}\Big\{ (2p-1)(\lambda^2 - A^2)^2 [A+\lambda\tanh\tau]^{p-2} - (2p+1)[A+\lambda\tanh\tau]^{p+2} \\
& +4p(3A^2 - \lambda^2)[A+\lambda\tanh\tau]^{p} + 2A(4p-1)(\lambda^2 - A^2)[A+\lambda\tanh\tau]^{p-1} \\
& -2A(4p+1)[A+\lambda\tanh\tau]^{p+1} \Big\} \Bigg] \mathrm{e}^{\mathrm{i}\varphi},
\end{aligned} \tag{30}
$$

$$
\begin{aligned}
\frac{\left\{\left(|q|^2\right)_x\right\}^2}{|q|^2 q^*} = \Bigg[& \frac{4p^2B^4}{\lambda^4}\Big\{ (\lambda^2 - A^2)^2 [A+\lambda\tanh\tau]^{p-2} + [A+\lambda\tanh\tau]^{p+2} \\
& +2(3A^2 - \lambda^2)[A+\lambda\tanh\tau]^{p} + 4A(\lambda^2 - A^2)[A+\lambda\tanh\tau]^{p-1} \\
& -4A[A+\lambda\tanh\tau]^{p+1} \Big\} \Bigg] \mathrm{e}^{\mathrm{i}\varphi}.
\end{aligned} \tag{31}
$$

Substituting (27–31) into (3), and then separating the real and imaginary parts, we can obtain the following pair of equations:

$$
\begin{aligned}
& \frac{\left\{ apB^2(p-1) - \alpha p^2B^2 - p\beta B^2(2p-1) - \beta p^2B^4 \right\}(\lambda^2 - A^2)^2}{\lambda^4}[A+\lambda\tanh\tau]^{p-2} \\
& + \frac{\left\{ 2apB^2A(2p-1) - 4\alpha p^2B^2A - 2\beta pB^2A(4p-1) - 4\beta p^2B^4A \right\}(\lambda^2 - A^2)}{\lambda^4}[A+\lambda\tanh\tau]^{p-1} \\
& - \frac{apB^2(p+1) + \alpha p^2B^2 - p\beta B^2(2p+1) + \beta p^2B^4}{\lambda^4}[A+\lambda\tanh\tau]^{p+2} \\
& \cdot \left\{ \frac{4\alpha p^2B^2A + 2\beta pB^2A(4p+1) + 4\beta p^2B^4A - 2apB^2A(2p+1)}{\lambda^4} \right\}[A+\lambda\tanh\tau]^{p+1} \\
& + \left\{ \frac{2\left\{a - \alpha - 2\beta - \beta B^2\right\} p^2B^2(3A^2 - \lambda^2)}{\lambda^4} - (a+\alpha)\kappa^2 - \omega \right\}[A+\lambda\tanh\tau]^{p} \\
& + b[A+\lambda\tanh\tau]^{3p} + c[A+\lambda\tanh\tau]^{5p} = 0
\end{aligned} \tag{32}
$$

and

$$\frac{Bp\left(v+2a\kappa\right)}{\lambda^2}\left[\left(A+\lambda\tanh\tau\right)^{p+1}-2A\left(A+\lambda\tanh\tau\right)^{p}-\left(\lambda^2-A^2\right)\left(A+\lambda\tanh\tau\right)^{p-1}\right]=0. \quad (33)$$

From the imaginary part equation (33), the soliton velocity is the same as in the case of bright solitons that is given by (15). From the real part equation (32), equating the exponents of $\left[A+B\tanh\tau\right]^{5p}$ and $\left[A+B\tanh\tau\right]^{p+2}$ terms in Eq. (36), one again obtains the same value of p as in (17).

Now, from (32), the linearly independent functions are $\left[A+B\tanh\tau\right]^{p+j}$ for $j=0,\ \pm1,\ \pm2$. Therefore, setting their respective coefficients to zero yields the following system of algebraic equations:

$$\left\{apB^2\left(p-1\right)-\alpha p^2B^2-p\beta B^2\left(2p-1\right)-\beta p^2B^4\right\}\left(\lambda^2-A^2\right)^2=0, \quad (34)$$

$$\left\{2apB^2A\left(2p-1\right)-4\alpha p^2B^2A-2\beta pB^2A\left(4p-1\right)-4\beta p^2B^4A\right\}\left(\lambda^2-A^2\right)=0 \quad (35)$$

