DESIGN TECHNIQUES FOR ALL-DIELECTRIC POLARIZING BEAM SPLITTER CUBES, UNDER CONSTRAINED SITUATIONS

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Abstract. In this paper, a guide to design optical coatings for polarizing beam splitter cubes, starting from certain imposed situations, is presented. For this purpose, will be presented two main situations: the criterion by which we choose materials for interferential package if the glass prisms is imposed and, on the other hand, how to choose the glass for prisms, depending on the materials for interferential package that we need to build.

Key words: optical coatings, thin layers, polarizing beam splitter cube.

1. INTRODUCTION

An optical polarizer beam splitter cube, made by coating of interferential thin films, is composed of two cemented prisms, one of them having on the hypotenuse a thin films package with indices and thickness so chosen to maximize the effect of polarization in the spectral range of interest. Figure 1 represents such a beam splitter that transmit radiation polarized type "p" (electric field intensity vector is parallel with the plane of incidence) and will reflect “s” polarized radiation (electric field intensity vector is perpendicular to the plane of incidence). The advantages of this type of polarizers are they have a very good efficiency (negligible absorption), resistance and reliability, negligible diffusion etc. Because of these qualities, beam splitters obtained by thin layers coating are commonly used in complex optical systems for image processing as well as in optical systems with laser radiation.

Based on the electromagnetic theory of light [1, 2] and the theory of thin optical layers [3–6], past research presents a series of solutions for different formulas of interferential package and different prism forms in order to maximize the polarizing effect and to get a broad spectral range of polarization [7–14], but
this article will show how to broach the design of such a beam splitter, in the cases of two types of imposed situations, frequently encountered.

2. THEORY

In this paper will be approximate that the optical materials have a negligible absorption, so it will be considered the equality [1, 2]:

\[ n = \frac{c}{v}. \]  

(1)

In the theory of thin optical layers was introduced the term named “effective index of refraction” [3, 4]. This is the refractive index of a medium, relative to the state of polarization of the radiation that crosses it. Thus, for the two linearly polarized components “p” and “s”, we have:

\[ n_p = \frac{n}{\cos \theta}, \]

\[ n_s = n \cos \theta, \]  

(2)

where: \( n_p \) and \( n_s \) are the “effective refractive indices” of the layer considered for the two components (“p” and “s”) of radiation; \( n \) is the nominal refractive index of the layer; \( \theta \) is the angle under the radiation passes through the layer. This is the angle of refraction, resulted from the Snell's equation: \( n_1 \sin i = n_2 \sin \theta \).

Also, optical thickness of each layer is multiplied by \( \cos \theta \) and is named “the effective thickness” [3, 4].

Using the Snell law, we obtain [3]:

![Image](image.png)

Fig. 1 – A polarizing beam splitter made with thin layers coating.
\[ \cos \theta = \left( 1 - \frac{A^2}{n^2} \right)^{1/2}, \quad (3) \]

where: \( A \) is \( n_0 \sin \theta_0 \) (numerical aperture) and is a constant in entire thin layers package; \( n \) is the nominal refractive index; \( \theta_0 \) is the angle of incidence; \( n_0 \) is the refractive index of air. And from equations (2) and (3) we obtain:

\[ n_p = \frac{n}{\left( 1 - \frac{A^2}{n^2} \right)^{1/2}}, \quad (4) \]

\[ n_s = n \left( 1 - \frac{A^2}{n^2} \right)^{1/2}. \quad (5) \]

Figure 2 represents the variations of these effective refractive indices \( n_p \) and \( n_s \) as functions of nominal index of refraction \( n \), in the case that incident light is coming from a medium having \( n_0 = 1.52 \), under an incidence angle \( \theta_0 = 45^\circ \).

It can be observed that \( n_s \) has a positive slope, while the \( n_p \) has a minimum even form \( n = n_0 = 1.52 \).

Indeed, the solution of equation \( \frac{dn_p(n)}{dn} = 0 \), is:

\[ n = \sqrt{2} A = \sqrt{2} n_0 \sin 45^\circ = n_0. \]

![Fig. 2 – Effective indices of refraction \( n_p \) (up) and \( n_s \) (down) as functions of nominal index \( n \), for 45° incidence angle and from BK7 optical glass \( n_0 = 1.52 \) as incidence medium.](image)
3. THE HYPOTHESIS

The above observations can be exploited to design a polarizing beam splitter. Thus, choosing two materials with indices of refraction in one and the other side of the minimum of function $n_p(n)$, we will record a much bigger difference between effective indices $n_s$ of these two materials, than effective indices $n_p$. This would mean that if these materials could form a package of thin films, there will have a big influence (maximizing reflectivity) for the component “s” of the incident light and a smaller influence for the component “p”. Thus, components “s” and “p” of the incident light will be separated, meaning that the “s” polarized radiation will be quasi-total reflected, while the “p” polarized radiation, in most, will pass through the cube (Fig. 1).

For example, if we choose for an optical coating package, TiO$_2$ ($n_{TiO2} \approx 2.3$) and SiO$_2$ ($n_{SiO2} \approx 1.46$), that are the most used materials in optical coating technologies, and BK7 optical glass, that is the most used glass in optics, with a specialized software, we can trace the spectral variation curves of transmission and reflection for each “s” and “p” component.

Note. 1. In the present article, we will note "H" and "L" the materials used to describe the thin layers package formula, representing "High" and "Low" index material. Optical thickness of each layer has the value obtained by multiply the factor before the symbol with the reference wavelength ($\lambda_0 = 550$ nm) and divided by four. Thus, "0.5H" represents a layer of material (substance) "H" with optical thickness $0.5 \frac{550}{4}$ nm. When a structure is repeated, that structure can be written in parentheses and number of repetitions is written on the right side-up, e.g. \(L H L H L H\) \(\equiv (L H)^3\).

2. ATTOL (Applied Theory of Thin Optical Layers) is the software application, made by the author of this article, used for numerical simulations and optimizations. Over the time, this software has demonstrated a high accuracy in the design of optical coatings.

Thus, for a beam splitter made of two glued BK7 optical glass prisms, one of them with a coating of 20 thin layers on the hypotenuse (Fig. 1), as in the following formula:

\[
BK7 - (L H)^{10} - BK7
\]

we obtain, by numerical simulation, the spectral transmission curves for $T_s$ and $T_p$, as shown in Fig. 3.
Fig. 3 – Theoretical spectral transmission curves for $T_p$ (up) and $T_s$ (down) of a polarizing beam splitter cube, according to (6), where $n_0 = 1.52$ (BK7); H is TiO$_2$ and L is SiO$_2$.

In Figs. 2 and 3 we can see how from the bigger difference between $n_{TiO_2}$ and $n_{SiO_2}$ results a strong blocking in transmission for component $T_s$, while from the lower difference between $n_{pTiO_2}$ and $n_{pSiO_2}$, results only a partial blocking for transmission of component $T_p$. Of course, this graph does not look too good for an advanced polarizing beam splitter cube. However, based on Fig. 2, the designer must find a solution in order to maximize the efficiency of the polarizing beam splitter cube, but, in practice, the designer is often found in one of two constrained situations, which are listed below:

**Case 1.** For reasons of minimizing the optical path in the system, beam splitter cube often is imposed to be made of BK7 glass ($n_{BK7} = 1.52$). In this case, we have to search for two coating materials that, according with Fig. 2, they have approximately equal values of $n_p$.

**Case 2.** If the designer is allowed to choose the optical glass for prisms, the two materials (TiO$_2$ and SiO$_2$) will be kept (we will see why). In this case, we will can consider TiO$_2$ and SiO$_2$ as imposed materials. Now we must find a glass whose index of refraction $n_0$ generate another variation $n_p = n_p(n)$ so as to obtain quasi-equal values for effective indices ($n_p$) of SiO$_2$ and TiO$_2$, respectively.

### 4. DESIGN TECHNIQUES

In Case 1, Fig. 2 helps us to find the two materials we need. We can observe that they should have the following approximate values: $n_L \approx 1.4$; $n_H \approx 1.7$. As available materials with indices of refraction in those areas are: MgF$_2$
(\(n_{\text{MgF2}} \cong 1.38\)) and SiO\(_2\) (\(n_{\text{SiO2}} \cong 1.46\)) for material L and Al2O3 (\(n_{\text{Al2O3}} \cong 1.63\)) for material H. The graphs obtained (Fig. 4) by numerical simulation with ATTOL confirm expectations in both situations. Thus, in both cases we can observe a significant increasing of the difference between \(T_s\) and \(T_p\), compared with the situation illustrated by the graph of Fig. 3.