$$-\frac{apB^2\left(p+1\right)+\alpha p^2B^2-p\beta B^2\left(2p+1\right)+\beta p^2B^4}{\lambda^4}+c=0, \quad (36)$$

$$\frac{4\alpha p^2B^2A+2\beta pB^2A\left(4p+1\right)+4\beta p^2B^4A-2apB^2A\left(2p+1\right)}{\lambda^4}+b=0, \quad (37)$$

$$\frac{2\left\{a-\alpha-2\beta-\beta B^2\right\}p^2B^2\left(3A^2-\lambda^2\right)}{\lambda^4}-\left(a+\alpha\right)\kappa^2-\omega=0. \quad (38)$$

The dark soliton solutions study will now be split into the following two cases:

Case I: $\lambda^2-A^2\neq0$. In this case,

$$B=\sqrt{-\frac{a+\alpha}{\beta}}, \quad (39)$$

$$B=\sqrt{-\frac{\alpha+\beta}{\beta}}, \quad (40)$$

$$\omega=\frac{\left(a-\alpha-2\beta-\beta B^2\right)\left(3A^2-\lambda^2\right)B^2-2\left(a+\alpha\right)\lambda^4\kappa^2}{2\lambda^4}, \quad (41)$$

$$A=\frac{b\lambda^4}{\left(5a-7\beta\right)B^2-4c\lambda^4}. \quad (42)$$

Equating the two values of B from (39) and (40) shows that dark solitons will exist provided that $\beta = a$ and $\beta(a+\alpha) < 0$. Furthermore, free parameter A can be obtained in terms of the model coefficients by substituting (39) or (40) into (42) to yield

$$A = \frac{b\beta\lambda^4}{4c\beta\lambda^4 - (5a - 7\beta)(a+\alpha)}, \tag{43}$$

which exists provided that

$$4c\beta\lambda^4 - (5a - 7\beta)(a+\alpha) \neq 0. \tag{44}$$

Case II: $\lambda^2 - A^2 = 0$. Here, (36–38) yields

$$A = \pm\lambda, \tag{45}$$

$$A = \frac{b\lambda^4}{(5a - 7\beta)B^2 + 4c\lambda^4}, \tag{46}$$

$$\omega = \frac{(a - \alpha - 2\beta - \beta B^2)B^2 - (a+\alpha)\lambda^2\kappa^2}{\lambda^2}. \tag{47}$$

Eq. (46) shows that the soliton will exist for

$$4c\beta\lambda^4 \neq (5a - 7\beta)(a+\alpha). \tag{48}$$

Hence, finally, the dark soliton solution to the mCGL equation with parabolic law nonlinearity (3), for the two cases together, is given by

$$q(x,t) = \left[A + \lambda \tanh\left[B(x - vt)\right]\right]^{\frac{1}{2}} \mathrm{e}^{\mathrm{i}(-\kappa x + \omega t + \gamma t + \theta)} \tag{49}$$

where the velocity v is given by (15) and the wave number ω is given by (41) if $A \neq \pm\lambda$ and by (47) when $A = \pm\lambda$. It is worth noting that the existence of the dark soliton solution (49) depends on the characteristics of the nonlinear medium, which satisfy the conditions given above.

4. DUAL-POWER LAW NONLINEARITY

In this subsection, the mCGL equation with dual power law nonlinearity will be studied. In this case, the governing equation is given by [4]

$$iq_t + aq_{xx} + \left(b|q|^{2n} + c|q|^{4n}\right)q = \alpha \frac{|q_x|^2}{q^*} + \frac{\beta}{4|q|^2 q^*}\left[2|q|^2\left(|q|^2\right)_{xx} - \left\{\left(|q|^2\right)_x\right\}^2\right]. \quad (50)$$

Notice that Eq. (50) reduces to the mCGL equation with parabolic law nonlinearity (3) if $n = 1$.

4.1. BRIGHT SOLITONS

For searching bright optical soliton solution, we use the same starting hypothesis as in (4). This leads to the same imaginary part equation as given by (13) and thus the velocity is still as in (15). The real part equation reveals

$$\begin{aligned}
&-\frac{\omega A + aA\kappa^2 - aAp^2B^2 + \alpha A\left(p^2B^2 + \kappa^2\right) + \beta Ap^2B^2}{\left[D_1 + \cosh\tau\right]^p} \\
&+\frac{2\alpha Ap^2B^2D_1 + (\beta - a)\,p\left(2p+1\right)AB^2D_1}{\left[D_1 + \cosh\tau\right]^{p+1}} \\
&+\frac{ap\left(p+1\right)B^2A\left(D_1^2 - 1\right) - \alpha Ap^2B^2\left(D_1^2 - 1\right) - \beta p\left(p+1\right)AB^2\left(D_1^2 - 1\right)}{\left[D_1 + \cosh\tau\right]^{p+2}} \\
&+\frac{bA^{2n+1}}{\left[D_1 + \cosh\tau\right]^{p(2n+1)}} + \frac{cA^{4n+1}}{\left[D_1 + \cosh\tau\right]^{p(4n+1)}} = 0.
\end{aligned} \quad (51)$$

Now, from (51), matching the exponents $p\left(4n+1\right)$ and $p+2$ gives

$$p(4n+1) = p + 2 \quad (52)$$

which implies

$$p = \frac{1}{2n} \quad (53)$$

for

$$n \neq 0. \quad (54)$$

The same value of p is obtained on equating the exponents $p\left(2n+1\right)$ and $p+1$. Similarly, from (51) as in the parabolic law case, the linearly independent functions are $1/\left[D_1 + \cosh\tau\right]^{p+j}$ for $j = 0,1,2$. Thus, setting their respective coefficients to zero yields the following parametric equations:

$$\omega A + aA\kappa^2 - aAp^2B^2 + \alpha A\left(p^2B^2 + \kappa^2\right) + \beta Ap^2B^2 = 0, \tag{55}$$

$$2\alpha Ap^2B^2D_1 + (\beta - a)\,p(2p+1)AB^2D_1 + bA^{2n+1} = 0, \tag{56}$$

$$(a-\beta)\,p(p+1)B^2A\left(D_1^2 - 1\right) - \alpha Ap^2B^2\left(D_1^2 - 1\right) + cA^{4n+1} = 0. \tag{57}$$

Solving the above equations gives

$$\omega = \frac{(a-\alpha-\beta)B^2 - 4n^2(a+\alpha)\kappa^2}{4n^2}, \tag{58}$$

$$D_1 = -\frac{nb}{\left[\alpha + (\beta - a)(n+1)\right]B}\left[\frac{b^2n^2}{\left[\alpha + (n+1)(\beta - a)\right]^2 B^2} + \frac{c}{(a-\beta)(2n+1) - \alpha}\right]^{-\frac{1}{2}}, \tag{59}$$

$$A = B^{\frac{1}{2n}}\left[\frac{4b^2n^4}{\left[\alpha + (n+1)(\beta - a)\right]^2 B^2} + \frac{4n^2c}{(a-\beta)(2n+1) - \alpha}\right]^{-\frac{1}{4n}}. \tag{60}$$

As seen from (59) and (60), the soliton pulse will exist for

$$\alpha + (n+1)(\beta - a) \neq 0, \ \& \ (a-\beta)(2n+1) - \alpha \neq 0. \tag{61}$$

Finally, the bright soliton of the complex modified mCGL equation with dual power law nonlinearity (50) is

$$q(x,t) = \frac{A}{\left[D_1 + \cosh\left[B(x - vt)\right]\right]^{\frac{1}{2n}}}\,\mathrm{e}^{\mathrm{i}(-\kappa x + \omega t + \gamma t + \theta)}, \tag{62}$$

where the velocity v is given by (15), the wave number ω is shown in (58), the parameter D_1 is given by (59) while the amplitude A of the soliton is given by (60). Note that this solution exists provided that the constraint equations between the model coefficients α, β and a given in (61) are satisfied.

4.2. DARK SOLITONS

For dark solitons, we use the same solitary wave ansatz as in (26). This leads to the same imaginary part equation as in (33) and thus the velocity of the soliton is the same as given by (15). Now, the real part equation is