The effect of polarization can be simply increased by adding more layers. Thus, with a package of 32 layers with optical thickness \(\lambda_0/4\), according with (7), where H is Al2O3 and L is SiO2, will obtain the graph from Fig. 5.

\[
\text{BK7} - (\text{L H})^{16} - \text{BK7}
\]
In Case 2 if the designer is allowed to choose the optical glass for prisms, a good choice would be to keep the pair $H = \text{TiO}_2$ and $L = \text{SiO}_2$ for the coating materials. They have a good compatibility in terms of internal stress, forming very strong structures [5, 6]. In addition, the big difference between the two indices of refraction ($n_{\text{SiO}_2} \cong 1.46$ and $n_{\text{TiO}_2} \cong 2.3$) will allow us to build a beam splitter with a very good degree of polarization, on a broad spectral domain. In this respect, we must find a new glass for prisms, having such index of refraction ($n_0$) to help us to redraw the graph of Fig. 2 in order to obtain quasi-equality between $n_p$ for SiO$_2$ and $n_p$ for TiO$_2$.

Using the ATTOL software, several graphics were generated for different $n_0$ indices. The most favorable situation, in which the two effective indices ($n_p$) were placed on the same value, was obtained for $n_0 = 1.75$ (Fig. 6). A found glass with refractive index of refraction close to this value is N-LAF7 ($n_{\text{N-LAF7}} = 1.7495$).

![Fig. 6 – Effective indices of refraction $n_p$ (up) and $n_s$ (down) as functions of nominal index ($n$), for 45° incidence angle and from N-LAF7 optical glass ($n_0 = 1.75$) as incidence medium.](image)

Thus were created all conditions for obtaining an efficient polarizer on a broad spectral domain. The new structure is:

$$\text{N-LAF7} – (L \ H)^{10} – \text{N–LAF7}$$

(8)

where $H$ is TiO$_2$ and $L$ is SiO$_2$.

Figure 7 represents the variations of $T_p$ and $T_s$, over a wide spectral domain, obtained with a thin layers package according to (8).
The last step of design is numerical optimization. This can be performed with specialized software and aims to increase optical efficiency of coating in the spectral domain of interest. Figures 8 and 9 represent the spectral transmissions $T_p$, $T_s$ and $T_t$ for visible domain $\lambda \in [400 \text{ 700}]$nm, as a result of ATTOL numerical optimization of (8). Optimized package is presented in (9):

$$
\text{NLAF7 - 0.49L 0.85H 0.95L 0.87H} \\
0.90L 0.87H 0.94L 0.95H \\
0.97L 0.86H 0.96L 1.38H \\
0.80L 0.77H 0.89L 1.25H \\
1.03L 0.94H 1.06L 1.30H - \text{NLAF7}
$$

Fig. 8 – Theoretical spectral transmission curves for $T_p$ (up) and $T_s$ (down) of a polarizing beam splitter cube, according to (9), where $n_0 = 1.75$ (N-LAF7); H is TiO$_2$ and L is SiO$_2$. 
Degree of polarization can be calculated using the well-known formula [1, 2]:

\[ P = \frac{I_M - I_s}{I_M + I_s} \quad (10) \]

Now, based on Fig. 9, we can approximate the average values of \( T_p \) and \( T_s \) on the spectral domain [400 - 700] nm. Thus, if we consider \( T_p \text{avg} \approx 99.7\% \) and \( T_s \text{avg} \approx 0.05\% \), according with (10), will result:

\[ P \approx \frac{99.65}{99.75} > 0.99. \]

5. CONCLUSIONS

In this article, a reliable strategy for designing of a polarizing beam splitter cube, in the cases of two main constrained situations, is presented. In this respect, we have established the following: first – how to find the coating materials, if the optical glass is imposed; second – how to find the glass for prisms if we are allowed to choose them, using the best materials for coating.

The main applications of this type of polarizing beam splitter refers to microscopy, spectroscopy, ellipsometry [15], photometry etc.

Finally we can mention the fact that every stage of designs was explained and justified from theoretical point of view and the result meets the theoretical predictions. The last presented example belongs to the broad band polarizing beam splitter category and the same structure remaining valid for other spectral bands, centered on different wavelengths. Translation can be done by varying the quarter-wavelength structure (8), increasing or decreasing the layer thickness in the same ratio, then must run a new numeric optimization.
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