$$\frac{\left\{apB^2\left(p-1\right)-\alpha p^2B^2-p\beta B^2\left(2p-1\right)-\beta p^2B^4\right\}\left(\lambda^2-A^2\right)^2}{\lambda^4}\left[A+\lambda\tanh\tau\right]^{p-2}$$
$$+\frac{\left\{2apB^2A\left(2p-1\right)-4\alpha p^2B^2A-2\beta pB^2A\left(4p-1\right)-4\beta p^2B^4A\right\}\left(\lambda^2-A^2\right)}{\lambda^4}\left[A+\lambda\tanh\tau\right]^{p-1}$$
$$-\frac{apB^2\left(p+1\right)+\alpha p^2B^2-p\beta B^2\left(2p+1\right)+\beta p^2B^4}{\lambda^4}\left[A+\lambda\tanh\tau\right]^{p+2}$$
$$\left\{\frac{4\alpha p^2B^2A+2\beta pB^2A\left(4p+1\right)+4\beta p^2B^4A-2apB^2A\left(2p+1\right)}{\lambda^4}\right\}\left[A+\lambda\tanh\tau\right]^{p+1}$$
$$+\left\{\frac{2\left\{a-\alpha-2\beta-\beta B^2\right\}p^2B^2\left(3A^2-\lambda^2\right)}{\lambda^4}-\left(a+\alpha\right)\kappa^2-\omega\right\}\left[A+\lambda\tanh\tau\right]^p \tag{63}$$
$$+b\left[A+\lambda\tanh\tau\right]^{p(2n+1)}+c\left[A+\lambda\tanh\tau\right]^{p(4n+1)}=0.$$

By equating the exponents of $\left(A+B\tanh\tau\right)^{p(4n+1)}$ and $\left(A+B\tanh\tau\right)^{p+2}$ terms in Eq. (63), one obtains the same value of p as in (53) along with the same restriction as (54). By collecting the coefficients of the linearly independent functions $\left(A+B\tanh\tau\right)^{p+j}$ for $j=0,\ \pm1,\ \pm2$ in Eq. (63), and setting them to zero, yields

$$\left\{apB^2\left(p-1\right)-\alpha p^2B^2-p\beta B^2\left(2p-1\right)-\beta p^2B^4\right\}\left(\lambda^2-A^2\right)^2=0, \tag{64}$$

$$\left\{2apB^2A\left(2p-1\right)-4\alpha p^2B^2A-2\beta pB^2A\left(4p-1\right)-4\beta p^2B^4A\right\}\left(\lambda^2-A^2\right)=0, \tag{65}$$

$$-\frac{apB^2\left(p+1\right)+\alpha p^2B^2-p\beta B^2\left(2p+1\right)+\beta p^2B^4}{\lambda^4}+c=0, \tag{66}$$

$$\frac{4\alpha p^2B^2A+2\beta pB^2A\left(4p+1\right)+4\beta p^2B^4A-2apB^2A\left(2p+1\right)}{\lambda^4}+b=0, \tag{67}$$

$$\frac{2\left\{a-\alpha-2\beta-\beta B^2\right\}p^2B^2\left(3A^2-\lambda^2\right)}{\lambda^4}-\left(a+\alpha\right)\kappa^2-\omega=0. \tag{68}$$

As in parabolic law, the two cases are

Case I: $\lambda^2-A^2\neq 0$. Here,

$$B=\sqrt{\frac{a-\alpha-2\beta+2n\left(\beta-a\right)}{\beta}}, \tag{69}$$

$$B=\sqrt{\frac{a-\alpha-2\beta+n\left(\beta-a\right)}{\beta}}, \tag{70}$$

$$\omega=\frac{2\left(a-\alpha-2\beta-\beta B^2\right)\left(3A^2-\lambda^2\right)B^2-4\left(a+\alpha\right)n^2\kappa^2\lambda^4}{\lambda^4}, \tag{71}$$

$$A=\frac{n^2 b\lambda^4}{aB^2\left(3n+2\right)-\beta B^2\left(3n+4\right)-4c\lambda^4 n^2}. \tag{72}$$

Equating the two values of B from (69) and (70) shows that dark solitons will exist provided that $\beta=a$ and

$$\beta\left[a-\alpha-2\beta+2n\left(\beta-a\right)\right]>0. \tag{73}$$

Additionally, we can determine the parameter A of the soliton in terms of the model coefficients by substituting (69) or (70) into (72) for $\beta=a$ which gives

$$A=\frac{n^2 b\lambda^4}{\left[a\left(3n+2\right)+\beta\left(3n+4\right)\right]\left(\alpha+\beta\right)-4c\lambda^4 n^2}. \tag{74}$$

The second case is given by

Case II: $\lambda^2-A^2=0$. Here, Eqs. (64)-(68) yields

$$A=\pm\lambda, \tag{75}$$

$$A=\frac{n^2 b\lambda^4}{aB^2\left(3n+2\right)-\beta B^2\left(3n+4\right)-4c\lambda^4 n^2}, \tag{76}$$

$$\omega=\frac{\left\{a-\alpha-2\beta-\beta B^2\right\}B^2-\left(a+\alpha\right)\kappa^2\lambda^2 n^2}{\lambda^2 n^2}. \tag{77}$$

Hence, finally, the dark soliton solution to the mCGL equation with dual-power law nonlinearity (50) is given by

$$q(x,t)=\left[A+\lambda\tanh\left[B\left(x-vt\right)\right]\right]^{\frac{1}{2n}}\,\mathrm{e}^{\mathrm{i}(-\kappa x+\omega t+\gamma t+\theta)} \tag{78}$$

where the velocity v is given by (15) and the wave number ω is given by (71) if $A\neq\pm\lambda$ and by (77) when $A=\pm\lambda$. It is worth noting that the existence of the dark soliton solution (77) depends on the characteristics of the nonlinear medium, which satisfy the conditions given above.

5. CONCLUSIONS

In this paper, the *soliton ansatz method* is used to integrate the mCGL equation. There are two types of nonlinearity that are considered in this paper. They are the parabolic law and its generalization, namely the dual-power law nonlinearity. Both bright and dark 1-soliton solutions were obtained. The parameter restrictions in all four cases were laid down that fell out automatically from the soliton parameters.

These results are going to be very useful in carrying out further analysis. Later these relations will be applied to soliton perturbation theory and to formulate the quasi-particle theory. The quasi-stationary soliton solutions will also be studied with the perturbation terms present. The stochasticity will also be taken into consideration. All such results will be reported gradually in future publications.

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REFERENCES

1. A. Biswas, Progress in Electromagnetic Research Letters, **96**, 1–7 (2009).
2. K. W. Chung and Y. Y. Cao, Submitted.
3. W. P. Hong, Z. Naturforsch., **62A**, 368–372 (2007).
4. K. Maruno, A. Ankiewicz, N. Akhmediev, Physics Letters, A **347**, 231–240 (2005).
5. D. Mihalache, Romanian Journal of Physics, **50**, 357–364 (2005).
6. D. Mihalache, D. Mazilu, Romanian Reports in Physics, **61**, 587–608 (2009).
7. D. Mihalache, Romanian Reports in Physics, **62**, 697–709 (2010).
8. D. Mihalache, Proceedings of the Romanian Academy, A **11**, 142–147 (2010).
9. D. Mihalache, Romanian Reports in Physics, **63**, 9–24 (2011).
10. D. Mihalache, Romanian Reports in Physics, **63**, 325–338 (2011).
11. A. Mohamdou, A. K. Jiotsa, T. C. Kofane, Chaos, Solitons & Fractals, **27**, 957–966 (2005).
12. A. Mohamdou, F. Ndzana, T. C. Kofane, Physica Scripta, **73**, 596–600 (2006).
13. F. B. Pelap, A, J. Kenfack, M. M. Faye, Nonlinear Oscillations, **10**, 270–276 (2007).
14. L. H. Shehata, H. M. Taha, Physica status solidi (b) Basic Solid State Physics, **242**, 2076–2088 (2005).
15. S. Shwetanshumala, Progress in Electromagnetic Research Letters, **3**, 17–24 (2008).
16. V. Skarka, N. B. Aleksic, Physical Review Letters, **96**, 013903 (2006); V. Skarka, N. B. Aleksic, H. Leblond, B. A. Malomed, D. Mihalache, Physical Review Letters, **105**, 213901 (2010); L.-C. Crasovan, B. A. Malomed, D. Mihalache, Physics Letters, A **289**, 59-65 (2001); L.-C. Crasovan, B. A. Malomed, D. Mihalache, Physical Review, E **63**, 016605 (2001).
17. J. M. Soto-Crespo, N. Akhmediev, Mathematics and Computers in Simulation, **69**, 526–536 (2005).
18. J. M. Soto-Crespo, P. Grelu, N. Akhmediev, N. Devine, Physics Letters, A. **362**, 31–36 (2007).
19. E. Yomba, T. C. Kofane, Physica, D **125**, 105–122 (1999).
20. E. Yomba, T. C. Kofane, Chaos, Solitons & Fractals, **15**, 187–199 (2003).